This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at http://books.google.com/

NYPL RESEARCH LIBRARIES
3 3433 06634062 5

VDM
Dunkerl.

HYDRAULICS

WORKS BY

S. DUNKERLEY, D.Sc., M.Inst.C.E.

MECHANISM

With 416 Diagrams.

(Second Edition.)

8vo, ***9s.*** *net.*

HYDRAULICS

2 Vols., 8vo.

Vol. I.—HYDRAULIC MACHINERY.

With numerous Diagrams.

10s. 6d. *net.*

Vol. II.—THE RESISTANCE AND PROPULSION OF SHIPS.

With numerous Diagrams.

LONGMANS, GREEN, AND CO.

LONDON, NEW YORK, BOMBAY, AND CALCUTTA

HYDRAULICS

BY

S. DUNKERLEY

D.Sc., M.Inst.C.E., M.Inst.M.E.

PROFESSOR OF CIVIL AND MECHANICAL ENGINEERING IN THE UNIVERSITY OF MANCHESTER
AND DIRECTOR OF THE WHITWORTH LABORATORIES

IN TWO VOLUMES

VOL. I.—HYDRAULIC MACHINERY

WITH NUMEROUS DIAGRAMS

LONGMANS, GREEN, AND CO.

39 PATERNOSTER ROW, LONDON

NEW YORK, BOMBAY, AND CALCUTTA

1907

All rights reserved

410923

PREFACE

THE subject-matter of this book treats of Hydraulics and Hydraulic Machinery. It is intended for use in the Universities, in the Royal Navy, and for designers of hydraulic machinery.

The notes, which form the foundation of the book, were written at the Royal Naval College, Greenwich. The lectures given on Hydraulic Machinery, and also on the Resistance and Propulsion of Ships, were a very important part of the course for Engineers and Constructors. This book deals with the former subject; a second volume will treat of the latter subject.

The order of the book is: In the first chapter, the flow of a perfect fluid; in the second chapter, fluid friction; in the third chapter, pressure machines; in the fourth chapter, reciprocating pumps; in the fifth chapter, turbines; and in the sixth chapter, centrifugal pumps. In the first part of the chapter the theoretical considerations are fully discussed; and in the second part, numerous illustrations of machines have been given. A number of numerical examples are added at the end of the book.

The chief features of the present volume are:

In Chapter III., very full details of the Hydraulic Gun Brake, as applied in His Majesty's ships, have been given. I received permission from the Lords Commissioners of the Admiralty to publish these details; but it would have been impossible to have presented the subject in a concise form for publication had it not been for the great kindness of Captain Emdin, R.N., Chief

Mechanical Engineer at Devonport. Two types are discussed—the *Royal Sovereign* type, with recoil valves, and the *Hindustan*, with gradually decreasing area of passages.

In Chapter V., Professor Osborne Reynolds's Four-stage Turbine has been discussed and results of trials given.

In Chapter VI., very full details with sectional drawings have been given of Professor Osborne Reynolds's "Hydraulic Brake" and "Four-stage Centrifugal Pump."

The last chapter is reserved for the consideration of Professor Reynolds's two works: (1) "An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels." (2) "On the theory of lubrication and its applications." I thought it advisable to give a full treatment to these papers, as the original references are not always available for students and others interested in the higher study of hydromechanics.

I am indebted to the Council of the Royal Society, to the Council of the Institution of Civil Engineers, to the Council of the Institution of Mechanical Engineers, and to the Editor of *Engineering*, for kindly granting permission to use the papers appearing in their *Transactions*. I also acknowledge the kindness of many engineering firms for supplying me with drawings and descriptive matter—of which mention is made in the proper place.

S. DUNKERLEY.

MANCHESTER,
March 4, 1907.

CONTENTS

HYDRAULICS

CHAPTER I

THE FLOW OF A PERFECT FLUID

§ 1. **Bernouilli's Equation.**—When water flows along a pipe there is usually a procession of particles following each other along the same path in exactly the same manner, and considering a particular point, such as P, each particle reaches that point in the same direction and with the same velocity. The motion is then said to be *steady*, and the lines each particle describes, whether straight or curved, are called *stream lines.* The whole current of fluid may be imagined divided, by means of stream lines, into *stream tubes,* as shown in Fig. 1. If the thickness of these tubes be so arranged that the flow along them is the same, then, in plane motion, the velocity will be inversely as the distance between the lines.

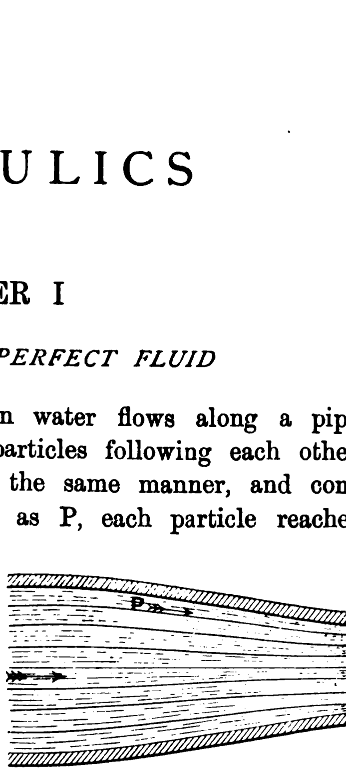


FIG. 1.

Now, consider one such tube, and how the pressure and velocity vary as a particle moves along it. To obtain a solution, certain assumptions have to be made. These are (1) that the fluid is perfect, so that the pressure is the same in all directions; (2) that the fluid is incompressible; (3) that there are no frictional resistances; and (4) that the motion

is steady, that is, that the velocity at any one point is always the same, so that there are no forces of acceleration to consider.

Let a denote the cross-sectional area in square feet at any point P of the tube; p the intensity of pressure in pounds per square foot; v the velocity in feet per second; z the elevation of P above same datum, say, sea level; and σ the weight per cubic foot of the fluid. Consider the energy one pound of the water contains on passing the section P, Fig. 1. The energy may be subdivided into three parts.

(1) The kinetic energy per pound is $\frac{v^2}{2g}$ foot-pounds.

(2) The energy given out in dropping to the sea level is z foot-pounds per pound.

(3) The pressure p is equivalent to a "head" $\frac{p}{\sigma}$; and hence the pressure per pound is $\frac{p}{\sigma}$ foot-pounds.

Thus the total energy at P is

$$\frac{v^2}{2g} + z + \frac{p}{\sigma}$$

foot-pounds per pound.

Now, since it is assumed that no work is done against friction, or in altering the volume of the fluid, or in causing acceleration, the energy cannot alter in going from P to, say, a second point Q. Hence the total energy at every section of a tube is the same; therefore

$$\frac{v^2}{2g} + z + \frac{p}{\sigma} = \text{constant} = h, \text{ say,}$$

in which h is the total head.

In addition, the volume of fluid passing every section of a tube per second is the same; therefore

$$\begin{aligned} va &= \text{constant} \\ &= Q \end{aligned}$$

where Q is the flow in cubic feet per second.

If all the stream lines are parallel, the above equation may be taken to apply to the whole mass of the fluid, and not only to the individual streams. But in curved pipes, the pressure and velocity will not, in general, be uniform over a section on account of the pressures necessary to cause centripetal accelerations. In dealing with pipe problems, such variations of pressure and velocity across a section may be neglected, and the pressure and velocity at every point of a section assumed given by the above equation. The expression is usually known as *Bernouilli's equation*.

§ 2. Experimental Representation of Bernouilli's Equation.—If the section of the pipe vary, where the area is least the velocity is greatest and, therefore, in a horizontal pipe (for which z is constant) the pressure is least. This may be experimentally verified by having pressure columns let into the pipe at certain points. In Fig. 2, a horizontal pipe of varying section projects from a tank; and pressure columns are inserted so as to be just flush with the pipe, and to present no burr to the inner surface. Under these circumstances, since the pressure is the same in all directions, the height to which the column rises is a measure of the pressure head. Thus, at different sections, the various quantities will be as shown. The pressure will be least at the smallest section. By making that section small enough, the pressure there may be made less than atmospheric pressure, and instead of using a pressure gauge a suction gauge would be required. Theoretically, the pressure might be reduced to absolute zero, but before this state of affairs is reached, the air held in solution is liberated, and the pressure may become so small that water vapour is formed.

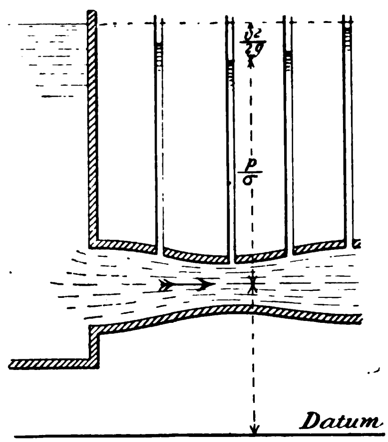


FIG. 2.

The mixture would become an elastic fluid, and the flow would be entirely modified. Taking the theoretical limit, if h be the depth of the least section below the level of the water in the tank, the limiting velocity at the smallest section is given by

$$h = \frac{v^2}{2g} + 34$$

the barometric pressure being equivalent to a head of 34 feet of water

$$\text{or} \quad v = \sqrt{2g(h + 34)}.$$

The velocity v, multiplied by the least sectional area, is the maximum quantity of water which can possibly flow out of the tank.

As illustrations of the flow through pipes in Fig. 3, water is discharged from a curved pipe from a tank, the discharge being into the atmosphere. If the bottom of the tank be taken as datum, then, denoting sections A and B by corresponding suffixes,

$$\frac{v_a^2}{2g} + z_a + \frac{p_a}{\sigma} = \frac{v_b^2}{2g} + z_b + \frac{p_b}{\sigma}.$$

If the water level be kept constant,

$$v_a = o \quad \text{and} \quad p_a = p_b$$

since at A and B the pressure is atmospheric. Also

$$(z_a - z_b) = h$$

h being the depth of the orifice below the water level. Hence

$$v_b^2 = 2gh$$

which gives the velocity of discharge into the atmosphere. The discharge per second is then the velocity v_b multiplied by the area at B.

Suppose the issuing pipe is bell-mouthed, as shown in Fig. 4, that is, the pipe is enlarged. In that case, the pressure at C is atmospheric, not at B, and

$$v_c^2 = 2gh.$$

Thus, by simply bell-mouthing, the discharge is increased in the ratio of

$$\frac{\text{sectional area at C}}{\text{sectional area at B}}.$$

The limiting state of affairs is reached when the pressure at B is zero; a condition which makes the limiting velocity at B given by the equation

$$v_b^2 = 2g(h + 34).$$

This represents a limiting velocity, and at a less velocity the stream will break up and the pipe runs partially full.

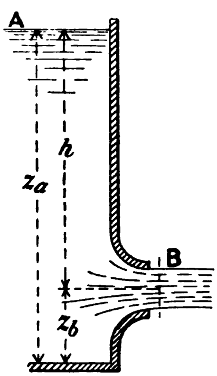


FIG. 3.

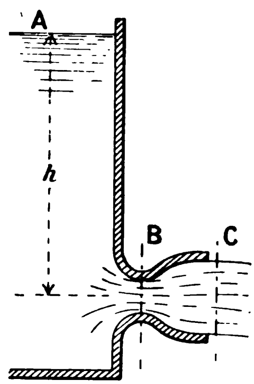


FIG. 4.

§ 3. **Venturi Meter.**—A further application of Bernouilli's equation is the *Venturi meter*, used to measure the amount of water flowing along a pipe. In the main length of pipe is inserted a length having a contracted throat, the pipe being enlarged again in order that the velocity should return to its original slow velocity (Fig. 5).

If suffixes (1) and (2) refer to the normal and contracted sections

$$\frac{v_1^2}{2g} + \frac{p_1}{\sigma} = \frac{v_2^2}{2g} + \frac{p_2}{\sigma}$$

or

$$\frac{v_2^2 - v_1^2}{2g} = \frac{p_1 - p_2}{\sigma} = h_1 - h_2$$

where h_1, h_2 are the heights of the columns in the gauges. If a_1 and a_2 are the sectional areas at the throat and pipe

$$v_1 a_1 = v_2 a_2$$

and

$$\therefore \frac{v_2^2}{2g}\left(1 - \frac{a_2^2}{a_1^2}\right) = h_1 - h_2$$

$$\therefore v_2 = \sqrt{2g \frac{a_1^2}{a_1^2 - a_2^2}(h_1 - h_2)}$$

$$\therefore v_2 a_2 = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2g(h_1 - h_2)}$$

whence $\quad Q = v_2 a_2 =$ discharge per second in cubic feet.

Thus the amount of water flowing in a closed pipe is obtained, provided the sectional areas are known and the difference of level

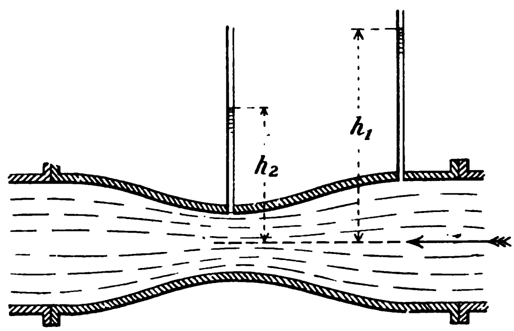


FIG. 5.

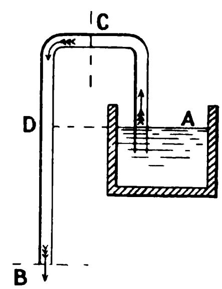


FIG. 6.

of the gauges observed. In practice this expression has to be multiplied by a coefficient which is slightly less than unity. The Venturi meter is discussed in greater detail in § 47.

§ 4. **Syphon.**—A third illustration which may be noticed is the *Syphon* (Fig. 6). It is well known that water will flow from a tank A through an overhead pipe, and discharge into the atmosphere at B, provided the flow is initially started and B is below A. Neglecting all losses, the velocity of discharge at B is given by

$$\frac{v_b^2}{2g} = \frac{v_a^2}{2g} + \text{the distance BD}$$

obtained by equating the energy at B with that of A. If the velocity at the surface A is so small as to be neglected, then

$$v_b = \sqrt{2g \times \mathrm{BD}}$$

which gives the velocity of discharge. At the section C,

$$\frac{v_c^2}{2g} + \frac{p_c}{\sigma} + \text{height of C above B} = \frac{p_b}{\sigma} + \frac{v^2}{2g}.$$

If the pipe is of uniform diameter, $v_c = v_b$, and, therefore,

$$\frac{p_b}{\sigma} - \frac{p_c}{\sigma} = \text{depth of B below C}$$

or

$$\frac{p_b}{\sigma} = \frac{p_c}{\sigma} + \text{depth of B below C.}$$

Now, $\frac{p_b}{\sigma}$ is the head equivalent to atmospheric pressure, since the discharge takes place into the atmosphere, and is, therefore, equal to 34 feet. The pressure at C can never be less than absolute zero, and, hence, the distance between B and C cannot exceed 34 feet. If it is greater it is impossible to syphon the water. Before this limiting pressure is reached, the pressure will be so reduced at C that the air held in solution will be liberated and will accumulate at C, and so stop the flow. The maximum head at which the syphon works is about 20 feet.

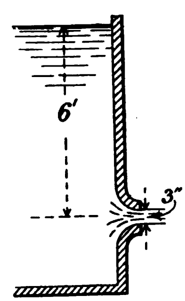


FIG. 7.

§ 5. **Numerical Illustrations.**—To illustrate these principles take one or two examples:

Ex. 1.—*A pipe projects from the side of a tank, the section gradually getting less until it reaches a diameter of* 3 *inches* (Fig. 7). *The pipe is* 6 *feet below the level of the water in the tank. Estimate the velocity of discharge, and the discharge in cubic feet per second.*

In this case

$$v^2 = 2gh$$

$$\therefore\ v = \sqrt{2 \times 32{\cdot}2 \times 6}$$

$$= 19{\cdot}66 \text{ feet per second.}$$

Discharge in cubic feet per second

$$= \frac{\pi}{4}d^2v = \frac{\pi}{4} \times \left(\frac{1}{4}\right)^2 \times 19{\cdot}66$$
$$= 0{\cdot}964.$$

Ex. 2.—*If in the preceding question the projecting pipe was bell-mouthed to a diameter of 4 inches; find the velocity of discharge, the discharge in cubic feet per second, and the pressure at the section where the diameter is* 3 *inches* (Fig. 8).

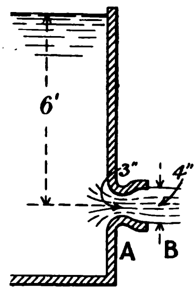


FIG. 8.

Since the discharge takes place into the atmosphere at B, the velocity of discharge at B is 19·66 feet per second. The discharge in cubic feet per second is

$$\frac{\pi}{4}d^2v = \frac{\pi}{4}\left(\frac{1}{3}\right)^2 \times 19{\cdot}66 = 1{\cdot}715,$$

nearly 80 per cent. greater than in the previous case. Again, if the pipe run full, the velocity at A is

$$\left(\frac{4}{3}\right)^2 \times 19{\cdot}66$$
$$= 35 \text{ feet per second.}$$

The pressure head at A, is

$$\frac{p_a}{\sigma} + \frac{v_a^2}{2g} = \frac{v_b^2}{2g}$$
$$\therefore \frac{p_a}{\sigma} = -\frac{1}{2g}(35^2 - 19{\cdot}66^2)$$
$$= - 13 \text{ feet.}$$

Thus the pressure at A is equivalent to a head of 13 feet below atmosphere, or to 5·64 pounds per square inch below atmosphere.

§ **6. Orifices deeply immersed.**—A further application of Bernouilli's Theorem is the discharge through small orifices deeply immersed (Fig. 9). The forms of the stream-lines are as shown, the outer stream-lines escaping tangentially to the walls. The lines

are curved and become sensibly horizontal at a short distance from the orifice, after which they descend in a parabolic path. The discharge cannot be estimated by a consideration of the section at the orifice for the following reasons :—

(*a*) The stream-lines cut the orifice section obliquely, and the discharge depends on the horizontal component of the velocity. The inclination of the different stream-lines to the vertical is not known.

(*b*) The stream-lines at the orifice section are curved, and there will be centripetal pressures acting normally to the stream-lines, which cannot be estimated.

(*c*) Side contractions take place, and modify the flow.

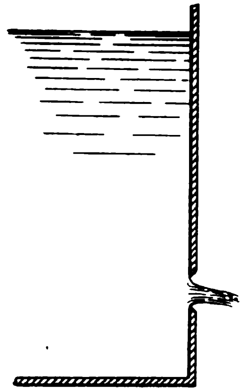


FIG. 9.

Thus, it is impossible to calculate the flow from the orifice section. But at the smallest contracted section the stream-lines are sensibly horizontal. This section is called the *vena-contracta*, the pressure being atmospheric at every section. If suffix (1) refer to the upper surface of water in the tank, and suffix (2)—refer to the *vena-contracta*, z—being measured above some datum, say, the bottom of the tank

$$\frac{v_1^2}{2g} + \frac{p_1}{\sigma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\sigma} + z_2.$$

But $p_1 = p_2$, and v_1 is very small compared to v_2, and

$$\frac{v_2^2}{2g} = z_1 - z_2 = h$$

where h is the distance of the orifice below the water surface; hence

$$v^2 = 2gh.$$

§ 7. **Empirical Coefficients.**—On account of air resistance and viscosity, the actual velocity of discharge is slightly less than the *theoretical* velocity. If the *actual* velocity of discharge be

represented by v_a, and c_v be an experimental coefficient, termed the coefficient of velocity, then

$$v_a = c_v\sqrt{2gh}.$$

The coefficient c_v may be expressed in terms of a second coefficient. Let h' be the loss of head in feet of water due to viscosity and air resistance. Then the effective head flow is $h - h'$; and therefore

$$v_a^2 = 2g(h - h')$$

$$\text{or } h' = h - \frac{v_a^2}{2g} = \left(\frac{1}{c_v^2} - 1\right)\frac{v_a^2}{2g}.$$

Since $\frac{v_a^2}{2g}$ is the actual head at discharge, the equation becomes

$$\frac{\text{loss of head}}{\text{available head}} = \left(\frac{1}{c_v^2} - 1\right) = c_r$$

where c_r is an empirical coefficient called the *coefficient of resistance*, and which depends on c_v.

In estimating the discharge, the actual velocity of discharge (v_a) must be multiplied by the sectional area of the *vena-contracta*. The product is the flow in cubic feet per second, and is usually denoted by Q. If a be the sectional area of the *vena-contracta*, the flow per second is

$$av_a.$$

The area a is not definitely known; the area of the orifice, say A, is given. It is generally assumed that the ratio of a to A is constant for all sizes of orifices within limits, and that ratio is called the *coefficient of contractor*, and denoted by c_c. Thus

$$Q = av_a = c_c A c_v\sqrt{2gh}.$$

The two constants c_c c_v are combined in one, the product being termed the *coefficient of discharge*, and denoted by C, so that, finally

$$Q = CA\sqrt{2gh}.$$

Although there are four coefficients, only two are independent.

§ 8. **Values of Empirical Coefficients.**—These constants can only be determined experimentally, and their values depend on a variety of circumstances. In particular, they depend upon whether the water is allowed to flow in a free, unobstructed manner to the orifice. When that is the case, that is to say, when the orifice is so deeply immersed, and so far from the sides and bottom, that these have no effect on the flow, the contraction is said to be *complete;* if, the sides and bottom are near enough to affect the flow, the contraction is said to be *imperfect;* and if, on one or more sides, the water is guided to the orifice (Fig. 10) the contraction is said to be imperfect.

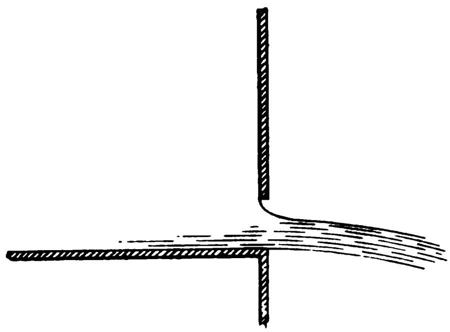


FIG. 10.

It is impossible to give any quantitative values to the coefficients in the last two cases; they depend entirely upon the degree of imperfection. But, if the contraction be complete, the values can be definitely obtained. They depend on the form of the orifice, and in the most important cases are as follows:—

Sharp-edged orifice (Fig. 11).

$$c_v = 0{\cdot}97, \quad c_r = 0{\cdot}0628, \quad c_c = 0{\cdot}64, \quad C = 0{\cdot}62.$$

Projecting pipe (Fig. 12).

$$c_v = 0{\cdot}82, \quad c_r = 0{\cdot}49, \quad c_c = 1{\cdot}00, \quad C = 0{\cdot}82.$$

Re-entrant pipe, the water escaping clear of the pipe (Fig. 13).

$$c_v = 0{\cdot}97, \quad c_r = 0{\cdot}0628, \quad c_c = 0{\cdot}5, \quad C = 0{\cdot}48.$$

This orifice is called *Borda's* mouthpiece.

The value of c_r in the first case shows that about 6 per cent. of the available head is lost in frictional and other losses. In

the second case, the small value of c_v, and the corresponding large value of c_r, is due to a cause which does not hold in the first and third cases, namely, a sudden enlargement of the stream, which will be considered later. It will also be noticed that the coefficient of discharge in the second case is 0·82 as against 0·62 in the first case; so that with a given head and area of orifice, the discharge in the second case is over 30 per cent. greater than in the first. The reason for this, in a bell-mouthed pipe, has been already pointed out (§ 2); but a secondary action, referred to above, takes place.

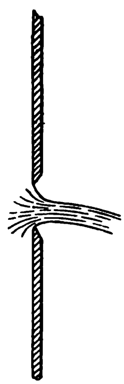


Fig. 11.

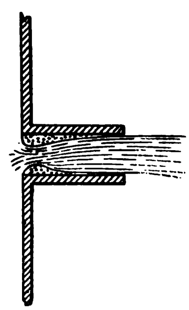


Fig. 12.

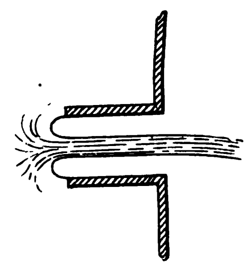


Fig. 13.

§ 9. **Experimental Determination of Empirical Coefficients.**— To determine the coefficient of contraction c by actual measurement, imagine two lugs attached to the side of the tank (Fig. 14). Let finely divided screws mesh with these lugs. The two points of the screws would first be brought into contact, and, the flow having been allowed to take place, the number of turns of both screws would be taken in the position when each point first touched the upper and lower surfaces of the jet. The measurements must be taken at the *vena-contracta*. This gives $c_c = \frac{a}{A}$, a being the area of the *vena-contracta*, determined as above described, and A the area of the orifice, which is known.

If the flow per second in cubic feet, Q, be estimated by running

the water into a measuring tank and noting the volume discharged in a given time, the coefficient of discharge c is given by

$$C = \frac{Q}{A\sqrt{2gh}}$$

whence $$c_v = \frac{C}{c_c}.$$

To measure c_v, the coefficient of velocity directly, the curve which the discharged jet assumes, is, neglecting air resistance—which is negligible—a parabola (Fig. 15). Let x and y be the horizontal

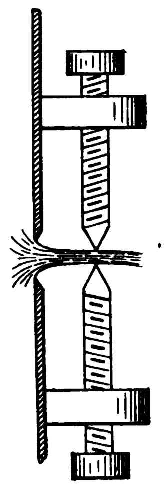


Fig. 14.

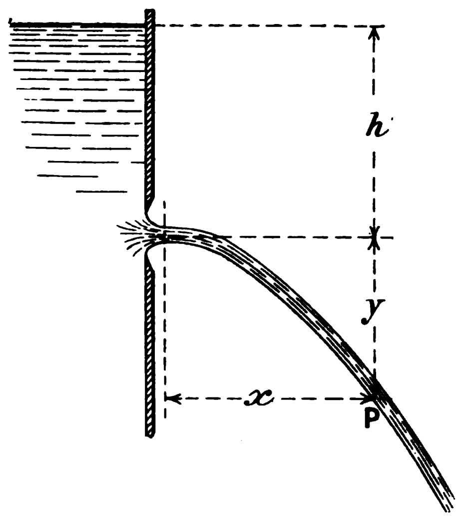


Fig. 15.

and vertical ordinates measured from vertical and horizontal lines through the *vena-contracta*. If a particle of water takes t seconds to describe its parabolic path to P, and if v_a be the actual velocity of discharge in a horizontal direction, then

$$x = v_a t$$
$$y = \tfrac{1}{2}gt^2$$
$$\therefore v = \sqrt{\frac{gx^2}{2y}}$$

or $$c = \frac{v_a}{\sqrt{2gh}} = \sqrt{\frac{x^2}{4yh}}$$

Thus, if x and y be measured, c_v is determined. A number of observations should be taken, and the mean results taken. The coefficient for greater depths than six inches is constant.

A second method is to suspend a tank about a horizontal axis, so that it is free to tilt. The water level is kept constant, and the water allowed to flow out. There will be a reaction tending to tilt the tank. This may be resisted by attaching an arm to the tank, and weighting it until the tank remains vertical. By taking moments about the point of suspension, the equivalent horizontal force at the orifice may be calculated. This force must be equal to the horizontal momentum in the jet. Now if, with previous notation, mass of water discharged per second is

$$\frac{\sigma}{g} a v_a$$

and the horizontal velocity impressed upon the water is v_a: hence the horizontal momentum per second

$$\frac{\sigma a v_a^2}{g}$$

which is known. Hence $a v_a^2$ is known. If the discharge, Q, is measured, the value of $a v_a$ is known. Therefore a and v_a are determined; and c_c, c_v are obtained from

$$c_c = \frac{a}{A}, \qquad c_v = \frac{v_a}{\sqrt{2gh}}$$

A number of observations should be taken with different values of h.

The same principle applies to a sudden stoppage of water in a pipe due to the sudden closing of the valve. If the length of pipe be h, and the velocity of flow v, and the time of closing t, assuming the retardation to be uniform, the pressure per square foot of section is

$$\frac{\sigma v}{gt}$$

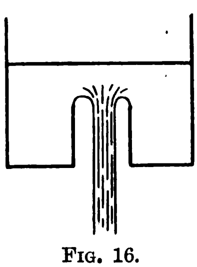


FIG. 16.

§ 10. Discharge through a Re-entrant at the Bottom of a Tank.—If the mouthpiece, or orifice, be in the bottom of a tank, the jet will get less and less in sectional area, owing to the increased velocity (Fig. 16).

But if gravity be neglected, and the water is projected under a pressure p above atmosphere, then if a_1, a_2, a_3 be the sectional area of the jet, tube, and cylinder,

$$a_1 v_1 = a_3 v_3$$

$$\text{increase in momentum} = \frac{\sigma a_1 v_1}{g}(v_1 - v_3)$$

$$= \frac{\sigma_1 a_1 \,.\, v_1^3}{g}\left(1 - \frac{a_1}{a_3}\right)$$

$$\text{unbalanced pressure} = pa_2$$

$$\text{and } \frac{v_1^2}{2g} = \frac{p}{w} + \frac{v_3^2}{2g} \quad \therefore \quad \frac{v_1^2}{2g} = \frac{\frac{p}{w}}{1 - \frac{a_2^2}{a_3^2}}$$

$$\therefore\ pa_2 = \frac{\sigma a_1 \,.\, 2p}{w} \quad \frac{1}{1 + \frac{a_1}{a_3}}$$

$$\text{or } a_2 = \frac{2}{\frac{1}{a_1} + \frac{1}{a_3}} \quad \text{or} \quad \frac{1}{a_1} + \frac{1}{a_3} = \frac{2}{a_2}$$

or, the section of the tube is a harmonic mean between the sections of the cylinder and jet.

§ 11. **Large Orifices and Notches.**—A large orifice or notch is such that its dimensions are comparable with the dimensions of the tank and depth of water. The method of treatment must, therefore, be quite different to small orifices. The objections pointed out in connection with small orifices (§ 6) are more important in large orifices. The first consideration is that the size of the orifice being commensurate with the other dimensions, the velocity will vary at different depths. If it were possible to consider the section at the orifice, the curve of velocities would be a parabola (Fig. 17), *abc*. The part *bc* would refer to the orifice, and, barring all effects, the area *bcde* would represent the theoretical flow on the assumption of no end contractions. As a matter of fact, the area of the orifice cannot be taken, because (1) the streams are oblique to the section, (2) the curvature of the streams, which now is important, (3) end contractions. At

the lips e and d, the flow is vertical, and consequently no discharge takes place. As the orifice is entered, the obliquity becomes less and the centre stream line is fully effective. The effective horizontal velocity could be represented by the ordinates to the curve efd; and the ratio of the area of that curve to the area of $ebcd$ would be the coefficient of discharge. The only way in which

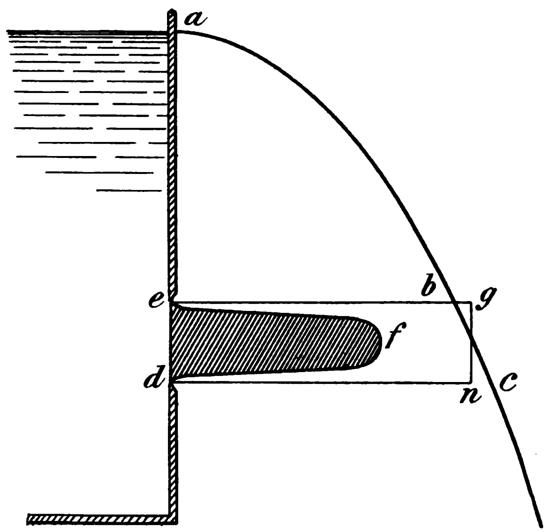


FIG. 17.

the discharge may be estimated is to consider the *vena-contracta* and integrate throughout the depth.

§ **12. Rectangular Orifice.**—Let large letters (Fig. 18) refer to the *vena-contracta*, and small letters to the orifice. Let a strip of breadth B, and thickness d, depth H, be taken. The area of the strip is BdH, the actual velocity of discharge is $c_v\sqrt{2gH}$, and, therefore the flow per second over the elementary area is

$$dQ = c_v B dH \sqrt{2gH}$$

Integrating between limits of H_2 and H_1, and the result is

$$Q = c_v B\sqrt{2g} \cdot \tfrac{2}{3}(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})$$

It is impossible to evaluate this expression, because B, H_2, H_1 refer

to the *vena-contracta*, and are therefore not known. But if a somewhat irrational assumption is made, namely, that

$$c_v \frac{B(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})}{b(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}})} = C = \text{coefficient of discharge}$$

then
$$Q = CB\tfrac{2}{3}\sqrt{2g}(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}).$$

This not only assumes that the *vena-contracta* is of the same shape as the orifice, but that this complex expression is constant for all depths and widths. This statement cannot be strictly true, especially in notches, and it can only be true provided the orifices are commensurable with the other dimensions.[1]

Perhaps an equally accurate formula is, when the above condition is satisfied, to take the mean depth to give the average velocity. In that case the expression for discharge becomes

$$Q = C\sqrt{2g}\,.\,b(h_2 - h_1)\sqrt{\frac{h_1 + h_2}{2}}.$$

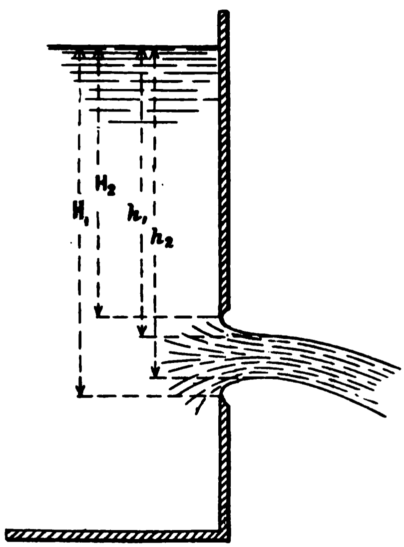


FIG. 18.

Expressed graphically, the discharge is represented by the area $eghd$ (Fig. 17). The value of c may be taken as 0·62, so that the first equation becomes

$$3{\cdot}3b(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}})$$

and the second

$$3{\cdot}5b(h_2 - h_1)\sqrt{h_1 + h_2}.$$

In a notch (Fig. 19), h_1 is zero, and b is the width of the

[1] By some writers the orifice is taken to give the discharge directly, and the results deduced from $cb\sqrt{2gh}\ .\ dh$. Needless to say, the argument is fallacious.

notch, and h the height of still water above the sill, in the first case

$$Q = 3{\cdot}3bh^{\frac{3}{2}}$$

and in the second $$Q = 3{\cdot}5bh^{\frac{3}{2}}.$$

The difference is about six per cent.

In a notch, the conditions are as shown in Fig. 19. The parabolic curve is *ab*, and the area *abc* would represent the theoretical discharge. At *d* there is a *velocity* of approach equal to $\frac{(ad)^2}{2g}$ and the actual curve is like *dec*. The ratio of the two is the coefficient of discharge. In all cases the head h must be measured some distance up-stream, or else the velocity of approach must be observed.

§ 13. **Francis's Formula.**—It is obvious that although the variation in the general coefficient may be found experi-

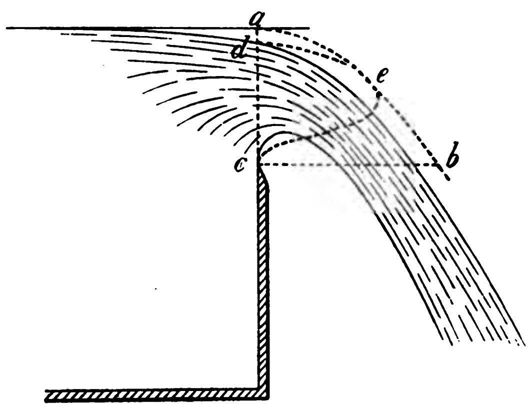


Fig. 19.

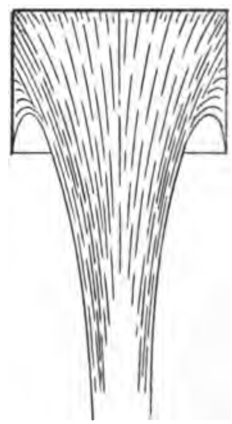


Fig. 20.

mentally to be small, provided the orifice have commensurable dimensions with the other dimensions; in notches this cannot be true (Fig. 20). It is evident that in some way, the discharge must depend on the ratio of depth to breadth

Mr. Francis conducted an extensive and valuable research on this point. He tried notches varying in width from 3 to 24

inches, and he adopted a perfect end contraction. The important feature brought out by these experiments was that from the breadth (b) a certain fraction of the depth had to be subtracted. For two end contractions the fraction was one-fifth, for one end contraction it was one-tenth, for a parallel stream it was zero. (Figs. 21, 22, 23.)

The formula therefore takes the form

$$Q = 3\,3(b - nh)h^{\frac{3}{2}}$$

in which $n = 2$, 1, or 0, according as there are two, one, or no end contractions. This is referred to in § 17.

Expressed generally, the formula assumed by Mr. Francis was of the type

$$Q = a(b + \beta h)h^{\frac{3}{2}}$$

in which Q is the discharge per second, b and h the breadth and

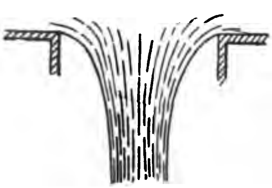


FIG. 21.

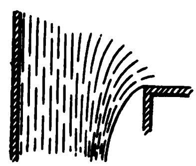


FIG. 22.

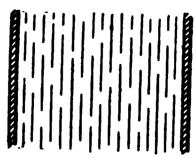


FIG. 23.

depth, in feet, respectively of the weir. Two observations are really sufficient to determine the empirical coefficients a and β. But for each orifice it would be necessary to determine the discharge for a number of depths and for a number of breadths, and it is then necessary to solve a number of simultaneous linear equations in two unknowns.

§ **14. Discharge through V-Notches.**—The case of a triangular notch is easily solved (Fig. 24). Logically the method adopted for a rectangular notch ought to be followed; but as a triangular notch has a special property, it will be sufficient to take the notch itself.

Thus, proceeding as before, if x be the width and dy the thickness of a strip

$$dQ = cxdy\sqrt{2gy}$$

and
$$\frac{x}{b} = \frac{h - y}{h}$$

$$Q = c\sqrt{2g}\frac{b}{h}\int_0^h (h - y)y^{\frac{1}{2}}dy$$

$$= c\sqrt{2g}\frac{b}{h}\left(\frac{2}{3}h^{\frac{5}{2}} - \frac{2}{5}h\right)^{\frac{5}{2}}$$

$$= \tfrac{4}{15}ab\sqrt{2g} \, . \, h^{\frac{3}{2}}$$

$$= \tfrac{8}{15}cb \tan \theta \, . \, \sqrt{2g} \, . \, h^{\frac{5}{2}}$$

where θ is the semi-angle of the triangle. For $\theta = 45°$,

$$Q = 4{\cdot}28ch^{\frac{5}{2}},$$

and taking
$$c = 0{\cdot}62$$

gives
$$Q = 2{\cdot}65h^{\frac{5}{2}}.$$

Thomson, as a more accurate estimate, gives 2·635.

It will be noticed that in a triangular notch, whatever the head over the sill, the sections of the notch are in geometrical ratio, and so is every other section of the issuing jet. In a rectangular notch it has been shown how the variation of coefficient has been taken into account (using Francis's formula); but in a triangular notch, the coefficient of discharge is *strictly* constant. Thus, for accurate determinations, a rectangular notch is to be preferred to a rectangular arc.

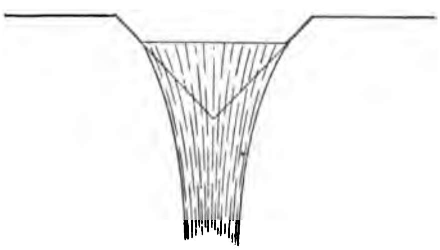


FIG. 24.

§ **15. Special Cases.**—One or two special cases may be noticed.

Drowned Orifices.—In the case of a lock, the sluices, whether

in the lock-gate or built in the walls, are always "drowned," *i.e.* they are fully immersed on each side (Fig. 25). In the case of sluices in a lock-gate, the stream contracts, as in orifices, and the problem has to be treated in the manner already described. But when the lock is supplied with culverts, passing from the upper reach to the lock, and thence to the lower reach, the entrances and exits to the lock are well rounded off and the sides are lined with smooth bricks, so that the culverts always run full. It is not necessary to make any deduction, but the full area of all the culverts may be taken.

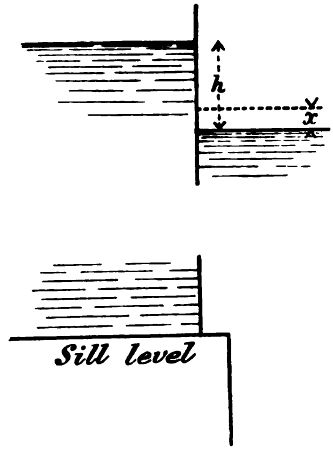


FIG. 25.

The calculation for the discharge is simple. The "effective" head is the difference between the water levels on the two sides, and is represented by h in Fig. 25.

A point of importance bearing on locks is the time required to fill and empty the lock. The water levels in the upper and lower reaches may clearly be assumed to remain constant. In that case if the difference of level at time t is x, the velocity is $\sqrt{2gx}$, and if A be the water surface of the lock and a the effective area of the water passages, then

$$\mathrm{A}dx = a\sqrt{2gx}\ .\ dt$$

$$t = \frac{2\mathrm{A}\sqrt{h}}{a\sqrt{2g}} \text{ between limits.}$$

Between any two heights, h_1 and h_2, it is

$$t = \frac{2\mathrm{A}}{a\sqrt{2g}}(\sqrt{h_1} - \sqrt{h_2}).$$

In this problem, the area A has been assumed constant. It may, however, vary, and in that case a second integration would

be required. As a problem which may be left to the reader, if T = time to discharge a quantity of water equal to the contents of a lock or reservoir on the assumption that the latter remained filled, so that the discharge takes place under a constant head; and if t_1, t_2 be the times necessary to discharge the same quantity of water, that is to say, to empty the reservoir, then when the reservoir is gradually emptied, in the cases where (1) the reservoir has vertical sides (2) is wedge-shaped,

$$t_1 : t_2 : T = 2 : \tfrac{4}{3} : 1.$$

Discharge through Sluices and under Arches.—In the case of a canal, sluices have to be constructed to take off surplus water, and to regulate the levels in the two reaches (Fig. 26). If the sluice-gate

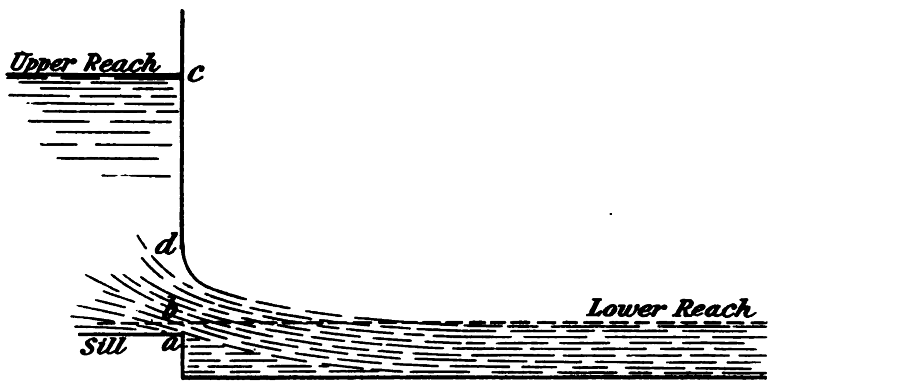


FIG. 26.

be raised to a height not exceeding the depth of water in the tail-race, the case is the same as § 15; but if the gate be raised above this level, as it would be in times of excessive flood, the case is different. The discharge may be divided into two parts, from the sill a to the lower water b. At all points of ab the usual assumption is to assume the head constant and equal to the difference of level between b and c. This will probably not be the strict truth, on account of the agitated state of the water. From b to dc the usual assumption is that the velocity head is bd at b, and zero at d. In other words, it is considered as a notch. It is unnecessary to give

a formula, but the coefficient of discharge may be taken in each case as 0·62.

§ **16. Forms of Issuing Jets.**—It has been assumed, in all the cases considered, that the shape of the *vena-contracta* is the same as that at the orifice. But the statement is not true at some distance from the orifice. Thus, consider two stream lines issuing from an orifice (Fig. 27). The velocity of projection of a will be greater than that at b, and the latter is inclined to the horizontal. Consequently the stream lines will cross at c, and the section at c must therefore be quite different to that of the orifice.

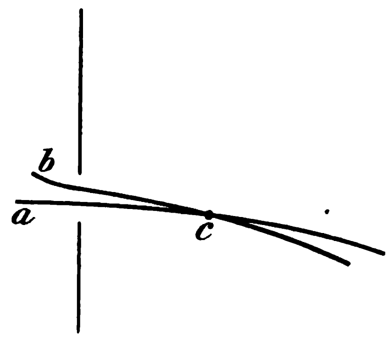


FIG. 27.

The peculiar shapes which the issuing jet assume have been experimented on by different observers, including Bidone, Magnus, and Lord Rayleigh. To show the peculiar character of the changes of section of the issuing jet, Fig. 28 represents an elliptic section, and Fig. 29 a triangular section. The distances from the orifices are given. The variation in section is remarkable. The same formation takes place in all cases, the jet splitting up in thin sheets perpendicular to the sides of the orifice.

As regards the reasons for these peculiar formations, Bidone ascribed to viscosity, Magnus to cohesion, and Buff to capillary force. Lord Rayleigh[1] has discussed the subject mathematically, and confirms Buff's theory. Under the action of a capillary force the fluid behaves as if enclosed in an envelope of constant tension, and the recurrent force of the jet is due to vibration of the fluid column about the central figure of equilibrium superposed upon the general progressive motion. Since the phase of vibration depends upon the time elapsed, it is always the same at one point in space, and thus the motion is steady in the hydrodynamics sense, and the boundary of the jet is a free surface.

[1] *Proceedings of the Royal Society*, xxix. p. 71.

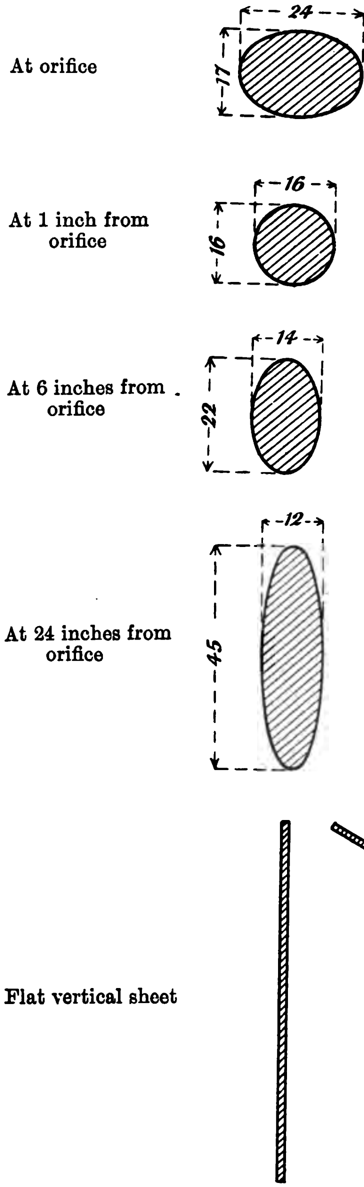

24
17
At orifice
16
16
At 1 inch from orifice
14
22
At 6 inches from orifice
12
45
At 24 inches from orifice
Flat vertical sheet

FIG. 28.

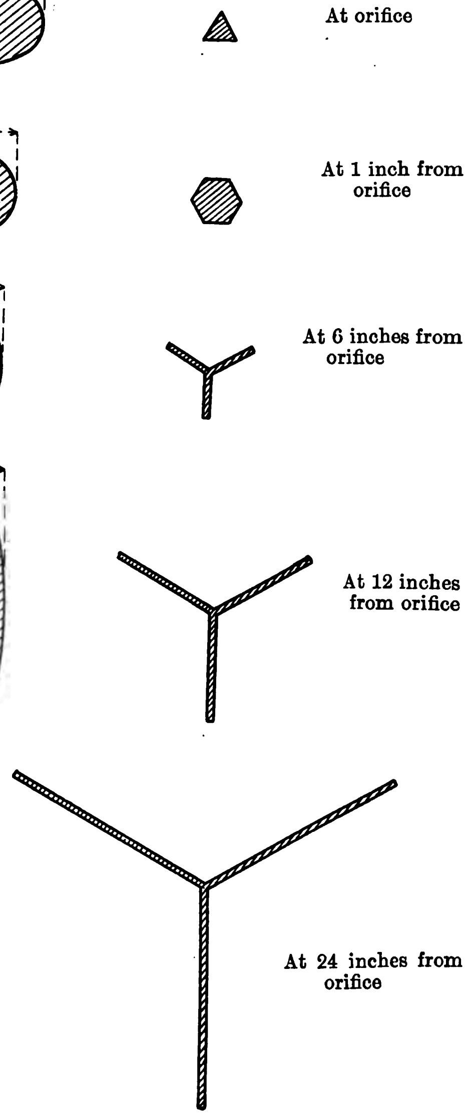

At orifice
At 1 inch from orifice
At 6 inches from orifice
At 12 inches from orifice
At 24 inches from orifice

FIG. 29.

§ 17. **Law of Comparison.**—Take two similar-shaped orifices, similarly situated, so that if one has double the linear dimensions of the other, it is likewise at double the depth; then the forms of the corresponding stream lines will likewise be similar—or the same diagram would represent each system on different scales. Let the linear dimensions of one be n times that of the other. Let symbols without suffixes refer to any pair of arbitrary corresponding points, and symbols with suffix o to a pair of particular corresponding points on the same stream line as the first pair. Then, taking the bottom of the tank as datum, if σ be the density of the fluid—use large letters for the large tank, and small ones for the small tank—

$$\frac{P}{\sigma} + \frac{V^2}{2g} + Z = \frac{P_o}{\sigma} + \frac{V_o^2}{2g} + Z_o$$

$$\frac{p_o}{\sigma} + \frac{v^2}{2g} + z = \frac{p_o}{\sigma} + \frac{v_o^2}{2g} + z_o$$

and $$Z = nz,\ Z_o = nz_o,$$

also, $$\frac{A}{a} = \frac{A_o}{a_o} = n^2,$$

the stream lines being similar.

$$\therefore \left(\frac{P}{\sigma} - n\frac{p}{\sigma}\right) - \left(\frac{P_o}{\sigma} - n\frac{p_o}{\sigma}\right) + \frac{V^2 - nv^2}{2g} - \frac{V_o^2 - nv_o^2}{2g} = 0$$

which reduces to

$$\frac{P - P_o}{\sigma} - n\left(\frac{p - p_o}{\sigma}\right) + \frac{\left(1 - \frac{v_o^2}{v^2}\right)}{2g}(V^2 - nv^2) = 0.$$

This is true for any pair of corresponding stream lines; in particular for the boundary tubes, that is to say, the tubes which are bounded on one side by the tank, and, in the escaping jet, by the atmosphere. For this particular pair of tubes, consider the two sections as being at the outer surface of the issuing jet; then, whatever be the absolute pressure of the atmosphere, or whether it be the same in the two cases,

$$p = p_o,\ P = P_o$$

and $$V^2 = nv^2.$$

Thus, the velocity ratio at any two corresponding tubes is $\sqrt{n}$. Moreover, from the above equation, the difference of pressure head between any two points in the large tank is n, that the same conditions hold for the second stream tube as for the surface stream line. The argument can be extended to a third stream line, and so throughout the whole liquid.

Thus, at every pair of corresponding points, the velocity ratio is proportional to the square root of the linear dimension; and the pressure ratio to the linear dimension. Hence, the volumetric discharge is proportional to $n^2\sqrt{n} = n^{\frac{5}{2}}$.

In a *V-notch*, the stream lines are always similar and similarly situated (§ 14). Hence the volumetric discharge is strictly proportional to $h^{\frac{5}{2}}$.

Verification of Francis's Formula.—In § 13 the formula

$$Q = 3{\cdot}3\,(b - 0{\cdot}2h)\,h^{\frac{3}{2}}$$

was given for the discharge of a rectangular. A general expression would be

$$Q = C\,(b - xnh)\,h^{y},$$

in which

Q = quantity per second in cubic feet.
C = a constant coefficient.
b = total length of weir.
x = a second coefficient.
n = number of end contractions. In a single weir, with complete contraction, $n = 2$; when the length of the weir is equal to the width of the canal leading to it, then $n = 0$; if the water is guided direct to one side of the weir, and the other is free, $n = 1$.
h = depth of water over the weir, taken far enough up stream from the weir to be unaffected by the curvature in the surface caused by discharge.
y = a constant index.

This formula is a rational formula, that is to say, the *form* of the expression can be deduced from the law of comparison. A notch may be made so long relatively to its depth that for any additional length the increase of the flow will be proportional to

the additional length. Or, to express it differently, the stream lines near the extremities of the notch are very much curved, but become straighter and straighter as the centre of the weir is approached. Beyond a certain distance, the flow is practically in parallel lines. With a given head, increasing the length of the weir will not alter the point at which the stream lines become sensibly parallel, and so will merely increase the length over which the flow is in parallel lines. The distance of this parallel portion from the edge of the notch will be proportional to the depth of water. Hence the flow may be divided into two portions, the parallel flow parts and the end flow. Take the width of the parallel flow part as b, and the width of each end, being proportional to h, $\frac{mh}{2}$. Imagine the central portion separated from the two ends. The two end portions form a rectangular notch of depth h and width mh; so that the width varies as the depth, and the law of comparison holds. The discharge from the two ends will accurately be given by

$$Q_1 = ah^2\sqrt{h}$$

where a is some coefficient.

The flow over the central portion is proportional to the length for a given depth. If a portion of breadth some fraction of the depth be taken, the flow over that portion will also be proportional to $h\sqrt{h}$: and therefore, over the whole central portion proportional to

$$b'h\sqrt{h}$$

or

$$= \beta b'h\sqrt{h}.$$

Thus the whole discharge will be equal to

$$\begin{aligned} &a\,h^2\sqrt{h} + \beta b'h\sqrt{h} \\ &= ah^2\sqrt{h} + \beta(b - mh)h\sqrt{h} \\ &= \beta\left(b - \frac{m\beta - a}{\beta}h\right)h^{\frac{3}{2}}. \end{aligned}$$

This refers to two-end contractions. For only one-end contraction

both m and a must be halved, whilst for no end contraction both m and a are zero. Thus, the formula becomes

$$\beta(b - nxh)\,h^{\frac{3}{2}}$$

in which $n = 2, 1, 0$, for two, one, or no end contraction.

To determine the two constants, β and x, necessitates two observations of flow; but for accurate determinations a number of observations must be taken. The formula will not apply if no central part exists, over which the flow is in sensibly parallel streams. The values given by Francis are $\beta = 3{\cdot}33$, and $x = \frac{1}{10}$. He states that this formula is not applicable to cases in which the height, h, from the crest to the surface level exceeds one-third of the length, nor to very small depths. In the experiments from which it was determined, the depths varied from 7 inches to 19 inches.

CHAPTER II

FLUID FRICTION

So far it has been assumed that the fluid is perfect, in which case the relation between pressure and velocity, in the case of a pipe laid horizontally, is given by the equation

$$\frac{p}{\sigma} + \frac{v^2}{2g} = \text{constant.}$$

If the sectional area is everywhere the same, that is to say, if the pipe is of uniform diameter, then the velocity is the same at every section; and, therefore, from the above equation, so is the pressure. In other words, when a perfect fluid flows along a horizontal pipe of uniform diameter, there is no change of pressure.

§ 18. **Viscous Resistance.**—Now, in practice, the fluids are not perfect, but are capable of withstanding a tangential resistance. If the fluid is very viscous, the size of the tube small, and the speed of the flow also very small, the fluid flows in parallel layers, and each layer slides over the next one (Fig. 30). This sliding action gives rise to what is called a viscous force, which causes one layer to act as a drag on its neighbour, and, for all practical purposes, the effect may be taken to be the same as sliding friction, as, for example, when one flat surface slides over a

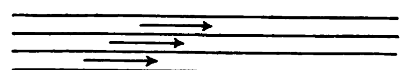


FIG. 30.

second. The result is that along a stream line, in the direction of flow, the pressure gets less and less—the loss of pressure being required to overcome this viscous resistance. The laws which govern this loss may be deduced from mathematical principles, and have been verified experimentally. It will be discussed at length in Chapter VII. In the case taken (namely, a viscous fluid, a small tube, and slow velocities), the loss of pressure is found to be proportional to the length of tube, to be directly proportional to the velocity, to be inversely proportional to the square of the diameter, and to depend on temperature, being less the higher the temperature. Expressed otherwise, under the conditions assumed,

$$\text{loss of pressure} \propto \frac{vl}{d^2}$$

where v is velocity, l the length, and d the diameter. This formula, for example, would apply to a fluid like glycerine flowing through a capillary tube.

§ 19. **Critical Velocity.—Loss of Head.**—Now, in ordinary hydraulic problems, the fluids are not so viscous, the pipe is of appreciable size, and the velocities are much larger. In such a case, the problem is entirely different from that just discussed. At a certain speed—depending on the diameter of pipe and temperature—the straight-line motion changes, and the water is broken up into a large number of rotating eddies of water, which are carried on with the general body of the stream. These eddies represent so much energy, and are ultimately absorbed by viscosity; and their energy reappears as heat which, from a hydraulic point of view, is not available. Thus, if the flow takes place above this critical velocity, there is a much greater loss of energy, and therefore pressure, than before.

The critical velocity at which the steady straight-line motion becomes eddy motion, and the law of loss of pressure above this velocity, is essentially a matter of experiment. It is sufficient to state that in all practical problems the velocity is well above this critical velocity, and that the law giving the loss of head is quite different from that already quoted. In the majority of

cases, the loss of pressure due to the formation and absorption of these eddies is found to be proportional to the length of tube, to be directly proportional to the square of the velocity of flow, to be inversely as the diameter of the pipe, and independent of temperature and pressure. Expressed otherwise, in all ordinary problems the

$$\text{loss of pressure} \propto \frac{v^2 l}{d}.$$

This law is not strictly true, but it is sufficiently approximate for practical purposes. A more exact expression would be

$$\frac{v^n l}{d^{3-n}}$$

where n is some index to be determined by experiment, and which depends on the material of the pipe. Below the critical velocity, $n = 1$, and the loss of pressure $\propto \frac{vl}{d^2}$; for a cast-iron pipe, above the critical velocity, $n = 2$, and loss of pressure $\propto \frac{v^2 l}{d}$.

§ **20. Determination of Index of Velocity. — Logarithmic Plotting.**—The value of n for a pipe may be found by taking a horizontal lead pipe, about 5 or 6 feet long, and boring two small holes at a certain distance apart. Insert in these holes, so as to be flush with the inside, two small tubes, and connect these tubes by means of rubber tubes with a mercury gauge, as shown in Fig. 31. In passing from A to B there will be a loss of pressure, and the mercury in one leg will stand higher than in the other. Suppose, for example, that the difference of level is m inches of mercury. Since mercury is 13·6 times as heavy as water, this is equivalent to 13·6m inches of water. But one leg contains m inches of water more than the other, and therefore the true difference of pressure between the sections A and B is $(13{\cdot}6 - 1)m$ inches $= 12{\cdot}6m$ inches of water $= 1{\cdot}05m$ feet of water. Thus the loss of head, expressed in feet of water, is found by multiplying the reading m in inches by 1·05.

Suppose, then, that in any experiment with the water flowing

along the pipe—the pipe being full—is measured, and the water is collected, the weight of the water flowing through per second, divided by the sectional area of the pipe in square feet will give the average velocity of flow v. Each experiment gives one relation between v and the loss of head, which will be denoted by h' ($= 1{\cdot}05m$). By varying the supply of water, and so the speed of flow, a number of values of v are obtained for the corresponding

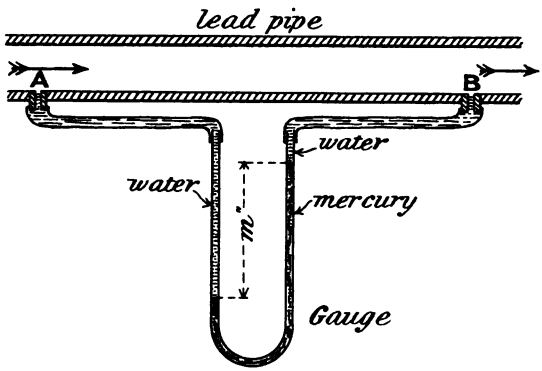


FIG. 31.

values of h'. What relation exists between h' and v? *Assume* the relation to be

$$h' \propto v^n.$$

This formula must be verified; and, if true, the value of n must be found. *If* the formula is true, it may be written

$$h' = av^n$$

where $a =$ constant; and, therefore, taking logarithms,

$$\log h' = \log a + n \log v.$$

Call $\log h'$, y, and $\log v$, x; the equation then reads

$$y = \log a + nx.$$

Hence, plot $\log h'$ and $\log v$, if all the points so plotted lie on a straight line, or very nearly so; n will depend on the inclination of that straight line. If the points obtained by

logarithmic plotting, as it is called, do not lie on a straight line, then the experimental results cannot be expressed by the law assumed.

§ 21. **Numerical Illustration.**—*A lead pipe is* 3·48 *feet long between gauge points and is* 0·4 *inch diameter. The discharge of*

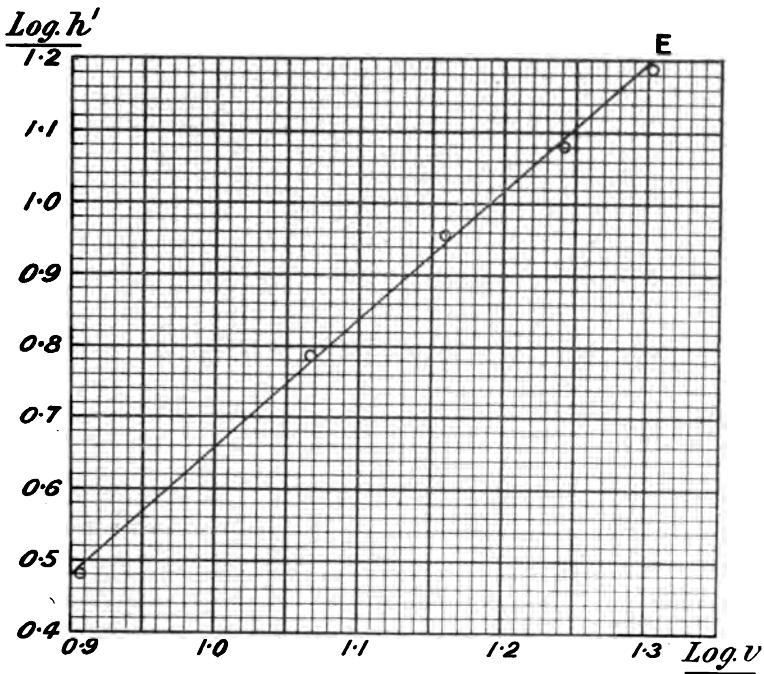


FIG. 32.

water in cubic feet per second, and the corresponding difference of gauge reading m, *is experimentally found to be as given in the following table:—*

Discharge in cubic feet per second	0·07	0·1018	0 1259	0·152	0·1786
Value of m (inches of Hg) . . .	2·89	5 83	8·65	11·62	14·90

When the necessary reductions have been made, the results are :

v	8·04	11·67	14·42	17·41	19·90
h'	3·03	6·11	9·07	12·20	15·62
log v	0·9053	1·0671	1·159	1·2407	1·2989
log h'	0·4814	0·7860	0·9576	1·0864	1·1937

The five points, plotted logarithmically, are as shown in Fig. 32.

As might be expected they do not quite lie on a true straight line, but the average line is shown—thus justifying the formula. Now take two points *exactly* on the straight line. Thus, when

$$\log v = 1{\cdot}0 \text{ and } 1{\cdot}3$$
$$\log h' = 0{\cdot}658 \text{ and } 1{\cdot}2 \text{ respectively.}$$

Then,
$$\log h' = \log a + n \log v$$
$$\therefore\ 0{\cdot}658 = \log a + n + 1{\cdot}0$$
$$1{\cdot}2 = \log a + n + 1{\cdot}3$$
$$\therefore\ n = \frac{0{\cdot}542}{0{\cdot}3}$$
$$= 1{\cdot}807.$$

§ 22. **Special Gauge.**—In making hydraulic experiments, the tanks are usually lead-lined, and it is very necessary to guard against any accidental discharge of mercury. A sudden turn of a cock, for example, may discharge in the gauge illustrated in Fig. 33 the whole of the mercury along the pipe. Special gauges have therefore to be provided.

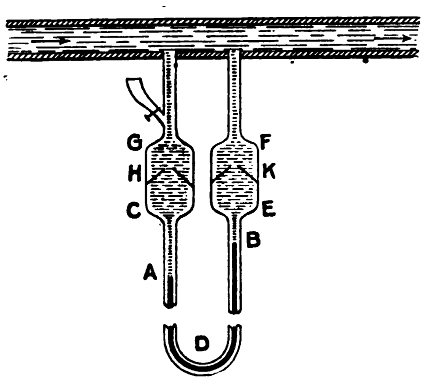


FIG. 33.

The mercury is contained in the U tube CDE, the ordinary level being at A and B; EF and CG are enlarged basins with a baffle plate at H and K; there is a funnel for

pouring in the mercury (Fig. 33). The tubes are filled with water above the levels, A and B. When a sudden rise of pressure takes place, even if the mercury rises to the basins, it is spread over such an area, that, with the baffles, it is impossible to rush past.

§ 23. **Quantitative Expression for Loss of Head in Ordinary Cases.**—The usual formula for loss of head is

$$h' \propto \frac{v^2 l}{d}.$$

This may be written in the form

$$h' \propto \frac{\pi dl \cdot v^2}{\pi d^2}$$

or

$$\propto \frac{Sv^2}{A}$$

where S is the wetted surface of the pipe, and A the sectional area of the stream.

or,

$$h' \propto \frac{S}{A} \cdot \frac{v^2}{2g}$$

whence,

$$h' = f \cdot \frac{S}{A} \cdot \frac{v^2}{2g}$$

where f is an experimental coefficient called the *coefficient of friction*. For a circular pipe of diameter d feet, running full, $S = \pi dl$ and $A = \pi d^2$, whence

$$h' = \frac{4fl}{d} \cdot \frac{v^2}{2g}.$$

§ 24. **D'Arcy's Formula.**—The value of the coefficient f is not strictly constant, but it depends, in some way, on the velocity of flow and on the diameter of the pipe. The variation of f with velocity is so slight that it may be neglected; but it depends considerably on diameter. A series of extensive experiments by D'Arcy on the Paris water mains showed that the variation of f with diameter d could be expressed by the equation

$$f = 0{\cdot}005\left(1 + \frac{1}{12d}\right)$$

d being the diameter in feet.

Thus, for different values of d, the value of f is given by the table:

Diameter in inches.	1	2	3	4	5	6	9	12	18	Very large
Value of f	0·01	0·0075	0·00655	0·00625	0·006	0·00588	0·00555	0·00541	0·00526	0·005

These results refer to a clean cast-iron pipe; for an old, incrusted, pipe the values must be doubled. It refers to pipe friction only; other additional losses will be noticed later. If the diameter of the pipe is not given, a provisional value of f is 0·0075.

§ **25. Line of Virtual Slope.**—For closed pipes, running full, the equation is

$$h' = 4f\frac{l}{d}\frac{v^2}{2g}$$

and f depends on the diameter of the pipe. In open channels the pressure is constant and hence the head necessary to overcome the frictional resistances is obtained by the fall in the river bed. In pipes, the pressure may vary, and the necessary head may be obtained either by a drop in pressure or a slope of the pipe, or both. In fact, the variation of pressure may be conveniently represented by means of pressure gauges, as already described.

Take, for example, the case of a straight pipe, of uniform diameter, projecting from a tank (Fig. 34).

If the pipes were plugged, so that there was no discharge, the water in each gauge would rise to the level of the water in the tank, and would therefore be at the level AB. If flow takes place, and there was no loss of head in friction, the velocity at every section would be the same, and the water in all the gauges would be at the level CD, and the difference between the levels AB and CD would be the kinetic head $\frac{v^2}{2g}$. But there is a loss of head in friction which is proportional to the

length of pipe; and the water in the gauge glasses will rise to such heights that the tops of the columns lie along a sloping straight line CE. Thus, in any gauge glass, the distance BD will represent $\frac{v^2}{2g}$, the distance DE the loss of head $4f \cdot \frac{l}{d} \cdot \frac{v^2}{2g}$, and the distance FE will represent the pressure head. The line CE is usually called the "line of virtual slope"; if it is parallel to the line of pipe, the pressure at every section of the pipe is the same and the slope of the pipe is just sufficient to overcome frictional resistances; if CE has a greater slope than the pipe, the pressure becomes less as the discharge is approached;

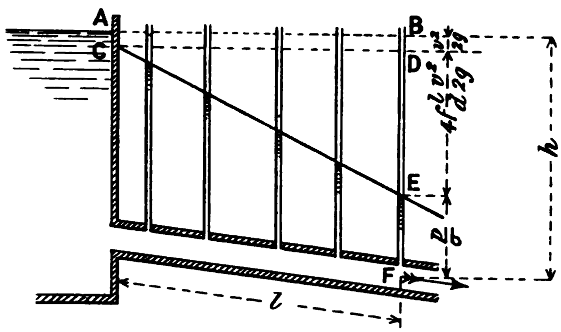


FIG. 34.

if the slope of CE is less than that of the pipe, the pressure gets greater.

The pipe may be curved either in elevation, in plan, or both. This Fig. 35 shows a pipe curved in elevation. The line of virtual slope is curved also, and may fall below the pipe line. There may be, therefore, negative pressure at some part of the pipe, but this negative pressure can never be greater than the equivalent of 34 feet of water, for reasons already stated (§ 2). In this case, the gauges are equally spaced along the centre line of the pipe.

§ **26. Discharge into Atmosphere.** — Ordinarily, the discharge takes place into the atmosphere, in which case the pressure at

the end of the pipe is zero. If the end of the pipe be at a depth h below the level of the water in the tank, then

$$\frac{v^2}{2g} + f4\frac{l}{d} \cdot \frac{v^2}{2g} = h$$

knowing l, d, and f, the velocity of discharge v is at once determined, and so the quantity of water discharged per second can be estimated.

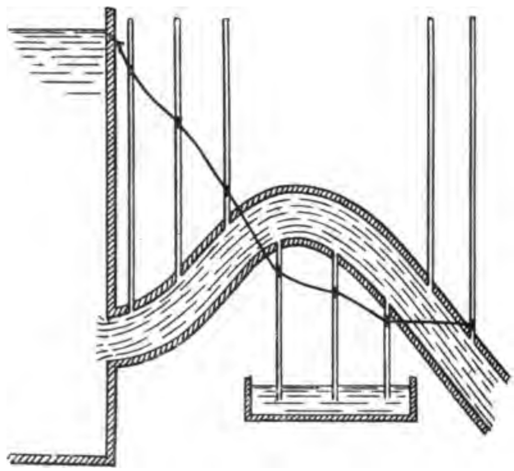


FIG. 35.

If, at discharge, the water be projected vertically, the theoretical height to which the jet will rise will be

$$\frac{v^2}{2g} = \frac{h}{1 + f \cdot \frac{4l}{d}}$$

and the discharge in cubic feet per second is

$$\frac{\pi}{4} d^2 v = \frac{\pi}{4} d^2 \sqrt{\frac{2gh}{1 + f \cdot \frac{4l}{d}}}$$

neglecting friction, the jet would rise to the height h.

Usually, the term $f \cdot \frac{4l}{d}$ is so great, compared to unity, that the

latter may be neglected, and the discharge in cubic feet per second becomes

$$\frac{\pi}{4}\sqrt{\frac{ghd^5}{2fl}} = Q, \text{ say.}$$

The diameter of pipe necessary to discharge Q cubic feet per second is, then, given by

$$d = \sqrt[5]{\frac{32flQ^2}{gh\pi^2}}.$$

Since f depends on d, the value of f is not known before d is determined. As a provisional value it may be taken as 0·0075. As a matter of fact, a considerable error in estimating f will only make a slight error in d. Thus, for example, if f be doubled, the diameter is increased $\sqrt[5]{2}$ times, that is, 1·15 times, or the increase is 15 per cent.

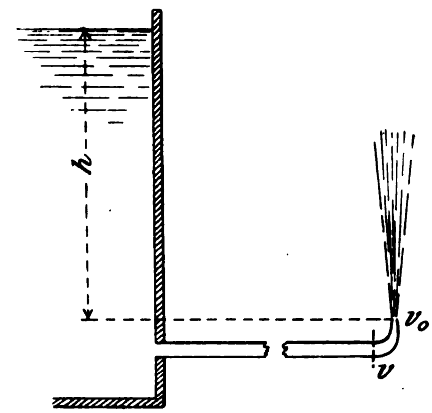


FIG. 36.

§ 27. Discharge through Nozzles.—The effect of putting a nozzle at the end of the pipe may be easily seen. Let v_o be the velocity of efflux through the nozzle, v through the main pipe (Fig. 36). The nozzle is usually so short, compared to the pipe, that all the frictional loss may be assumed to take place in the pipe. In that case, if l be the length of the pipe, the loss of head in friction is

$$f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g}$$

and the kinetic head at discharge is now $\frac{v_o^2}{2g}$, not $\frac{v^2}{2g}$. Thus, if the discharge takes place in the atmosphere at a depth h below the level of the water in the tank,

$$h = f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g} + \frac{v_o^2}{2g}.$$

Let d_o be the diameter of the nozzle; then, since the same quantity of water flows through the nozzle as through the pipe,

$$d^2v = d_o^2v_o$$

whence
$$h = \frac{v_o^2}{2g}\left[1 + f \cdot \frac{4l}{d} \cdot \frac{d_o^4}{d^4}\right].$$

The theoretical height to which the jet could be projected vertically is, therefore,

$$\frac{v_o^2}{2g} = \frac{h}{1 + f \cdot \frac{4ld_o^4}{d^5}}.$$

On account of the spreading of the jet, the actual height is found to be about three-quarters of the theoretical.

It will be noticed that the height to which the water can be projected is greater than before. The reason is that putting on a nozzle creates a back pressure in the pipe, and therefore reduces the velocity of flow along the pipe. There is thus less loss in friction. The jet can rise to a greater height, but the quantity discharged is much reduced. In point of fact, the pressure at the end of the pipe just before the nozzle is, neglecting any losses in the nozzle, given by the equation

$$\frac{p}{\sigma} + \frac{v^2}{2g} = \frac{v_o^2}{2g}.$$

The quantity of water discharged in cubic feet per second is

$$\frac{\pi}{4}d^2v_o.$$

§ 28. **Pumping Horse-power.**—The pumping horse-power necessary to pump a given quantity of water a certain height through a given pipe, and the diameter of nozzle necessary, may be estimated as follows:

Let the height to which the water has to be projected be h_o. Then, since the actual height may be taken to be three-quarters the head due to the velocity of efflux, the velocity of efflux through the nozzle (v_o) is given by

$$\frac{v_o^2}{2g} = \frac{4}{3}\ h_o.$$

If Q is the known discharge in cubic feet per second, and d_o the diameter of the nozzle, then

$$\frac{\pi}{4}d_o^2 v_o = Q$$

thus giving d_o.

The head necessary at the pumping end of the pipe is then

$$\frac{v_o^2}{2g}\left(1 + f \cdot \frac{4ld_o^4}{d^5}\right) = \frac{8Q^2}{g\pi^2}\left(\frac{1}{d_o^4} + \frac{4fl}{d^5}\right)$$

by substitution.

Thus σQ pounds of water per second have to be pumped against this head, and, therefore, the pumping horse-power is

$$\frac{8\sigma Q^3}{550g\pi^2}\left(\frac{1}{d_o^4} + \frac{4fl}{d^5}\right)$$

since 550 feet pounds per second represent one horse-power.

If the diameter of the nozzle is known, this gives the pumping horse-power; if the height to which the jet rises is given, we must first calculate v_o, and then d_o, by the equations already given.

The horse-power given to the pump is greater than the above on account of losses in the pump, and the indicated horse-power of the engine is greater than that given to the pump on account of the mechanical losses in the engine.

§ 29. **Effect of varying Diameter of Nozzle.**—For a given nozzle, d_o is known, and the discharge in cubic feet per second (Q) so that the

$$\text{pumping horse-power} = \frac{8\sigma Q^3}{550g\pi^2}\left(\frac{1}{d_o^4} + \frac{4fl}{d^5}\right).$$

The velocity of efflux is then given by

$$\frac{\pi}{4}d_o^2 v_o = Q$$

and thus v_o is determined. The theoretical lift is $\frac{v_o^2}{2g}$, and the actual lift is $\frac{3}{4} \cdot \frac{v_o^2}{2g}$. Thus the actual lift is determined.

By taking different sized nozzles, the different quantities may be plotted on a base representing the diameter of the nozzle. The

calculations are shown in the accompanying table. The value of f has been taken as 0·15.

Suppose a pump gives out, say, 100 horse-power, and that it pumps water through 500 feet of 3-inch pipe. Let nozzles of varying diameters be placed at the end of this pipe. According to D'Arcy's formula, for a 3-inch pipe—clean pipe—it ought to be 0·00655; or for an encrusted pipe, 0·013. So far as pipe friction is concerned, we have, therefore, a slight margin.

d_o in inches	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	3
Q in cubic feet per second	$\frac{1}{13·46}$	$\frac{1}{5·35}$	$\frac{1}{2·182}$	$\frac{1}{1·402}$	$\frac{1}{1·139}$	$\frac{1}{1·00}$	$\frac{1}{0·972}$	$\frac{1}{0·962}$
G in gallons per minute	27·9	70·1	171·8	267·5	329·5	375·0	386·0	390·0
v_o in feet per second .	873	549	336	232·5	161·5	81·6	47·2	21·2
$\frac{3}{4}\frac{v_o^2}{2g}$ in feet	8865	3570	1316	630	304	76·6	26·0	5·25

When plotted, the general character of the curves is shown in Fig. 37.

Apparent Paradox.—It will be noticed that for nozzle diameters greater than $1\frac{1}{2}$ inch about, the discharge remains sensibly constant. In other words, for a long pipe, the quantity of water which flows through a full bore is only slightly greater than if the end were blanked, and a hole one quarter the area bored in the flange. This apparent paradox might have been anticipated from the general formula

$$h = \frac{8Q^2}{g\pi^2}\left(\frac{1}{d_o^4} + \frac{4fl}{d^5}\right).$$

For if d_o be a considerable fraction of d, the second term is large compared to the first, and the expression approximately becomes

$$h = \frac{32flQ^2}{9\pi^2 d^5}.$$

Hence, Q is independent of the nozzle diameter.

§ **30. Hydraulic Transmission of Power.**—An important application of the preceding principles is the hydraulic transmission of

power. In such cases the head is principally a pressure head, the velocity of flow being purposely kept low, in order to reduce the frictional losses to a minimum. Usually the velocity of flow varies from 3 to 6 feet per second.

Let p be the pressure in pounds, per square inch, at the high-

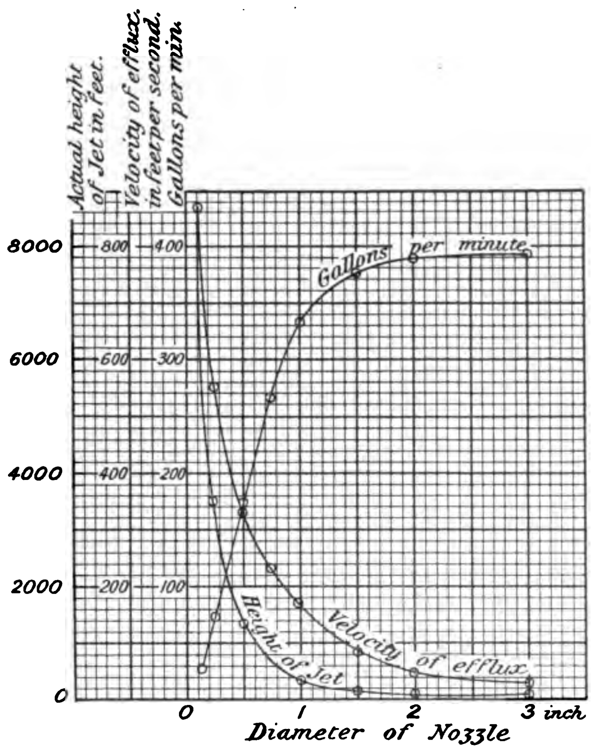


FIG. 37.

pressure end of the main, so that the available "head" in feet of water is $\frac{144p}{\sigma}$. This is expended in producing the flow, in overcoming resistances at the exit and entrance to the main, and in overcoming the resistance in the main itself. In a long main, the first two items are small, and the third only need be considered.

Thus, if the speed of flow is v feet per second the loss of head in pipe friction is

$$f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g}$$

and the available head at the motor is, therefore,

$$\frac{144p}{\sigma} - f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g}.$$

The efficiency of transmission is, therefore,

$$\frac{\frac{144p}{\sigma} - f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g}}{\frac{144p}{\sigma}} = 1 - \frac{\sigma fl}{72pgd} \cdot v^2.$$

This is zero if $v = \sqrt{\frac{72pgd}{\sigma fl}}$, which gives the speed of flow at which all the head is used upon overcoming friction, and none is available at the motor.

Again, the discharge in cubic feet per second is $\frac{\pi}{4}d^2v$, and therefore the horse-power transmitted from the high-pressure end,

$$= H_1, \text{ say} = \frac{144p \cdot \frac{\pi}{4}d^2v}{550} = \frac{pd^2}{4 \cdot 87}v$$

since the horse-power is equal to the pressure multiplied by the volume per second and divided by 550.

The horse-power wasted in friction is

$$\left(f\frac{4l}{d} \cdot \frac{v^2}{2g}\right)\left(w\frac{\pi}{4}d^2v\right)\frac{1}{550} = \frac{fld}{180 \cdot 2} \cdot v^3.$$

Substituting v in terms H_1, the horse-power lost

$$= \frac{fld}{180 \cdot 2} \times \left(\frac{4 \cdot 87}{pd^2}\right)^3 H_1^3$$

$$= 0 \cdot 64 \frac{flH_1^3}{p^3d^5}.$$

The horse-power delivered to the motor is

$$H_2 = H_1 - 0 \cdot 64 f \frac{lH_1^3}{p^3d^5}$$

and the efficiency of transmission is

$$\frac{H_2}{H_1} = 1 - 0{\cdot}64 f \frac{l H_1^2}{p^3 d^5}.$$

It will be noticed that the horse-power delivered to the motor is zero when $H_1 = 0$ (*i.e.* when no energy is transmitted) and when

$$H_1 = \sqrt[2]{\frac{p^3 d^5}{0{\cdot}64 f l}}.$$

When the horse-power sent out is equal to this expression, the velocity is such that all the energy is expended in overcoming the frictional losses. The energy delivered to the motor is a maximum when

$$\frac{\delta H_2}{\delta H_1} = 0$$

that is, when $$H_1 = \sqrt{\frac{p^3 d^5}{1{\cdot}92 f l}}.$$

When delivering maximum horse-power, the efficiency of transmission is two-thirds. Thus, then, as the velocity of flow increases, the horse-power sent out from the high-pressure end increases, and so also does the horse-power expended in overcoming friction. But the first varies as v, whilst the second varies as v^3. The second term becomes therefore increasingly important, and at a certain speed the whole of the energy sent out is used to overcome friction. At some intermediate speed the horse-power delivered to the motor is a maximum.

§ 31. Curves of Motor Horse-power and Efficiency. — These results will be better illustrated by taking a numerical example and plotting certain curves. *Thus, suppose the pressure at the high-pressure end is* 750 *pounds per square inch, the diameter of the pipe* 4 *inches, and the length of pipe* 1 *mile. Take* $f = 0{\cdot}00625$ *for a clean pipe.* Then, substituting in the formula

$$H_1 = \frac{p d^2}{4{\cdot}87} v$$

the equation is

$$v = 0{\cdot}0585 \text{ H}_1$$

and from the formula

$$\text{H}_2 = \text{H}_1 - 0{\cdot}64 f \frac{l\text{H}_1^3}{p^3 d^5}$$

$$\text{H}_2 = \text{H}_1 - 0{\cdot}00001218\text{H}_1^3.$$

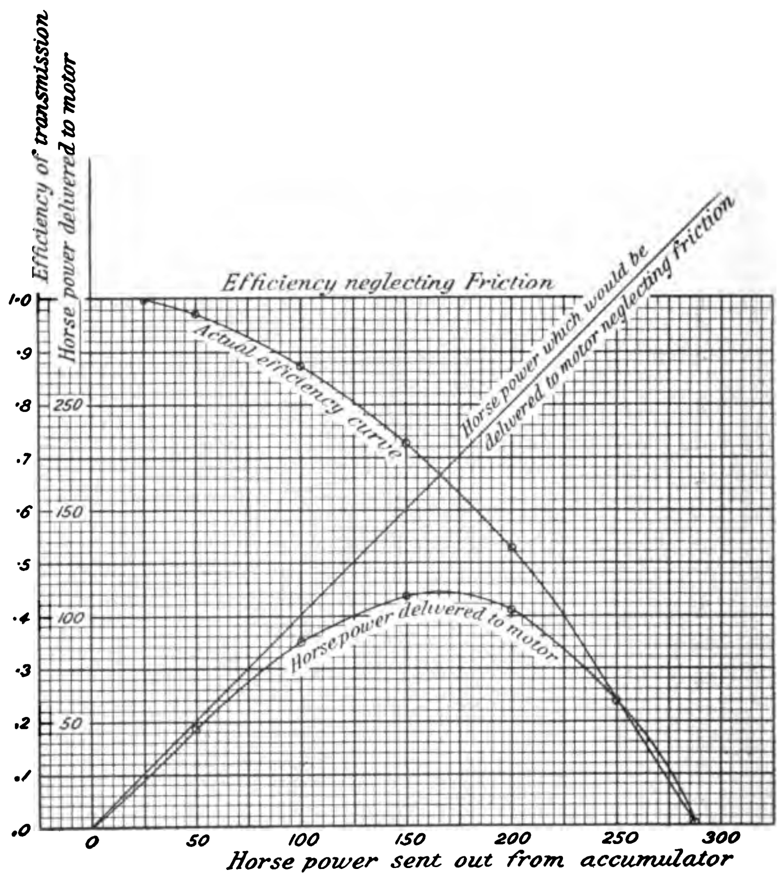


Fig. 38.

The efficiency is equal to H_2/H_1, that is, to

$$1 - 0{\cdot}00001218\text{H}_1^2.$$

Now, give H_1 different values, and estimate the values of

v, H_2, and efficiency. The results are given in the following table:

H_1	25	50	100	150	200	250	286·9
v	1·462	9·924	5·85	8·78	11·7	14·62	16·8
H_2	24·81	48·48	87·82	108·9	102·6	60	0
Efficiency . .	0·994	0·97	0·878	0·725	0·523	0·240	0

If these results be plotted on base of H_1, the general nature of the curves is as shown in Fig. 38.

Neglecting friction, $H_2 = H_1$, and the curve of H_2 would be a straight line inclined at 45°. With friction, it first follows the straight line, then bends over, reaches a maximum, begins to descend and becomes zero again, when $H_1 = 286{\cdot}9$. When the power delivered is a maximum, the power sent out is 165·5 and the power delivered 110·3. The curve of efficiency is a parabola.

This illustration shows that if the load on a hydraulic motor is thrown off, the speed of the motor will not indefinitely increase, but can only reach a certain limit; since, when this limit is reached, all the power is absorbed in friction. The frictional resistance, therefore, acts the part of a safety brake to a hydraulic motor.

§ **32. Numerical Illustration.**—As a further illustration of the foregoing take the following:—*The diameter of the ram of an accumulator loaded with* 100 *tons is* 19·6 *inches and* 48 *horse-power are delivered to the motor through* 2000 *yards of* 4-*inch pipe. Find the efficiency of transmission, taking* $f = 0{\cdot}007$.

Here the pressure at the high-pressure end is

$$p = \frac{100 \times 2240}{\frac{\pi}{4} \times 19{\cdot}6^2} = 742 \text{ pounds per square inch.}$$

The horse-power sent out from the accumulator is

$$\frac{144p \cdot \frac{\pi}{4}d^2v}{550} = \frac{144 \times 742 \times \frac{\pi}{4} + \left(\frac{1}{3}\right)^2 \times v}{550} = 17v.$$

The horse-power lost in friction

$$= \left(f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g}\right)\left(\sigma \frac{\pi}{4} d^2 v\right)\frac{1}{550}$$

$$= \frac{0{\cdot}007 \times 4 \times 6000}{\frac{1}{3}} \times \frac{v^2}{64{\cdot}4} \times 62{\cdot}5 \times \frac{\pi}{4} \times \left(\frac{1}{3}\right)^2 v \frac{1}{550}$$

$$= 0{\cdot}0777 v^3.$$

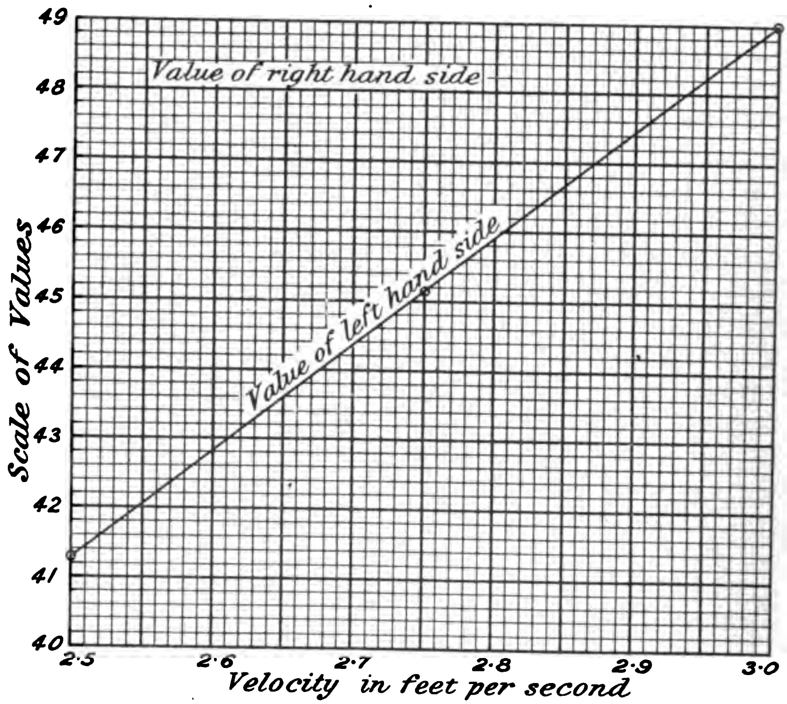


FIG. 39.

The horse-power given to the motor is, therefore,

$$17v - 0\ 0777v^3 = 48.$$

This is a cubic in v. It may be solved by the ordinary approximate methods or, better, by plotting (Fig. 39). The method about to be described is applicable to the solution of other equations besides cubics. Obtain, first, a rough solution by

considering the physical nature of the problem. The horse-power sent out is $17v$, and is somewhat greater than the horse-power delivered, namely, 48. Thus a first value of v may be taken as 3. Assume, then, $v = 3$, and calculate the value of the left-hand side of the equation: it is

$$17 \times 3 - 0{\cdot}0777 \times 3^2 = 51 - 2{\cdot}1 = 48{\cdot}9$$

and is, therefore, greater than the right-hand side, namely, 48. As a second value take $v = 2{\cdot}5$, and the left-hand side is

$$42{\cdot}5 - 1{\cdot}21 = 41{\cdot}29$$

which is less than 48. Take $v = 2{\cdot}75$, and the left-hand is

$$46{\cdot}8 - 1{\cdot}61 = 45{\cdot}19.$$

To obtain the exact value of v, plot the values of the left- and right-hand sides on a v base (Fig. 39). The correct value is at the intersection of the curves, and $v = 2{\cdot}94$. Thus the horse-power sent out from the accumulator is about $50v$, and the efficiency of transmission is 0·960.

§ 33. **Eddy Losses.**—The only loss so far considered has been what is usually termed "pipe friction." There are other losses in a hydraulic system, such as losses due to a partially closed valve, and to elbows and bends, etc. These losses can only, for the most part, be determined from experiment; but, in one particular case, the problem lends itself to theoretical treatment.

Suppose a pipe suddenly changes section, as shown in Fig. 40. Experiments show that, due to the sudden enlargement, eddies are formed, which move on with the fluid, and are ultimately absorbed by viscosity, their disappearance causing a loss of pressure. The energy necessary to create them reappears as heat, which, from a hydraulic point of view, is unavailable.

Energy Equation.—If the change of section takes place gradually, the relation between pressure and velocity is given by Bernouilli's equation; so that in a horizontal pipe, if (1) and (2) refer to the two sections marked,

$$\frac{p_1}{\sigma} + \frac{v_1^2}{2g} = \frac{p^2}{\sigma} + \frac{v_2^2}{2g}.$$

This equation is only true provided the fluid is perfect, and no eddies exist. If eddies exist, the energy equation cannot be applied, because the energy of the eddies must be included; and these cannot be calculated. When eddies are present, the *momentum* equation can be applied.

Momentum Equation.—It is important to distinguish between the energy equation and momentum equation. The first is the effect of a force acting over a certain distance; the second the effect of force acting over a certain interval of time. If P be force, x distance, v velocity, m mass, the expression of the first is $Px = \frac{1}{2}mv^2$, and of the second $Pt = mv$. When work is done on a number of bodies, or on a number of particles of the same body, work is spent in two ways, (1) increasing the kinetic energy of the system, (2) in overcoming mutual actions. In estimating each of these, no question of direction enters into the problem. In rigid bodies, the value of the second item is zero; but, in fluids, losses in friction and shock constitute a considerable fraction of the total energy necessary to produce a certain change. The energy thus necessary is to a great extent visible as rotating eddies or whirlpools of water.

In applying the second principle the forces act in a particular direction. "Change of momentum is proportional to the impressed force, and takes place in the direction in which the force acts." Thus, it is only necessary to know the motion of a body, or particles of a body, in the direction of force. If the accumulated momentum of a system has changed, a force must have operated to produce that change. In applying this principle, care must be taken to consider *all* the forces which act on the system, and not merely those which do work, as in the principle of work. Such forces are, for example, the pressures on the walls of a pipe or tube. It might happen that the kinetic energy of a system is great, whilst its momentum is very small; as, for example, a whirlpool or eddy. In estimating the *energy* of a moving mass of water, the energy of the eddies, as well as the translational energy of the whole mass, must be included; but in estimating the *momentum* it is not necessary to include the eddies, because their momentum in

every direction is zero. Thus, in many cases, the momentum equation may be used when the information or data do not permit the use of the energy equation; but, in some cases, either equation may equally well be used.

§ 34. **Loss of Head due to a Sudden Enlargement.**—Let, referring to Fig. 40, the area of the small pipe be a_1 and of the large pipe be a_2; and let the flow along either pipe be Q cubic feet per second. The momentum per second in the water flowing past the section a_1 is

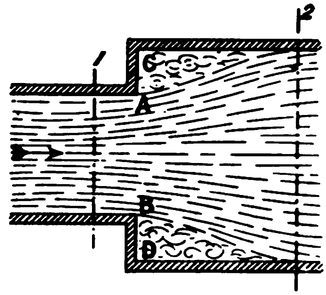


FIG. 40.

$$\frac{\sigma Q v_1}{g}$$

and in passing the section a_2 is

$$\frac{\sigma Q v_2}{g}$$

Thus the change of momentum per second is

$$\frac{\sigma Q}{g}(v_2 - v_1).$$

This is true notwithstanding the formation of eddies, because the momentum of an eddy is zero; and it must be equal to the total force in the direction of flow. The total force is made up of three components, namely, (1) the force $p_1 a_1$ acting over the area a_1 where p_1 is the pressure at a_1; (2) the force $- p_2 a_2$ at section a_2; (3) the force over the walls AC, BD. The pressures on the sides of the pipe have no component in the direction of motion. Hence, by the momentum equation,

$$p_1 a_1 - p_2 a_2 + \text{thrust over AC and BD} = \frac{\sigma Q}{g}(v_2 - v_1).$$

The thrust over the end walls AC, BD is uncertain. The usual assumption is that over AC, BD the pressure is the same

as at a section of the small pipe. Over the end walls the force is, therefore,

$$p_1(a_2 - a_1)$$

and so the momentum equation becomes

$$p_1a_1 - p_2a_2 + p_1(a_2 - a_1) = \frac{\sigma a_2 v_2}{g}(v_2 - v_1)$$

since $$Q = a_2v_2$$

whence $$\frac{p_1 - p_2}{\sigma} = \frac{v_2(v_2 - v_1)}{g}.$$

Thus the pressure head at the large section is

$$\frac{p_2}{\sigma} = \frac{p_1}{\sigma} - \frac{v_2(v_2 - v_1)}{g}.$$

If no eddies were formed, the pressures would be given by Bernouilli's equation, and the pressure head in the large pipe would be

$$\frac{p_1}{\sigma} - \frac{v_2^2 - v_1^2}{2g}.$$

The *loss* of pressure head due to the formation of eddies is, therefore, the difference of these expressions, and is equal to

$$\frac{v_2(v_2 - v_1)}{g} - \frac{v_2^2 - v_1^2}{2g} = \frac{(v_1 - v_2)^2}{2g}.$$

Thus the loss of head due to a sudden enlargement is the head due to the relative velocity in the two sections of the pipe. This result is based upon the assumption that the pressure over the end plates is the same as that at a section of the smaller pipe; and, if this assumption is incorrect, so is the result. Experiment shows, however, that the assumption is justified (see table).

The loss of head here investigated is quite distinct from, and is in addition to, the loss of head due to friction. It may be written in the form

$$\frac{v_2^2}{2g}\left(\frac{a_2}{a_1} - 1\right)^2$$

where v_2 is the velocity in the larger pipe. A more general expression is

$$m\frac{v_2^2}{2g}$$

where $m = \left(\frac{a_2}{a_1} - 1\right)^2$ and depends, therefore, on the ratio of areas. For different ratio of diameters, the value of m is given in the following table:—

$\frac{d_2}{d_1}$	1	1·414	1·73	2·0	2·24	2·65	3·0	3·16
$\frac{a_2}{a_1}$	1	2	3	4	5	7	9	10
m	0	1·00	4·00	9·00	16·0	36·0	64·0	81·0

§ 35. **Contraction followed by an Enlargement.**—If the direction of flow be reversed, the stream first contracts and then expands to fill the small pipe (Fig. 41). At the corners C and D the stream lines follow the boundaries, but at A and B they separate. The stream in passing into the small pipe contracts and afterwards expands again. The contraction causes no serious loss, but the subsequent expansion forms a sudden enlargement; and the loss of head may be obtained by the above formula. If suffix (1) refer to the contracted section and (2) to the small pipe, then the loss of head

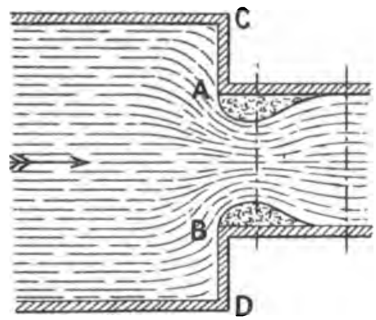


FIG. 41.

$$= \frac{(v_1 - v_2)^2}{2g} = \frac{v_2^2}{2g}\left(\frac{a_2}{a_1} - 1\right)^2.$$

The area a_2 is the sectional area of the small pipe; but a_1 depends on the area of the small pipe, and, also, on the area of the large pipe. If the diameter of the larger pipe is

great compared with that of the small pipe, experiment shows that the ratio of $\frac{a_2}{a_1}$ is 0·618.

§ **36. Obstruction in a Pipe.**—A common illustration of a sudden enlargement following a contraction is when an obstruction takes place in a pipe due to a diaphragm, sluice, throttle valve, or cock. In Fig. 42 there is a contraction followed by a sudden enlargement back to the original diameter of the pipe. Let a_2 be the sectional area of the pipe, a_o of the orifice, and a_1 of the contracted jet. The loss of head in enlarging from a_1 to a_2 is

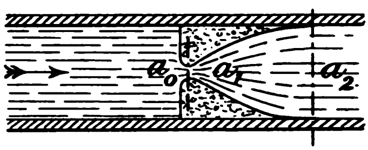


FIG. 42.

$$\frac{v_2^2}{2g}\left(\frac{a_2}{a_1} - 1\right)^2 = m\frac{v_2^2}{2g}, \text{ say.}$$

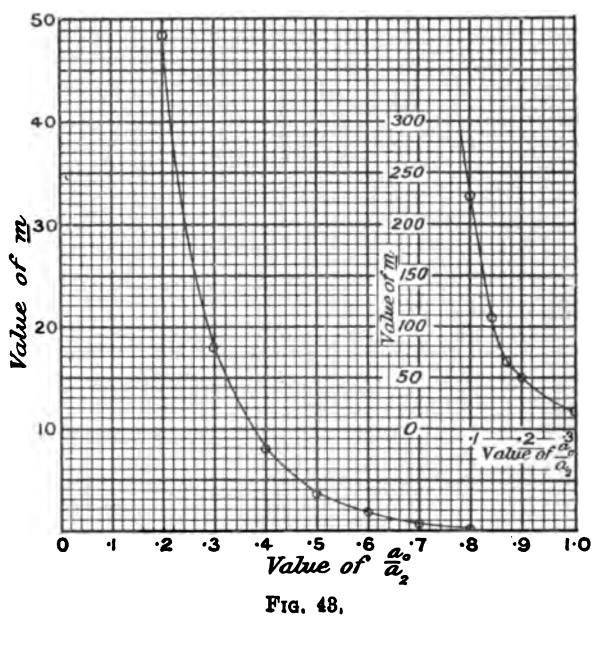


FIG. 43.

The value of $\frac{a_2}{a_1}$ will depend on $\frac{a_0}{a_2}$. Thus if $a_0 = a_2$, there is no obstruction, and $a_1 = a_2$; and if $\frac{a_0}{a_2}$ is very small, $\frac{a_1}{a_0} = 0.618$. For any other proportions, according to Rankine,

$$\frac{a_1}{a_0} = \frac{0.618}{\sqrt{1 - 0.618\frac{a_0^2}{a_2^2}}}.$$

Taking different values of the area ratio $\frac{a_0}{a_2}$, the corresponding value of $\frac{a_1}{a_0}$ can be estimated, and $\frac{a_2}{a_1}$ can be determined and m deduced.

The results are given in the following table:—

$\frac{a_0}{a_2}$	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
$\frac{a_1}{a_0}$	0·62	0·625	0·637	0·652	0·675	0·701	0·74	0·795	0·874	1·0
$\frac{a_2}{a_1}$	6·2	3·125	2·123	1·63	1·35	1·168	1·057	0·994	0·971	1·0
m	229	49·0	18·0	8·1	3·9	1·9	0·86	0·33	0·07	0·0

If these results be plotted on a base representing $\frac{a_0}{a_2}$, the general nature of the curve is shown in Fig. 43.

§ **37. Experimental Work.**—The experimental work on the subject is practically all due to Weisbach, who gave the results in the form of tables. The results so obtained agree very well with those in the previous table, which are deduced from Rankine's formula, except for the case of cocks. The table may be taken to apply, without modification, to sluices in rectangular and cylindrical pipes, and to throttle valves. These three cases are illustrated in Figs. 44–47.

For a cock in a cylindrical pipe (Fig. 47) the actual experimental

results appear to be from 75 per cent. to 100 per cent. greater than those given in the table.

Elbows.—Another source of loss, also due to an enlargement

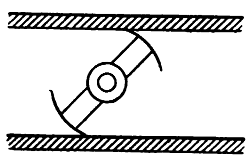


FIG. 44.

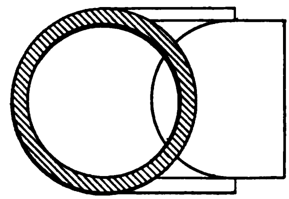


FIG. 45.

following a contraction, is in the case of elbows (Fig. 48). At the elbow there is a partial contraction and the water then fills the pipe again. The loss of head will depend on the ratio $\frac{a_2}{a_1}$, and this

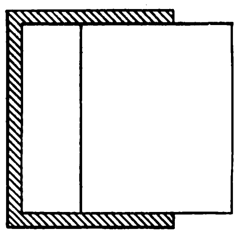


FIG. 46.

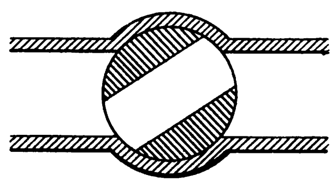


FIG. 47.

will depend on the angle θ. If v is the velocity of flow in feet per second, the loss of head due to this cause may be expressed by

$$m\frac{v^2}{2g}.$$

According to Weisbach, the values of m are given in the following table:—

=	20°	40°	60°	90°	120°	140°
m =	0·046	0·14	0·36	0·98	1·86	2·43

Bends.—A certain loss of head, quite apart from skin friction, takes place when water flows round a pipe bent to the form

of a circle. The loss of head depends on the diameter of the pipe, on the radius of the bend, and on the angle of the bend, and, as before, may be written $m\frac{v^2}{2g}$, where m depends on the three quantities stated. For a right-angled bend the experimental results give

$\frac{\text{Diameter of pipe}}{\text{Radius of bend}}$	0·2	0·6	1·0	1·2	1·6	2·0
m	0·06	0·08	0·14	0·22	0·49	1·0

and the value of m varies directly as the angle of bend.

§ **38. General Problems, Coefficient of Hydraulic Resistance.**— Thus, then, in dealing with the losses in a hydraulic system, the losses are: (1) skin friction, (2) partially closed valves, (3) elbows and bends. The first is, in general, the most important item; the third is usually so small as to be neglected. In any particular case of a hydraulic system, since all these losses vary as v^2, the total loss of head may be expressed in the form

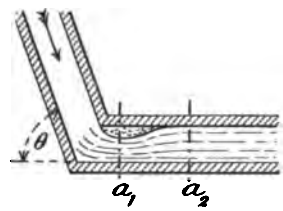


Fig. 48.

$$F\frac{v^2}{2g}$$

where F is some coefficient called the *coefficient of hydraulic resistance*, and which may be taken to be independent of speed. In the case of a straight, unobstructed pipe, so that skin friction is the only loss, the value of F is

$$f \cdot \frac{4l}{d}.$$

To compare the relative magnitudes of the various losses, consider a pipe 50 feet long and 3 inches diameter which delivers water in a cylinder 15 inches diameter, and let the velocity of the piston of the cylinder be 15 feet per minute. Suppose in the 50 feet

of pipe there are a half-closed valve, a right-angled elbow, and a right-angled bend of radius equal to the diameter of the pipe. Taking f for pipe friction to be 0·013 (double that given by D'Arcy), the different losses are as follows (in which v represents the velocity in the pipe):

(1) Skin friction $= \frac{4 \times 0{\cdot}013 \times 50}{\frac{1}{4}} \cdot \frac{v^2}{2g} = 10{\cdot}4\frac{v^2}{2g}.$

(2) Half-closed valve $= 3{\cdot}9\frac{v^2}{2g}.$

(3) Right-angled elbow $= 0{\cdot}98\frac{v^2}{2g}.$

(4) Right-angled bend of radius equal to diameter $= 0{\cdot}14\frac{v^2}{2g}.$

There is a fifth loss due to the water flowing out of a 3-inch pipe into a 15-inch cylinder. If v_2 be the velocity in the cylinder, the loss of head due to this cause is

$$\frac{v_2^2}{2g}\left(\frac{15^2}{3^2} - 1\right)^2 = \frac{v_2^2}{2g} \times 24.$$

Referred to the velocity in the supply pipe, the loss of head is

$$= 24 \times \left(\frac{3}{15}\right)^2 \times \frac{v^2}{2g} = \frac{24}{25}\frac{v^2}{2g} = 0{\cdot}96\frac{v^2}{2g}.$$

The total loss is

$$\Big(\underset{\text{Pipe friction.}}{\underset{\downarrow}{10{\cdot}4}} + \underset{\text{Valve.}}{\underset{\downarrow}{3{\cdot}9}} + \underset{\text{Elbow.}}{\underset{\downarrow}{0{\cdot}98}} + \underset{\text{Bend.}}{\underset{\downarrow}{0{\cdot}14}} + \underset{\text{Cylinder.}}{\underset{\downarrow}{0\ 96}}\Big)\frac{v^2}{2g}$$

$$= 16{\cdot}38\frac{v^2}{2g}$$

$$= 9{\cdot}95 \text{ feet, since } v = \frac{15}{60}\left(\frac{15}{3}\right)^2 = 6{\cdot}25 \text{ feet per second.}$$

For this particular case, therefore, the value of F, the coefficient of hydraulic resistance, is 16·38 when referred to the velocity in the supply pipe.

It must be evident that a precise determination of this coefficient F in any actual system is a matter of great difficulty. The internal state of the pipes, and the precise losses in valves of peculiar design, are not definitely known. The only satisfactory way is to make an experiment on the whole plant and so find the loss of head at a certain speed. At all other speeds, provided the state of the valves is unaltered, the coefficient of hydraulic resistance may be taken to be the same. A number of examples, as applied to pressure engines, which illustrate the principles laid down, are given in § 79.

§ **39. Branched Pipes.**—In a water-supply of a town there may be many storage reservoirs connected by one main. Thus, a single pipe running from one reservoir may be joined by others coming from other reservoirs. If so, the pipe is called a *branched pipe*. (Fig. 49.)

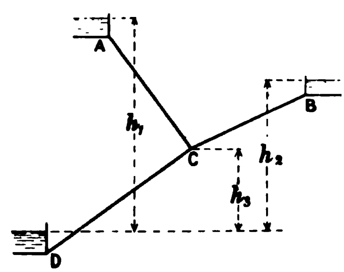


FIG. 49.

As an illustration, take three reservoirs, A, B, C, the discharge outlet being D (Fig. 49). Let suffixes 1, 2, 3, 4, refer to A, B, C, D; the heights about D being h_1, h_2, h_3; and let the pressure at C be p. Then clearly

$$h_1 - h_3 = \frac{p}{\sigma} + f \cdot \frac{4l_1}{d_1} \cdot \frac{v_1^2}{2g}$$

$$h_2 - h_3 = \frac{p}{\sigma} + f \cdot \frac{4l_2}{d_2} \cdot \frac{v_2^2}{2g}$$

$$h_3 + \frac{p}{\sigma} = f \cdot \frac{4l_3}{d_3} \cdot \frac{v_3^2}{2g}.$$

In addition

$$Q_1 = a_1v_1;\ Q_2 = a_2v_2;\ Q_3 = a_3v_3 = a_1v_1 + a_2v_2.$$

In these equations there are four unknown and four independent equations.

Illustration. Suppose

$$l_1 = 2000 \qquad d_1 = 2 \text{ inches} \qquad h_1 = 60$$
$$l_2 = 2500 \qquad d_2 = 3 \text{ inches} \qquad h_2 = 40$$
$$l_3 = 3000 \qquad d_3 = 4 \text{ inches} \qquad h_3 = 20$$
$$4f = 0{\cdot}03$$

$$40 = \frac{p}{\sigma} + \frac{f \,.\, 4}{2g} \times \frac{l_1 v_1^2}{d_1}$$

$$= \frac{p}{\sigma} + \frac{1}{2150} \times 2000 \times 6 \times v_1^2 = \frac{p}{\sigma} + 5{\cdot}59 v_1^2 \quad \ldots.(1)$$

$$20 = \frac{p}{\sigma} + \frac{1}{2150} \times 2500 \times 4 \times v_2^3 = \frac{p}{\sigma} + 4{\cdot}65 v_2^2 \quad \ldots.(2)$$

$$20 + \frac{p}{\sigma} = \frac{1}{2150} \times 3000 \times 3 v_3^2 = 4{\cdot}18 v_3^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$Q_1 = \frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} \times \frac{1}{36} \times v_1 = 0{\cdot}0218 v_1 \quad \ldots\ldots\ldots\ldots\ (4)$$

$$Q_2 = \frac{\pi}{4} d_2^2 v_2 = \frac{\pi}{4} \times \frac{1}{16} \times v_2 = 0{\cdot}049 v_2 \quad \ldots\ldots\ldots\ldots\ldots(5)$$

$$Q_3 = \frac{\pi}{4} d_3^2 v_3 = \frac{\pi}{4} \times \frac{1}{9} \times v_3 = 0{\cdot}0873 v_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$0{\cdot}0873 v_3 = 0{\cdot}049 v_2 + 0{\cdot}0218 v_1$$
$$v_3 = 0{\cdot}561\, v_2 + 0{\cdot}25 v_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Subtract (2) from (1)

$$20 = 5{\cdot}59 v_1^2 - 4{\cdot}65 v_2^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Add (2) and (3)

$$40 = 4{\cdot}65 v_2^2 + 4{\cdot}18 v_3^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Also from equation (8)

$$v_2^2 = 1{\cdot}2 v_1^2 - 4{\cdot}3 \quad \ldots\ldots\ldots\ \ldots..(10)$$

and from equation (9)

$$v_3^2 = 9{\cdot}58 - 1{\cdot}112 v_2^2$$
$$= 9{\cdot}58 - 1{\cdot}112(1{\cdot}2 v_1^2 - 4{\cdot}3) \quad \ldots\text{from } (10)$$
$$= 14{\cdot}37 - 1{\cdot}335 v_1^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

from (7)

$$
\begin{aligned}
v_3^2 &= (0{\cdot}561v_2 + 0{\cdot}25v_1)^2 \\
&= 0{\cdot}315v_2^2 + 0{\cdot}0625v_1^2 + 0{\cdot}28v_1v_2 \\
&= 0{\cdot}315(1{\cdot}2v_1^2 - 4{\cdot}3) + 0{\cdot}0625v_1^2 + 0{\cdot}28v_1\sqrt{1{\cdot}2v_1^2 - 4{\cdot}3}
\end{aligned}
$$

or

$$
\begin{aligned}
-1{\cdot}775v_1^2 + 15{\cdot}7 &= 0{\cdot}28v_1\sqrt{1{\cdot}2v_1^2 - 4{\cdot}3} \\
v_1^2 - 8{\cdot}85 &= -0{\cdot}158v_1\sqrt{1{\cdot}2v_1^2 - 4{\cdot}3} \quad \ldots\ldots(12) \\
v_1^4 - 17{\cdot}7v_1^2 + 78{\cdot}3 &= 0\,025v_1^2(1{\cdot}2v_1^2 - 4{\cdot}3) \\
&= 0{\cdot}03v_1^4 - 0{\cdot}1075v_1^2 \\
\therefore\ 0{\cdot}47v_1^4 - 17{\cdot}7v_1^2 &= -78{\cdot}3 \\
v_1^4 - 18{\cdot}28v_1^2 &= -80{\cdot}8 \\
(v_1^2 - 9{\cdot}14)^2 &= -80{\cdot}8 + 83{\cdot}7 = 2{\cdot}7 \\
v_1^2 - 9{\cdot}14 &= -1\,64 \\
v_1^2 &= 10{\cdot}79 \quad \text{or} \quad 7{\cdot}50 \\
v_1 &= 3{\cdot}28 \quad \text{or} \quad 2{\cdot}74
\end{aligned}
$$

When the first value of v_1 is substituted in equation (11), the second solution is the one to take, as might have been inferred from the negative sign attached to equation (12).

Thus, finally

$$
\begin{aligned}
v_1 &= 2{\cdot}74 \text{ feet per second} \\
v_2 &= \sqrt{1{\cdot}2v_1^2 - 4{\cdot}3} = 2{\cdot}17 \text{ feet per second} \\
v_3 &= \sqrt{14{\cdot}37 - 1{\cdot}335v_1^2} = 2{\cdot}09 \text{ feet per second.}
\end{aligned}
$$

Thus

$$
\begin{aligned}
Q_1 &= \frac{\pi}{4} + \frac{1}{36} \times 2{\cdot}74 = 0{\cdot}06 \text{ cubic feet per second} \\
Q_2 &= \frac{\pi}{4} + \frac{1}{16} \times 2{\cdot}17 = 0{\cdot}1062 \quad ,, \quad ,, \\
Q_3 &= \frac{\pi}{4} + \frac{1}{9} \times 2{\cdot}59 = 0{\cdot}182 \quad ,, \quad ,,
\end{aligned}
$$

The sum of the first two, namely, 0·166 ought to be equal to the second. But on account of the great amount of arithmetical and algebraic analysis, the agreement is as near as can be expected.

Finally,

From (3) $\frac{p}{\sigma} = 4{\cdot}18v_3^2 - 20 = -1{\cdot}7$ feet so that there is a small negative pressure at the junction-box.

Expressed in gallons per hour, the three pipes discharge 13,500, 24,000, 41,000.

§ **40. Projecting Mouthpiece.**—An interesting illustration is the projecting mouthpiece, already noticed in § 8, Fig. 12. If a_2 be the area of the pipe, and a_1 the area at the contracted part, the stream contracts and no appreciable loss takes place. But between sections a_1 and a_2 there is a sudden enlargement, and thus the loss of head is

$$\frac{v_2^2}{2g}\left(\frac{a_2}{a_1} - 1\right)^2$$

whence
$$h = \frac{v_2^2}{2g}\left(1 + \overline{\frac{a_2}{a_1} - 1}^2\right).$$

If the tank be large compared to the area of the orifice,

$$\frac{a_1}{a_2} = 0{\cdot}618$$

in which case
$$h = 1{\cdot}383\frac{v_2^2}{2g}$$

$$\therefore \frac{v_2^2}{2gh} = \frac{1}{1{\cdot}383} = 0{\cdot}72$$

$$\therefore c_v = 0{\cdot}85.$$

The value given in § 8 is 0·82, the difference being due to other slight losses. The difference of pressure between sections a_2 and a_1 is

$$v_2\frac{(v_1 - v_2)}{g} = \frac{v_2^2}{g}\left(\frac{a_2}{a_1} - 1\right)$$
$$= 0{\cdot}62\frac{v_2^2}{g}$$
$$= 0{\cdot}62 \text{ and } 1{\cdot}44h$$
$$= 0{\cdot}89h.$$

Thus, so long as a suction tank is placed at a less depth below the pipe, water will be pumped up and flow along the pipe.

§ **41. Jet Propeller.**—A more detailed analysis of the jet propeller is as follows: In Fig. 50 A is the suction pipe, B the supply, D the nozzle, and C the delivery. Fig. 51 is a section through the suction pipe. Let h_1, h_2 be the falling and suction heads, v_1, v_2, and v the velocities at the sections marked, namely B, A, and B, at which the areas are a_1, a_2, a_3. Let p be the absolute pressure in the mixing chamber, p_0 the absolute atmospheric pressure.

Then, neglecting frictional losses

$$\frac{p}{\sigma} + \frac{v_1^2}{2g} = \frac{p_0}{\sigma} + h_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{p}{\sigma} + \frac{v_2^2}{2g} = \frac{p_0}{\sigma} - h_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Let Q_1 be the discharge from the tank, and Q_2 the amount pumped up. Then $v_1a_1 = Q_1$, $v_2a_2 = Q_2$, $v_1a_1 + v_2a_2 = Q_1 + Q_2 = Q$, Q being the quantity carried away by the discharge pipe.

In the mixing chamber, Q_1 cubic feet per second is suddenly

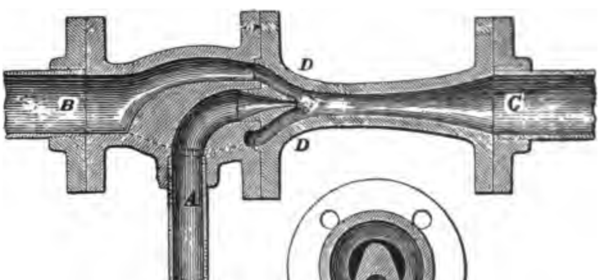


Fig. 50.

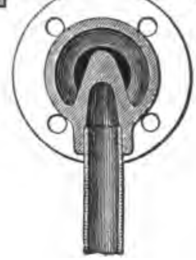


FIG. 51.

changed from v_1 to v; and Q_2 cubic feet per second from v_2 to v. The losses, per second, in foot pounds are

$$\frac{\sigma Q_1(v_1 - v)^2}{2g} \text{ and } \frac{\sigma Q_2(v_2 - v)^2}{2g}$$

respectively. Hence, assuming the discharge is into atmosphere

$$\sigma Q_1 h_1 = \sigma Q_2 h_2 + \sigma Q_1 \frac{(v_1 - v)^2}{2g} + \sigma Q_2 \frac{(v_2 - v)^2}{2g} + \frac{\sigma(Q_1 + Q)v^2}{2g}$$

or
$$Q_1\left(h_1 - \frac{v_1^2}{2g} + v\frac{v_1 - v}{g}\right) = Q_2\left(h_2 + \frac{v_2^2}{2g} - v\frac{v_2 - v}{g}\right)$$

and
$$\eta = \frac{Q_2 h_2}{Q_1 h_1}.$$

These four equations are sufficient to determine the three velocities v_1, v_2, v, and the pressure p.

Flow in Rivers and Canals.

§ **42. Chezy - D'Arcy - Bazin Formula.** — In dealing with the frictional resistance of the flow in pipes, it has been shown (§ 38) that the loss of head is

$$\propto \frac{s}{A} \cdot \frac{v^2}{2g}$$

which may be written $h = f \cdot \frac{s}{A} \cdot \frac{v^2}{2g}$.

This formula is true for not only pipes running full, but also for pipes running partially full, and for canals and rivers. It may be expressed as follows: If P be the wetted perimeter, and l be the length of wetted surface, then

$$s = \mathrm{P}l$$

and
$$h = f \cdot \frac{\mathrm{P}l}{\mathrm{A}} \cdot \frac{v^2}{2g}.$$

The ratio $\frac{\mathrm{A}}{\mathrm{P}}$ has the dimensions of a length, and it is called the *hydraulic mean depth*. Thus, the formula becomes—

$$h = \frac{fl}{m} \cdot \frac{v^2}{2g}.$$

If i be the slope per foot run of the river or canal bed,

$$v^2 = mi \cdot \frac{2g}{f}.$$

Since $\frac{2g}{f}$ is a constant, the equation reduces to

$$v = c\sqrt{mi}$$

in which $c = \sqrt{\frac{2g}{f}}$; and $\mathrm{Q} = \mathrm{A}v = c\sqrt{\frac{\mathrm{A}^3 i}{\mathrm{P}}}$.

This is *Chezy's Formula*, and enables the velocity to be calculated when the slope and dimensions of the channel are known.

The value of c depends on the dimensions and roughness of the channel. The co-efficient can be represented by a formula of the type

$$f = \alpha\left(1 + \frac{\beta}{m}\right)$$

where α and β are empirical coefficients. In different kinds of channels, the values of the coefficient, according to D'Arcy and Bazin are -

Kind of Channel.	α	β
I. Very smooth channel, sides of smooth cement or planed timber	0·00316	0·1
II. Smooth channels, sides of ashlar, brickwork, planks	0·00401	0·23
III. Rough channels, sides of rubble masonry or pitched with stone	0·00507	0·82
IV. Very rough channels in earth	0·00592	4·10
V. Torrential streams encumbered with detritus	0·00846	8·2

The coefficient also varies with velocity. Weisbach suggests the formula

$$f = \beta\left(1 + \frac{\delta}{v}\right)$$

in which $f = \cdot00709$ and $\beta = 0\cdot1920$.

§ **43. Ganguillet - Kulter Formula.** — Ogani Ganguillet and Kulter, in Switzerland, obtained a formula

$$c = \frac{a}{\frac{\beta}{\sqrt{m}}}$$

or, inverting,

$$\frac{1}{c} = \frac{1}{a} + \frac{\beta}{a\sqrt{m}}$$

an equation to a straight line having $\frac{1}{\sqrt{m}}$ for abscissa, $\frac{1}{c}$ for ordinate, and inclined to the axis of elevation at an angle of $\tan^{-1}\frac{\beta}{a}$. If the experimental values of $\frac{1}{c}$ and $\frac{1}{\sqrt{m}}$ are plotted,

they lie on a curve rather than a straight, showing that β must depend on a. After much comparison, the following form was arrived at

$$c = \frac{A + \frac{l}{m}}{1 + \frac{An}{\sqrt{m}}}.$$

Where n is a coefficient depending only on the roughness of the channel, and A and l are new coefficients, the value of which remains to be determined. The coefficient c, as already pointed out, depends on the inclination of the stream, decreasing as the slope inceases.

Let $$A = a + \frac{p}{i}$$

then $$c = \frac{a + \frac{l}{n} + \frac{p}{i}}{1 + \left(a + \frac{p}{i}\right)\frac{n}{\sqrt{m}}}$$

the form of the expression ultimately adopted by Ganguillet and Kulter for the constants a, l, p, they obtained the values 41·6, 1·811 and 0·00281 in English units. The coefficient of roughness n is found to vary from 0·008 to 0·050.

The most practically useful valves of the coefficient of roughness n—

Nature of Sides of Channel.	n.
Well-planed timber	0·009
Cement plaster	0·010
Plaster of cement with one-third sand	0·011
Unplaned planks	0·012
Ashlar and brickwork	0·013
Canvas on frames	0·015
Rubble masonry	0·017
Canals in firm ground	0·02
Rivers and canals in perfect order, free from stones or weeds	0·025
Rivers and canals in moderately good order, not quite free from stones and weeds.	0·030
Rivers and canals in bad order, with weeds and detritus	0·035
Torrential streams encumbered with detritus.	0·050

§ **44. Pipes running Partially Full.**—One or two interesting problems may be noticed—

First. *In a trapezoidal section, to find the shape of section which will cost a minimum on excavation and pitching* (Fig. 52).

Here $$A = sd^2 + 2bd$$
$$P = 2b + 2d\sqrt{1 + s^2}$$

$2b$ being the width of the water, d the depth, s the slope of the sides.

Using the formula of § 41, for a given discharge and slope A^3 and P, and the greater A, the greater P. If A is a minimum

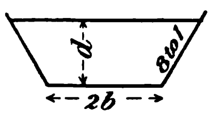


FIG. 52.

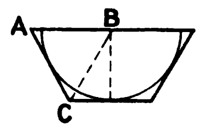


FIG. 53.

so is P. To find the proportions, differentiate A and P with respect to d and get

$$A = sd^2 + d(P - 2d\sqrt{1 + s^2})$$
$$\frac{\delta A}{\delta d} = 2sd + P - 4d\sqrt{1 + s^2} + d\frac{\delta P}{\delta d}$$
$$\therefore P = 4d\sqrt{1 + s^2} - 2sd$$

and also $$= 2b + 2d\sqrt{1 + s^2}$$
$$\text{i.e. } 2d\sqrt{1 + s^2} = 2(b + sd)$$
$$\text{and } AC = CB$$

or a circle with B as centre describes a circle touching the two sides and bottom (Fig. 53).

The width $$b = d\{\sqrt{1 + s^2} - s\}$$

and $$P = 2d\{2\sqrt{1 + s^2} - s\}$$
$$A = d^2(2\sqrt{1 + s^2} - s)$$

The value of $m = \frac{d}{2}$, and d is determined from

$$Q = c\sqrt{\frac{d^5}{2}\, 2\sqrt{1 + s^2} - s\}^2 i}.$$

§ 45. Circular Pipe running Partially Full.—

Here
$$P = \frac{d}{2}\theta$$

$$A = \frac{1}{8}d^2(\theta - \sin\theta)$$

whence
$$Q = c\sqrt{\frac{d^5}{256}\frac{(\theta - \sin\theta)^3}{\theta}i}$$

knowing θ, i, d, Q is determined. The only way of solving the equation is to adopt some graphical method. The work may, however, be simplified by drawing up a table. The formulæ are

$$v = \frac{Q}{A} \text{ and } v^3 = c^2\frac{Qi}{P}.$$

For a given value of θ, P and A are known, and therefore there are two equations to determine v.

If
$$P = \alpha d \text{ and } A = \beta d^2$$

in which

α	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8
β	0·00025	0·00137	0·0064	0·0105	0·020	0·034	0·052	0·075

α	0·9	1·0	1·2	1·4	1·6	1·8	2·0	2·5	3·0
β	0·103	0·136	0·203	0·307	0·406	0·505	0·594	0·745	0·785

As an example, suppose the channel is an open pipe 9 inches diameter, the discharge $\frac{1}{2}$ cubic foot per second, the slope 1 in 150.

Then $$v^3 = \frac{51}{a} \quad : \quad v = \frac{0 \cdot 89}{\beta}$$

To obtain a solution, corresponding values of a and β must be taken, and the values of v calculated.
Thus

when	$a = 1 \cdot 0$	$\beta = 0 \cdot 136$	$v = 3 \cdot 71$ and $6 \cdot 55$
	$a = 1 \cdot 2$	$\beta = 0 \cdot 203$	$v = 3 \cdot 49$ „ $4 \cdot 39$
	$a = 1 \cdot 4$	$\beta = 0 \cdot 307$	$v = 3 \cdot 32$ „ $2 \cdot 90$

Two perpendicular axes are taken, and the values of a are taken as abscissæ (Fig. 54), and the values of the velocity as

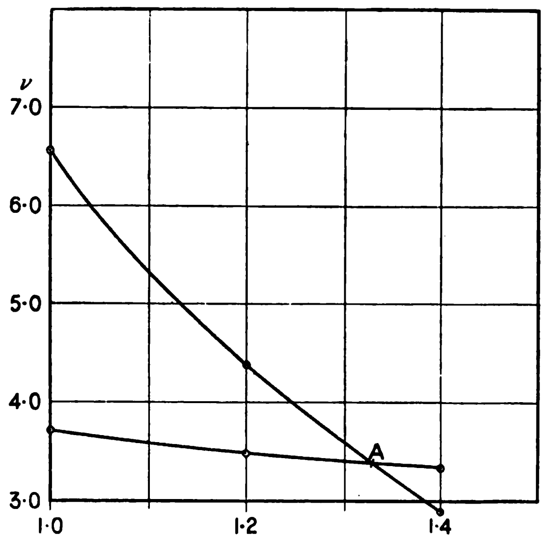


Fig. 54.

ordinates. The two curves cross at the point A, at which the velocity is $v = 3 \cdot 4$ feet per second. The corresponding value of $\theta = 151°$.

A further problem is to find, in a circular pipe, the angle which the water line subtends at the centre of the pipe. Using the formula just obtained when v is a maximum

$$\frac{\theta - \sin\theta}{\theta} \text{ is a maximum}$$

and

$$\therefore \theta = \tan\theta$$
$$\theta = 257° \, 37'.$$

In sewers, it is very essential that the sewers should be self-flushing, otherwise deposits will form and choke. In the case of small drains, of 6 to 9 inches diameter, the velocity ought not to be less than 3 feet per second; for larger pipes, 2 feet per second is sufficient, and for smaller pipes it ought to be increased.

In a circular pipe the discharge and velocity will not be constant. But in an egg-shaped sewer, except for small discharges, the velocity may be made sensibly constant for all discharges.

§ **46. Variation of Velocity at Different Points in a Stream.**— When a series of observations are taken at each depth and the results averaged, the mean velocities at each depth when plotted give a regular curve, agreeing very closely with the parabola. In experiments on the Mississippi the vertical velocity curve in calm weather was found to agree fairly with a parabola, the greatest velocity being at three-sixteenths the depth of the stream from the surface. With a wind blowing down stream the surface velocity is increased, and the axis of the parabola approaches the surface. On the contrary, with a wind blowing up stream the surface velocity is diminished, and the axis of the parabola is lowered, sometimes to half the depth of the stream (Fig. 55).

In the gaugings of the Mississippi the vertical velocity curve was found to agree well with a parabola having a horizontal axis at some distance below the water surface, the ordinate of the parabola at the axis being the maximum velocity of the section. During the gaugings the force of the wind was registered on a scale ranging from 0 for a calm to 10 for a hurricane. Arranging

the velocity curves in three sets—(1) with the wind blowing up stream, (2) with the wind blowing down stream, (3) calm or wind blowing across stream—it was found an up-stream wind lowered, and a down-stream wind raised, the axis of the parabolic velocity curve. In calm weather the axis was at three-sixteenths of the trial depth from the surface for all conditions of the stream.

§ **47. Venturi Meter.**—One method of measuring the quantity of water along a pipe is by means of a Venturi Meter[1] (§ 3). In cases where this is used to measure the flow, a recording apparatus is usually attached. In this article, it is stated that this particular meter was not invented by Venturi, but by Mr. Charles Herschel, the well-known hydraulic engineer.

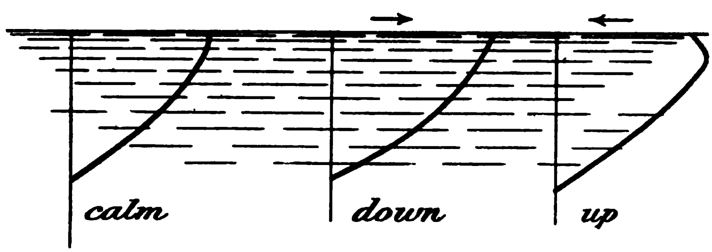


FIG. 55.

A section of the pipe forming the throat is shown in Fig. 56. The pipe is 5 feet diameter, and the throat 22½ inches diameter. At the throat the diameter was very accurately bored to the diameter mentioned, and through which are a series of small holes leading to an annular space formed in the main casting (Fig. 57). The pressure in this space is measured in the usual way. Similar arrangements for measuring this pressure are made at the outer end of the cones (Figs. 58, 59), and from a comparison of these pressures the velocity through the waist calculated. In practice, however, it is convenient to be able to read the quantity delivered through the throat direct to a dial, and to this end the pressure pipes from the meter are led to a recorder, in

[1] *Proceedings of the Institute of Mechanical Engineers*, 1900, i. 37.

which the recording dials are driven intermittently by clockwork,

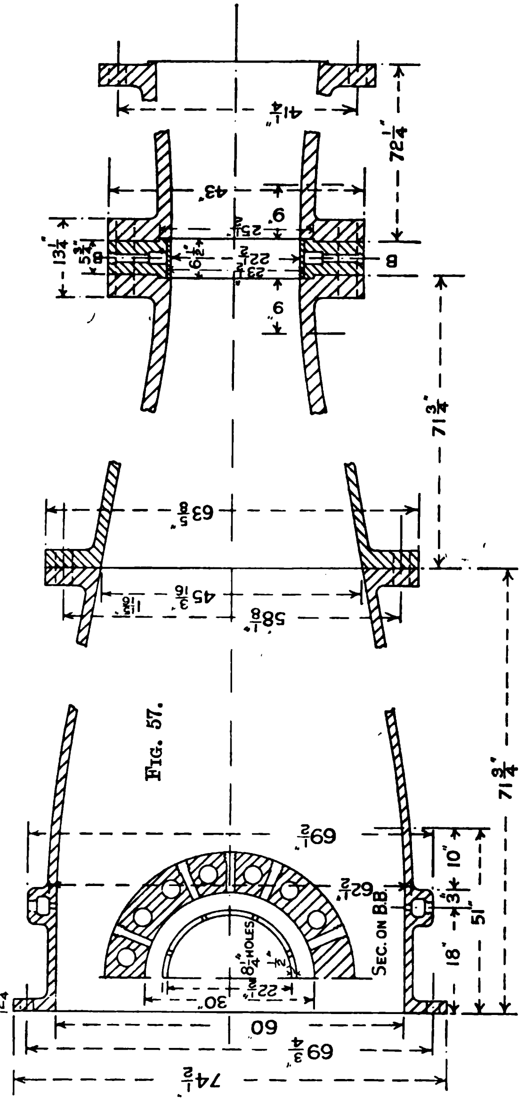


FIG. 56.

FIG. 57.

FIG. 59.

the amount they are moved forward being proportional to the velocity of the water through the throat as measured by the

pressure pipes. The recorder, therefore, is a kind of mechanical integrator, its indications being proportional to the area of the curve which would be obtained by plotting down at equal intervals of time the corresponding velocities through the throat. Experiments made on a meter for a 9-feet main showed variations of only one-half per cent., when carefully tested through a wide range of discharge. The loss of head occasioned by the narrowing of the pipe is but little greater than in equal length of straight pipe.

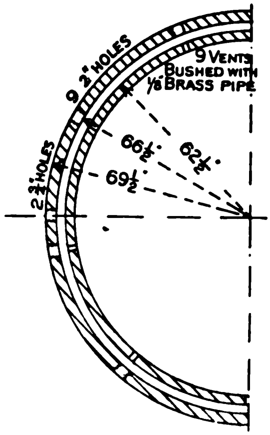


FIG. 58.

§ 48. **Water Meter.**—Meters, as usually understood, may be divided into three kinds—

(*a*) Inferential Meters.

(*b*) Volume or Capacity Meters.

(*c*) Positive Meters.

The importance of "dribbles" in water main is strongly emphasized by Mr. Schönheyder.[1] A ¼-inch hole in a pipe subjected to a pressure head of 100 feet would produce a leakage of 380 gallons an hour, or 9200 gallons a day, and would

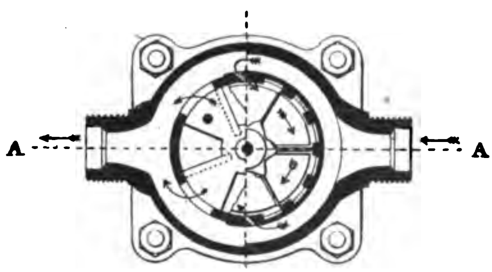


FIG. 60.

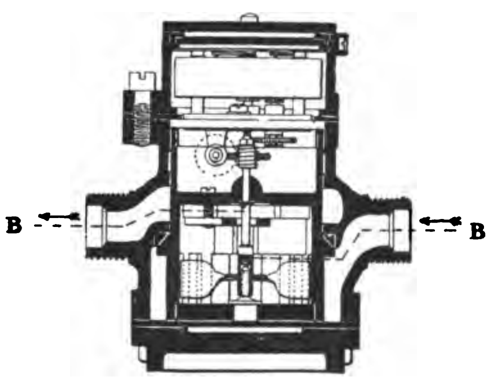


FIG. 61.

[1] *Proceedings of the Institute of Mechanical Engineers*, 1900, Part I.

supply to 600 persons at the rate of 15 gallons a day. This points to the desirability of measuring water by meter, and charging accordingly.

§ **49. Inferential Meter.**—As an illustration of an inferential meter, the one invented by Mr. Tyler may be described. The latest type is shown in Figs. 60 and 61. They act on the "turbine" principle. The water is not actually measured, but the quantity of water passed is inferred from the number of revolutions made by the fan or turbine, which is the only moving part. The bearings are metal and vulcanite. The water enters from the bottom and strikes the vanes of the fan meter at a constant velocity. The reaction of the water turns the wheel, which thereby actuates the clockwork of the dial.

§ **50. Volume or Capacity Meters.**—These meters are usually used in America. In construction they are, broadly speaking, all the same, as they consist of a casing of either gun-metal or vulcanite, in which works a vulcanite block, serving both as piston and valve (Fig. 62). They seldom possess any provision for taking up wear, and the parts are therefore difficult and expensive to repair. Hence, though they profess to measure the volume passing through them, they cannot measure small flows, as they are not tight even when new, and their leaky condition is necessarily augmented by wear. Their merits appear to be simplicity, small size, lightness, and cheapness; and for large flows they are said to be accurate. Only in the *Kent Uniform* has an attempt been made to compensate for wear, by means of a lever adjustable by a set-screw against the edge of the oval piston-block. The part of the lever, however, which bears against the piston is tipped with glass, a material quite unsuited for such work on account of its brittle nature; and, further, the lever is not automatically adjusted to wear. When the water is not clean, is rather hard, or for some

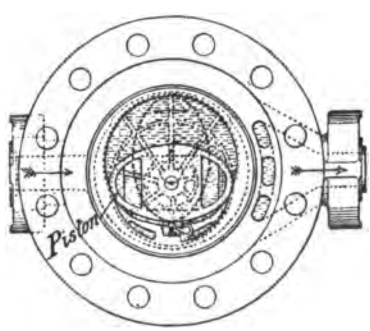


FIG. 62.

time stagnant, the meter will set fast, while still a large quantity of water passes unregistered.

The Kent meter is illustrated in Fig. 62. The elliptic disc rotates eccentrically to the casing, and the volume discharged is the volume of the casing less the volume of the roller.[1]

§ 51. **Positive Meters.**—These meters provide a space to be filled and emptied with water, and which have some contrivance for rendering them tight at varying pressures, and under diverse conditions of service. Hence they have each one or more cylinders (with their pistons and valves), which are alternately filled and emptied; and they have suitable counters.

The *Kennedy* meter has a single, vertical double-acting cylinder, with its piston packed with a rolling indiarubber ring, and a valve representing an ordinary plug-cock, but having a thin plug, which does not cover the inlet and outlet ports at half stroke (Fig. 63); hence little or no concussion is caused through the reversal of the valve, as the water is never quite stopped in its change of movement into one or other end of the single cylinder. The reversal of the valve is effected by a tumbling weight, which is raised by the piston-rod in both its up and down strokes, and which, in falling, forces down a double-ended lever or the end of a spindle of the plug valve; suitable spring buffers finally arresting its fall. As this method of reversing the valve does not produce uniform lengths of

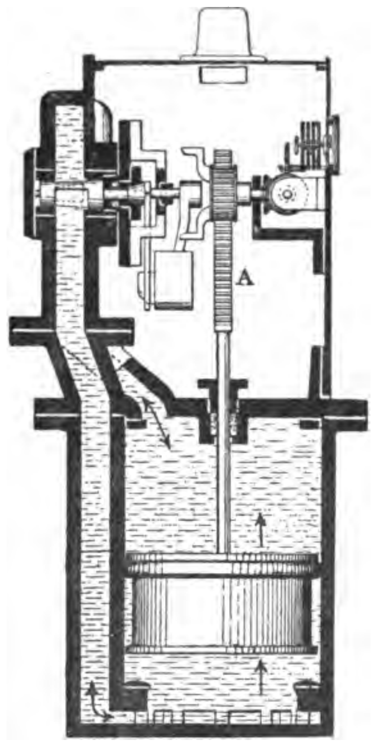


FIG. 63.

[1] For design of rollers, see author's book on "Mechanism."

stroke at varying speeds of the meter, an ingenious counter is employed, which by means of suitable ratchets and pawls practically measures the lengths of each stroke, and adds them up, thereby producing a very accurate registration. These meters are of large capacity, hence their movements are slow, so that the wear and tear of the parts are small; and, further, they discharge large quantities of water under small heads, or differences of pressure at the inlet and outlet. The rolling piston rings are said to be very durable, but heavy shocks will sometimes cause them to slip from their proper position, and become damaged. The meter is very bulky and rather noisy. The valve when at mid-stroke leaves the inlet and outlet passages momentarily open to each other, so that for a very short time water can pass from inlet to outlet without entering the meter, and therefore without being measured. However, when the meter has been properly packed, adjusted, and oiled, the interval of time is so exceedingly small as not sensibly to affect the registration; and this part of the construction has for the last few years been much improved by the adoption of Mr. Muirhead's arrangement, whereby the valve is started from rest by the movement of the piston before the tumbling weight falls over; hence the work of reversing the valve is not entirely dependent on the falling weight, and the action of the valve is said to be now much more certain. The meter requires to be cleaned, oiled, and generally overhauled every month, and it is not recommended to place it underground. This is the only single-cylinder positive meter in use at the present time.

§ 52. **Schönheyder's Positive Meter.**—This meter is of the three-cylinder horizontal type, having a single flat valve with a double movement which causes it to remain tight; and the three-cylinder arrangement produces a steady flow, as well as quiet working.

The meter consists of the following parts:—The lower portion or body, A (Figs. 64, 65), contains the cylinder, B, and the valve seating, C, with its three ports and passages, D, communicating with the bottom of the cylinders; and there is a discharge port and passage, E. Inlet and outlet connections, F and G, and

strainer, H, are also attached to the body portion, I, in the cover, with the rib, K, for holding down the strainer; and it has a prolongation at the top for receiving the counter-gear. The unequal spacing of the bolts prevents the cover from being wrongly fixed to the body. Though the counter-gear contains a few novelties—such as the entire absence of brackets, screws, springs, and small pins, and has a conveniently hinged glass cover—still essentially

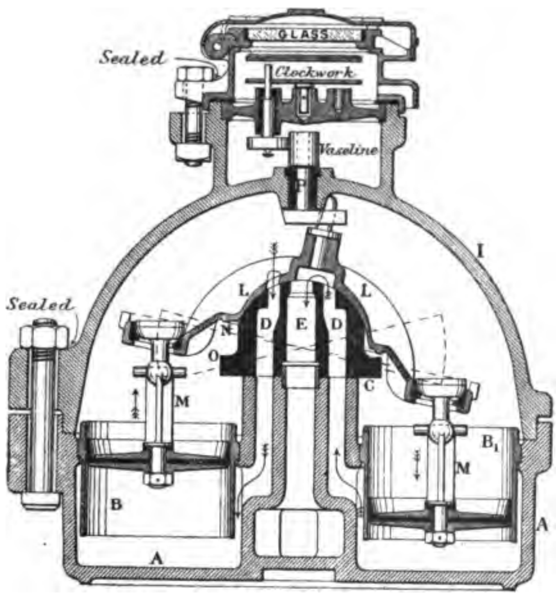


FIG. 64.

it does not differ much from the counters of other meters. L is the valve with its three arms, in the ends of cup-shaped bushes, for receiving the spherically shaped heads of the piston-rod, M; and to these are secured the pistons, composed of upper and lower piston-plates, rivets, and flexible piston packings. The water entering the meter, as shown by the arrows, passes up through the strainer into the upper portion of the casing, and presses equally downwards on all three pistons, and also on the valve. According to the position of the valve, the lower end of each cylinder in

succession is communicating with the outlet passage, and its piston is therefore forced down by the superior pressure above, and thus discharges the contents of the cylinder. At the same time one or other of both cylinders is having its piston raised, whereby water is drawn through the passages, and the lower part is filled. Thus each lower end of the three cylinders, B, is in due course filled and emptied, one or two pistons always supplying the active force, so that there is no dead-point. The length of the stroke is regulated by the flanged projection N, on the valve L, coming into rolling contact with a similar flange, O, on the valve seating C; a slight skew of the ports causes the pistons to endeavour to take a larger stroke than they should, and the roller paths restrict this tendency. The belt in the valve and the notches in the valve seating prevent the valve from turning round on its own axis. A pin in the upper part of the valve engages the crank of the crank-spindle, P, which communicates motion to the clockwork in the usual manner. The pins through the upper ends of the piston-rods prevent the pistons from falling out of the cylinders, should the meter happen to be turned upside down.

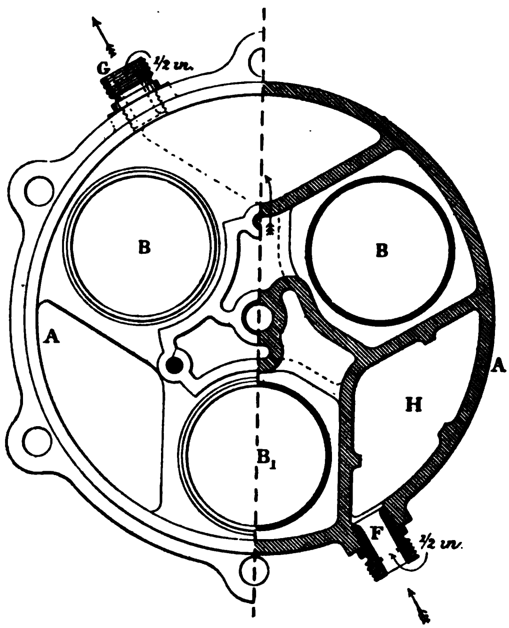


FIG. 65.

From the above description it will be seen that the meter is positive in its action, that the lengths of the stroke are definite, that the speed of water through it is practically uniform, as in a three-throw pump, so that there is no concussion or water-hammer;

that there is no back-lash between any of the working parts, and that the meter can therefore be run at any convenient speed without noise. It has few working parts, not a stuffing-box nor a spring among its details, and is self-lubricating. It contains no small parts, neither pins nor screws; the three studs and nuts (permanently securing the valve seating), the cover bolts, and the piston nuts are only the appliances of the sort used. As soon as the cover has been removed, the whole of the working parts can be taken out, examined, cleaned, new piston cups fitted, and other ordinary repairs effected, if necessary, even without removal from its position in the pipe line. The only joint which has to be made is between the body and the cover; and any leakage here is at once detected, as it is outwards. As to the durability, it is found that every two years the only things required are cleaning, painting, and retesting. The valve faces never require any attention, as they soon polish themselves bright like mirrors, and remain quite tight.

§ 53. **Experimental Results.**—Mr. Schönheyder[1] gives a comparative test of the following results of four meters, coupled in line on the supply to a country waterworks. The supply was varied from time to time by hand, and each meter was tested for accuracy before fixing.

TESTS OF FOUR WATER-METERS.

Meters.	"Positive."	"Inferential."	"Volume."	"Positive."
Initial errors.	– 3 per cent.	+ 7 per cent.	+ 2 per cent.	+ 1¼ per cent.
Trial I.	2780	1990	2346	2898
" II.	2800	3260	2454	2958
" III.	1310	880	850	1370
" IV.	1960	1410	1344	2840
" V.	2000	1310	578	2890
" VI.	1880	1200	2358	1980
" VII.	2060	1325	1405	2150
" VIII.	2080	1345	1465	2300
" IX.	2320	1680	1802	2346
" X.	2250	1850	2040	2360
Total registered	21,440	16,250	16,642	22,496

[1] *Proceedings of the Institution of Mechanical Engineers*, 1900, Part I. p. 55.

There seemed little to choose between an "Inferential" and a "Volume" meter; both were practically 25 per cent. slow on the average of the ten weeks' test.

As a further test of Mr. Schönheyder, which he calls the "Imperial," Professor Spooner, in the discussion, tested a meter having $3\frac{3}{4}$-inch cylinders, stroke $1\frac{7}{8}$ inches, 100 revolutions per minute, and delivering a maximum of 1350 gallons per hour, under a head of 5 feet 7 inches.

Outlet.	Gallons per hour.	Per cent. error.	
		Slow.	Fast.
0·75 inch	1468	—	—
0·75 cock	1100	—	0·5
0·5 inch	754	—	0·2
0·39 „	644	—	0·2
0·312 „	508	—	0·5
0·273 „	455	—	0·3
0·244 „	367	—	0·7
0·165 „	264	—	1·3
0·146 „	189	—	—
0·117 „	88	—	—
0·078 „	44	—	—
0·039 „	18	—	—

§ 54. **Method of gauging a Stream.**—For accurate determinations of the discharge of a stream, soundings must be taken at a considerable number of sections, and the area of each section calculated. The velocity at different depths must be observed for a number of longitudinal sections. The results in the Mississippi have been given in § 46. The Pilot tube is sometimes used to find the velocity. The best arrangement is to have two Pilot tubes with their mouthpieces at right angles. The upper part is made of glass, and they are provided with a brass scale and verniers. This instrument is supported on a vertical rod, the fixing allowing free revolution, so as to enable the instrument to set itself along the direction of the current. A two-way cock, which can be opened or shut, enables the instrument to be lifted up for reading. The coefficient in the equation $v = \sqrt{2gh}$, which is practically

constant. The errors involved may be considerable—the velocity may vary, the columns oscillate, and it cannot be used for low velocities.

§ 55. **Perrodil's Hydro-dynamometer.**—In Fig. 66, *ab* is a rod, of greater length than the depth of the river, which can be fixed in the bottom of the river. Over this slides a hollow rod, *cd*, which carries a frame, *efgh*. The two vertical members of this frame are connected by cross-bars, and united above water by a circular bar, situated in the vertical plane and carrying a horizontal graduated circle, *kl*. The whole system is movable round its axis, being suspended on a pivot at *g*. Other horizontal arms serve as guides. The central rod *mn* forms a torsion rod, being fixed at *n* to the frame *efgh*, and passing freely upwards through the guide; it carries a horizontal needle moving over a graduated circle, *kl*. The support *m*, which carries the apparatus, also receives, in a tubular guide, the end of the torsion rod when necessary. The impulse of the stream is received on a circular disc, *p*, in the plane of the torsion rod and *efgh*. To raise and lower the apparatus easily, it is not fixed directly to *ab*, but to a tube, *cd*, sliding on *ab*.

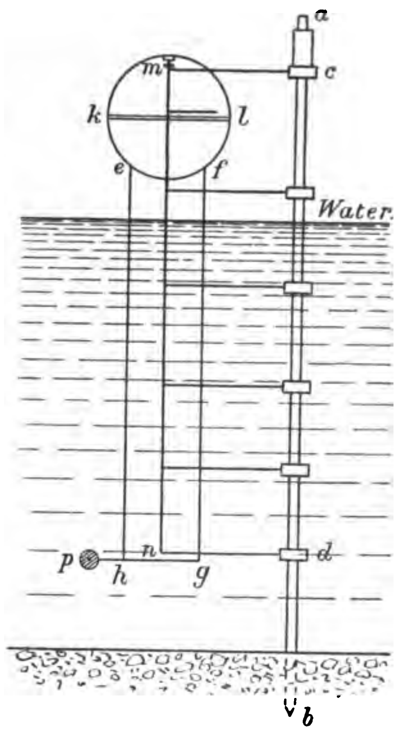


FIG. 66.

Suppose the apparatus arranged so that the disc is at that level in the stream where the velocity is to be determined. The plane *efgh* is placed parallel to the direction of motion of the

water. Then the disc p—acting as a rudder—will place itself parallel to the stream on the down-stream side of the frame. The torsion rod will be unstrained, and the middle will be at zero on the graduated circle. If, then, the instrument is turned by pressing the needle, till the plane $efgh$ of the disc, and the zero of the graduated circle, are at right angles to the stream, the torsion rod will be twisted through an angle which measures the normal impulse on the stream on the disc p. That angle will be given by the distance of the needle from zero. Observation shows that the velocity of the water at a given point is not constant. It varies between limits more or less wide. When the apparatus is nearly in its right position, the set screw at m is made to clamp the torsion spring. The needle is fixed, and the apparatus carrying the graduated circle oscillates. It is not difficult to note the mean angle reading marked by the needle.

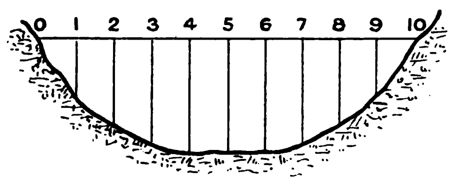


FIG. 67.

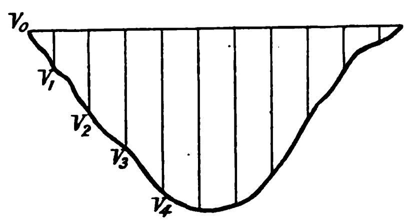


FIG. 68.

In this instrument, the velocity of the current is obtained by one observation, and the instrument remains immersed. In the current meter—consisting of a screw, the speed being measured by the revolutions on the counter—there is error due to "slip," so that it is difficult to determine the relation between the speed and revolutions, and it has to be lifted from the water at every observation.

§ 56. **Approximate Solution for Discharge of a River.**—For rough estimations, a simpler method would be necessary. A section of a river is shown in Fig. 67, the section being divided into ten equal divisions. The surface velocity, along the longitudinal sections (1, 2, 3, etc.) will depend on the position of the section, and may, roughly, be represented by Fig. 68. To a first approximation, it

may be assumed a parabola. Again, in any vertical section, such as 2, the velocity curve, in calm weather, agrees fairly well with a parabola (§ 46). Thus, if the mean velocity at the point of mid-stream be measured, by drawing a parabola with this as ordinate, and the surface breadth as abscissa, the velocity at the mid-points of the sections 01, 12, . . . may be scaled off. At any section such as 45, the average velocity is two-thirds the surface velocity; and thus the approximate discharge is

$$(v_0 + v_1)\tfrac{2}{3}bd_1 + (v_1 + v_2)\tfrac{2}{3}bd_2 + \ . \ . \ .$$

in which b is the width of a section $(-12_1 \ldots)$ and d_1, d_2 are the depths at the centre of each section.

§ **57. Thomson Theory of River Bends.**—In rivers running through soft or gravelly soil, the windings which already exist tend to become more acute by the scouring away of material from the outer bank and depositing it on the inner bank. The sinuosities sometimes increase until a loop is formed with only a narrow strip of land between the two encroaching branches of the river. Finally a "cut-off" may occur, a water-way being opened through the strip of land, and the loop left separated from the stream,

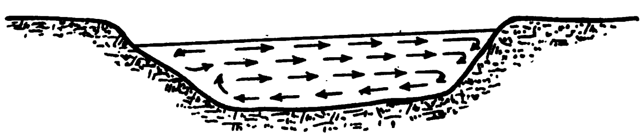


FIG. 69.

forming a horseshoe-shaped lagoon or marsh. The usual theory is that the water, flowing in a straight line, rushes against the outer bank, scours it, and deposits some of it on the inner bank. This is an imperfect view. What happens is that in flowing round a bend under gravity, the water follows the law of a free vortex. Its velocity is greatest, therefore, at the inner bank, and the difference in velocity in direction of the stream cannot account for it. But, on account of centrifugal force, the curvature of the stream causes a variation of pressure across the stream, the pressure increasing radially outwards, and the water surface

having a slope from the inner to the outer side is widened. Near the surface, and for a considerable depth, this causes no difference; but, near the bottom, the velocity gets checked, and its centrifugal force is insufficient to balance the pressure due to greater depth (Fig. 69). It flows, therefore, inwards, and that near the surface flows outwards to fill its place. In addition to this cross-motion, the stream has a forward motion, so that it tends to silt along a curved direction from the inner bank to the outer.

§ 58. **Testing Current Meters.**[1]—Mr. Robert Gordon made some interesting experiments on current meters in the Admiralty tank at Haslar. The apparatus described is used for testing the resistance of ships and the performance of ships' screws. The current meters to be tested are towed by a dynamometrical apparatus through still water in a large tank, which gives a parallel-sided water-space 278 feet long, 36 feet broad, and for the most part 10 feet deep, though it shallows up at the ends. It is roofed from end to end, the framework of the roof carrying a light railway with a clear space between the rails, which run the entire length of the building at about 20 inches above the normal water-level.

A stout-framed truck, T, Fig. 70, suspended from the axles of two pairs of wheels, runs on the railway, and carries the recording and measuring apparatus. A sheet of paper is wound round a cylinder, R, carried by the truck; and the cylinder is moved by a band from the hinder axle, so that the circumferential travel of the paper represents, on a reduced scale, the forward motion of the truck. A pen, A, actuated by clockwork, marks time on the cylinder as it revolves. A second pen, B, is moved electrically, and marks indents on the recording paper for every 25 feet run by the truck. A third pen, C, moved electrically, can be used for recording the number of revolutions made by the meter under trial, contact being made inside the meter in the method actually arranged for use. A fourth pen, D, is required for recording the force actually used for towing the object through the water. Generally not more than three or four pens are used at one time for recording: another occasional pen, E, is sometimes required to

[1] *Proceedings of the Institute of Mechanical Engineers*, May, 1884.

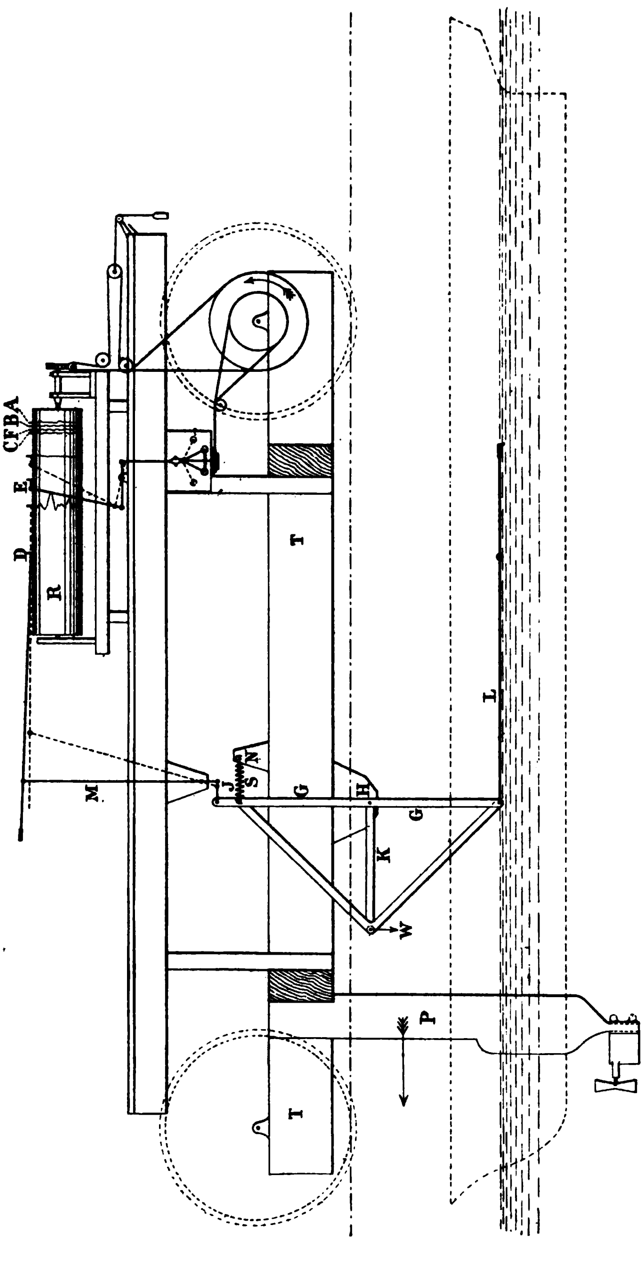


Fig. 70.

show slight quick or slow variations in the rate of speed of the truck and object during the experiment. The arrangement of all five pens is shown with the recording cylinder, R, and the resistance-measurer or dynamometer, G. A sixth pen, F, is used in ship-model experiments, as well as the pen C, so as to record the indentations of two very small and delicate current-meters, which precede the model some distance ahead, and show whether the speed of the model through the water differs from the measured speed over the ground, in virtue of any slight current set up by previous experiments.

The dynamometer consists of a vertical beam, G, Fig. 70, hung on a fixed centre, H, by a double knife-edged suspension. From its lower end a link, L, takes the towing strain of the model; while a knife-edge fixed in the beam, G, at an equidistant point above the centre, H, carries one end of the spring, S, whose extension measures the resistance of the model, the other end being attached to a knife-edge, N, fixed in the truck. The uppermost extremity of the beam is linked at J to the multiplying lever, M, which moves the resistance-recording pen, D, on the paper. An extensive collection of springs permits by substitution any resistance to be measured; but the actual strain corresponding with the indications of the recording pen is determined by weights, W, applied to the end of the horizontal lever, K, at a point whose distance from the centre, H, equals that of the towing link, L, and the resistance-spring, S.

The arrangement for testing the Deacon meter is shown in Fig. 70, where a round bar nearly an inch in diameter carried the meter; but it was found expedient in the rating experiments to substitute for the round bar one of tapering or fish-section, as shown at P, set with its edges in the direction of the run, as the round bar was observed to cause eddies and a disturbance in the water, likely to interfere with accuracy of record in the instrument when used for high velocities.

In Fig. 71 are given the rate-curves of this meter and of two others, in order to show the method of inferring the rate-curves of the different meters from the master curve shown by the strong line. The abscissæ represent the speed in feet per second, the

ordinates the travel of the log through the water for a given number of turns of the screw. The curves are *similar* curves, those of meters Nos. 10 and 11, shown by the dotted and full fine lines, being inferred by a process of trial and error from the

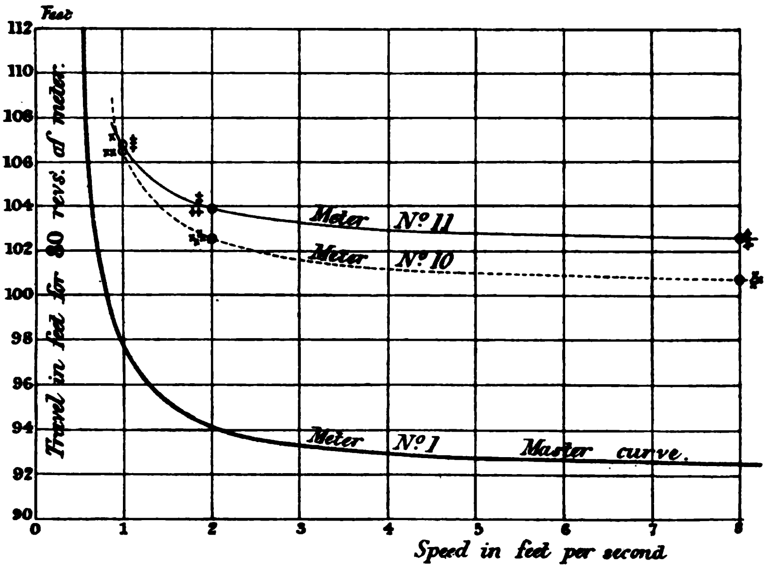


Fig. 71.

master curve of No. 1 by appropriate alterations in the scales for the ordinates and abscissæ.

§ **59. Resistance of Bends and Elbows.**[1]—Mr. Alexander (now Professor of Engineering at Cork) made some interesting experiments on the subject of bends. The only results available where those of Weisbach, being

$$h_b = \frac{m}{2} \cdot \frac{v^2}{2g}$$

where
$$m = 0{\cdot}131 + 1{\cdot}847\frac{r}{R}$$

[1] *Proceedings of the Institute of Civil Engineers.* Read at the meeting of the Birmingham Association of Students of the Institution, and published in the *Proceedings*, March 10, 1904.

h_b being the loss of head in a bend, $r - R$, the radii of the pipe and bend respectively. These experiments were made on a $1\frac{1}{4}$-inch pipe.

The object of the experiments was to find the loss of head for different degrees of curvature, but the experiments were all made on a right-angled bend. Experiments were first made on a straight length of wooden piping, varnished on the inside, three piezometers being fitted every fifteen inches, about. Readings were taken for the loss of head between the two extreme piezometers and between adjacent ones. From the tabulated results, the hydraulic gradients for different velocities were obtained, and a curve was plotted with the velocities as ordinates, and the slope as abscissæ. By comparing the curve with the logarithms of these velocities (§ 21) the law connecting the velocity and slope was obtained.

The formula for a straight pipe was

$$S = 0{\cdot}0407\ V^{\cdot 1777}$$

in which S represents the resistance, and V the velocity in feet per second.

Having made experiments on a straight pipe, experiments were then made on a right-angled bend, set to different angles. The length was the same as for a straight pipe. The curves are shown in Fig. 72. The difference between them represents the loss due to the bend. The total loss is represented by H, the loss in the straight pipe by h_0; h, the loss due to the two straight tangents, and h_b the loss directly due to the bend. The loss due to the bend was then plotted logarithmically on a velocity base, and it appeared that the law was of the same form as for a straight pipe and was of the same nature, and that until the bend becomes as sharp as a knee, there is nothing in the nature of a shock. The formula for a bend was

$$h = kv^{1\ 777}$$

where k has to be determined.

The loss appears to be dependent on (1) roughness of

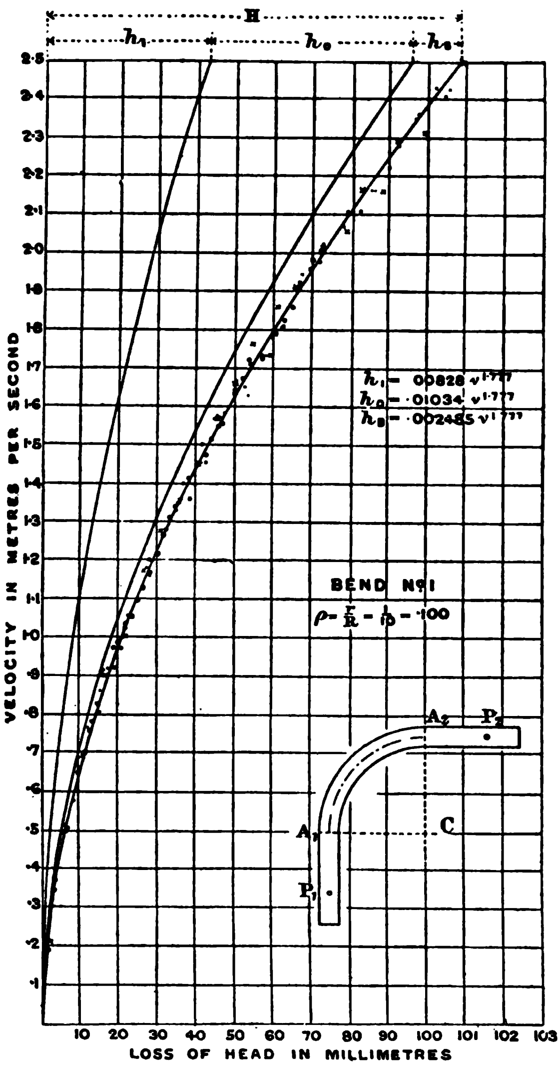


FIG. 72.

pipe, (2) radius of pipe, (3) curvature of the bend, (4) the

length of bend measured on the centre line. The formula deduced was

$$h_b = kv^n = \frac{al_b\rho}{(c)^t\left(\frac{d}{4}\right)^u}v^n$$

where h_b is the loss of head due to the bend, l_b is the length of the bend, ρ the curvature of the bend $\left(=\frac{r}{R}\right)$, c a coefficient obtained from $v = cR^{0.61}\,s^{0.56}$, where s has its value already given, d the diameter of the pipe, and v the velocity in feet per second. The estimated coefficients are

$$\begin{aligned} a &= 0.00034 \\ t &= 1.777 \\ n &= 1.777 \\ u &= 1.08 \end{aligned}$$

Apparently, up to a certain critical curvature, corresponding to 0·2, the value of m is 0·83; above this curvature, it is 2·5.

Mr. Alexander comes to the conclusion that the equation for the length of straight pipe offering the same resistance as the bend causes over and above the resistance that it would offer as an ordinary pipe, is

$$L = 12.85\left(\frac{r}{R}\right)^{0.83} l$$

where

r = radius of pipe,
R = radius of bend,
l = length of curve.

The length L, has to be added to l, and then the fall is calculated, as the given gradient, as $l + L$.

He also points out that experiments carried out at Detroit appear to verify that a bend of radius equal to 2½ diameters of the pipe offers less resistance than any other.

§ **60. Loss in Diverging Channels.**[1]—In the calculations for

[1] By Dr. Stanton: *Engineering*, November 21, 1902.

engineering purposes of the changes in pressure in water flowing through channels of varying cross-section, that any deviation from the law of pressure and velocity given by Bernouilli's equation

$$\frac{p}{\sigma} + \frac{v^2}{2g} + z = \text{constant}$$

can be accounted for by the addition of a "friction" term, in which, presumably, a coefficient of friction derived from experiments on parallel channels can be employed.

Thus, on the supposition that the friction varied as the square of the speed, the actual conditions of flow in a circular pipe would be approximately given by the equation—

$$\frac{p}{\sigma} + \frac{v^2}{2g} + z + \int \frac{f}{g} \frac{Q^2}{\pi^2 r^5} ds = \text{constant} \quad \text{...........(1)}$$

in which
p = pressure of water,
v = velocity of flow,
z = height of water above datum,
Q = quantity of flow per second,
r = radius of pipe,
f = coefficient of friction,
ds = element of length of pipe.

It will be noticed that the assumption generally made is that the loss of head due to friction is the same for a given length of pipe, such as AB in Fig. 73, whether the flow be from A to B or from B to A, provided that the quantity of the flow is the same in the two cases.

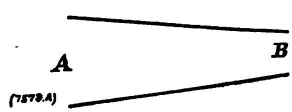


FIG. 73.

That this would not be the case might be predicted from the discovery made twenty years ago by Professor Osborne Reynolds,[1] that the tendency of the flow in a converging pipe is to be steady at all speeds, whereas that in a diverging pipe is to be "eddying" or turbulent at all speeds, which latter fact had been previously noticed by Mr. Francis in his experiments on the flow of water in

[1] Royal Institution lecture, March, 1884.

a diverging tube.[1] It follows as a consequence of this that, given steady motion of the water entering the converging channel, this state would be maintained up to the smallest section; and hence the loss due to friction would be much less than the loss taking place in the flow through a diverging channel of the same dimensions. In his lectures at Owens College, Professor Reynolds demonstrates experimentally the very considerable loss of energy which takes places in a diverging channel of rectangular cross-section.

Some points which still appear doubtful are: (1) The effect of the form of the channel, and (2) the effect of the angle of divergence on the loss of head involved.

The first observations were made on a channel of circular cross-section, 4½ inches long, the diameter varying from ¾ inch at one end to ¼ inch at the other. Experiments were made with water flowing first in one direction and then reversed, the pressures at the large and small sections for given velocities of flow being carefully measured.

Table I. gives the loss of head due to friction in the two cases at varying speeds.

TABLE I.

FLOW IN A PIPE OF CIRCULAR CROSS-SECTION.

Velocity in feet per second at least section.	Total Head $\left(\frac{p}{w}+\frac{v^2}{2g}\right)$ in feet.		Loss of Head.		Direction of flow.
	Entering pipe.	Leaving pipe.	In feet.	As percentage of initial head.	
11·50	2·27	2·07	0·20	8·8	Converging
13·12	3·00	2·70	0·30	10·0	„
16·80	4·86	4·42	0·44	9·0	„
25·70	11·45	10·35	1·10	9·6	„
11·90	2·22	1·64	0·58	26·1	Diverging
13·75	2·97	2·19	0·78	26·2	„
17·25	4·66	3·48	1·18	25·3	„
31·30	15·20	11·28	3·92	25·8	„

[1] Lowell, "Experiments in Hydraulics."

From these results it will be seen that the loss of head of the water flowing as a diverging current to a state in which the kinetic energy is practically negligible is approximately 25 per cent. of the initial velocity head,[1] and that when the direction of flow is reversed the loss is approximately 10 per cent. of the final velocity head, and that these ratios appear to be the same for considerable variations of speed. A possible explanation of the discrepancy might be found in the shape of the expanding nozzle; but further experiments with carefully made openings failed to bring about any agreement, the relative values of the losses being practically the same as in the above.

§ 61. The Effect of the Angle of Divergence on the Loss.—For the purpose of making a series of experiments on channels of varying angles of divergence the arrangement shown in Fig. 74 was used.

C, C is a casting having a parallel channel, of width 2 inches, and depth 0·375 inch running from end to end. In this were placed the strips F, F, so as to form a channel of uniform depth, the convergence and divergence of which could be varied by planing the strips to the required angle.

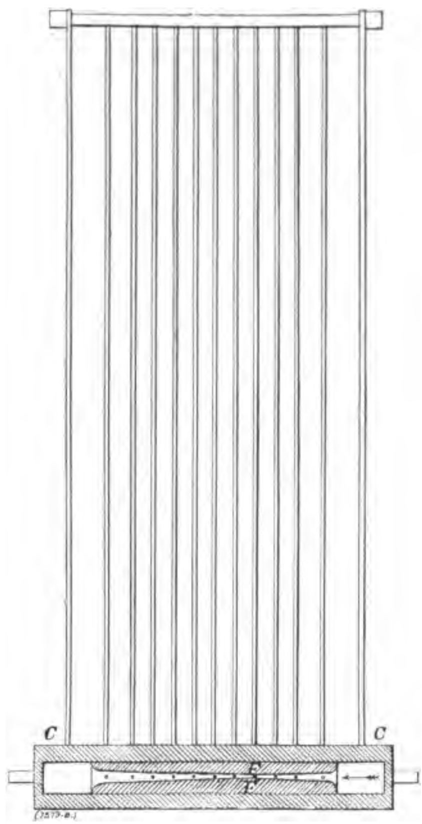


FIG. 74.

[1] In Tables I. and II. the value of p at the least section is taken as the datum from which the pressure is measured.

The pressures were measured at horizontal distances of 1·25 inch along the centre line, the pressure gauges consisting of vertical glass tubes, all connected at the top to a common air vessel, the heights of the water columns in the tubes giving the values of the corresponding pressures. From observations of the pressures along the channel, the quantity of the flow, and the dimensions of the channel, it was possible to calculate the losses of energy which took place. For a given angle of divergence a set of experiments were made at varying speeds of flow. The strips were then adjusted for another angle, and the observations repeated.

The loss of energy in the diverging channel was taken as the difference of the total head $\frac{p}{\sigma} \times \frac{v^2}{2g}$ at the neck and at the wide end.

The results of the experiments showed that the loss of energy in a diverging channel of rectangular cross-section is nearly constant for all angles of divergence ranging from 2° up to a certain value, after which it increases rapidly, the value of this limiting angle being for the cases in question approximately 6°. A particular feature of them is that with a considerable angle of divergence and low speeds of flow the diverging channel obviously did not "run full," the water on leaving the neck forming an approximately parallel channel surrounded by "dead" water, with the consequence that the rise in pressure was very small as the speed was increased. The tendency was for the channel to run more and more "full," and thus reduce the loss of head due to the formation of the dead water.

The answers to the above questions therefore seem to be that, even under the most favourable conditions, the loss of energy in a diverging channel is from two to three times the value of that which takes place when the flow is the same in magnitude, but reversed in direction, and that at a certain critical angle, depending apparently on the velocity of flow, this loss suddenly increases in value, tending to a condition in which the whole initial kinetic energy of the water is dissipated in eddies. Further, the results in Table I. show that, although there is such a wide difference

between the losses involved in diverging and converging flow, yet in both cases this loss varies approximately as the square of the velocity. This might appear to be at variance with the statement above—that the flow in a converging channel tended to be steady; but it must be remembered that in the cases in question the water entering the channel had a velocity greatly in excess of its critical value, and was consequently in an eddying condition, which state would be maintained throughout the channel.

Again, if the radius of the pipe can be written in the form

$$r = a + bs$$

the value of the friction term in equation (1) becomes [1]

$$H^1 = \frac{f}{4bg} \cdot \frac{Q^2}{\pi^2}\left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Applying the results in Table I. to this, the values of f for the cases of diverging flow in this particular channel are found to be

0·0312 0·0303 0·0315 0·0306

which are about four times its value for a parallel channel. It is also obvious that if the loss of head in the diverging channel is, as indicated in these experiments, within certain limits independent of the angle of divergence, the values of f obtained by such experiments will depend on the magnitude of this angle. That is, for a large angle of divergence there will be a large value of f, which will diminish as the angle of divergence diminishes. Instead, therefore, of using a variable value of f, the writer would suggest that in hydraulic calculations of the losses of head in diverging channels this loss should be taken as a definite percentage of the velocity head at the smallest section, the value of which, as determined by the experiments, is approximately 25.

For the case of converging channels the determination is more difficult, and will apparently depend on whether the velocity at the entrance to the channel is above or below its critical value for that particular diameter. In either case, however, it may be safely

[1] Bovey's "Hydraulics," p. 99.

assumed that the form of the channel will have considerable influence on the loss of head; and therefore, if the velocity exceeds the critical value, as will be the most usual case in practice, this loss may be calculated by means of equation (2).

The fact brought out in the above experiments, that in diverging streams of water there is a limiting angle of divergence at which the loss due to eddying becomes rapidly greater, affords an explanation of the well-known rule, which was stated by Mr. Froude twenty-five years ago—that in the design of a ship's lines greater care should be taken to insure fineness at the stern than forward, experience showing that the eddy resistance of ships is considerably increased by a full stern and abrupt termination of the water-lines.

Although, owing to the influence of the sides of the channel, no quantitative results could be obtained from the apparatus described above, which would apply to actual practice, it seemed possible to obtain comparative results which would indicate the effects of bluntness of stern in ships. For this purpose the four channels shown in Fig. 75 were made, the boundary on one side being straight, and the other curved to represent a ship-shaped contour. In these contours the lines forward are the same, but the aft ones vary in fulness, No. 1 having the most gradual changes of slope, and No. 4 the greatest.

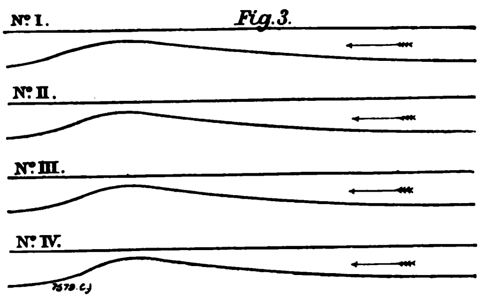


FIG. 75.

TABLE II.

FLOW IN A DIVERGING CHANNEL OF RECTANGULAR CROSS-SECTION.

Angle of Divergence.		Total head $\left(\frac{p}{w} + \frac{v^2}{2g}\right)$ in feet.		Loss of head.	
		Initial.	Final.	In feet.	As percentage of initial head.
deg.	min.				
2	2	0·93	0·68	0·25	27·0
		2·06	1·47	0·59	28·6
		3·38	2·39	0·99	29·3
		5·25	3·70	1·55	29·5
2	15	1·15	0·81	3·34	29·5
		2·08	1·53	0·55	26·4
		3·57	2·58	0·99	27·7
		5·30	3·77	1·53	28·9
3	8	0·72	0·52	0·20	27·8
		2·17	1·57	0·60	27·6
		3·81	2·75	1·06	27·8
4	2	0·82	0·59	0·23	28·1
		1·59	1·13	0·46	28·9
		4·12	3·00	1·12	27·3
		5·23	3·73	1·50	28·7
5	3	0·78	0·53	0·25	32·0
		1·61	1·17	0·44	27·3
		3·87	2·88	0·99	25·6
		5·00	3·70	1·30	26·0
5	57	1·49	1·00	0·49	32·9
		2·57	1·85	0·72	28·0
		3·90	2·79	1·11	28·5
		5·42	3·78	1·64	30·3
6	54	1·83	0·80	1·03	56·3
		2·55	1·67	0·88	34·5
		3·79	2·35	1·34	35·4
		6·55	3·74	2·81	42·9
7	35	1·53	0·48	1·05	68·7
		2·28	1·35	0·93	40·8
		4·20	2·58	1·62	38·6
		6·28	3·70	2·58	41·1

TABLE III.

FLOW OF WATER PAST SHIP-SHAPED FORMS.

	Head in feet of water.		Loss of head in feet.	
	At stem.	At stern.	Total.	Percentage of initial head.
No. 1	1·88 3·35	1·10 2·22	0·78 1·13	41·4 33·7
No. 2	1·82 3·03	1·07 1·92	0·75 1·11	41·2 36·6
No. 3	1·83 3·46	1·00 2·18	0·83 1·28	45·3 37·0
No. 4	1·73 3·67	0·62 1·71	1·11 1·96	64·0 53·4

Experiments were made on the flow of water through these channels, the losses of energy being calculated as before. The results are stated in Table III. It will be seen that although there is no very considerable increase in the loss due to eddying in passing from forms No. 1 to No. 3, yet the slight changes in the water lines between No. 3 and No. 4 is sufficient to produce an increase of 30 per cent. in the loss. As, however, the eddy resistance in ships is generally supposed to be less than 10 per cent. of the total resistance, this may not be of serious practical importance, except as a suggestion of the considerable change in the motion of the water brought about by small changes in the contour of the surface over which it flows.

§ 62. **Loss due to Sudden Enlargement.**—Experiments have been made in the hydraulic laboratory of the Manchester University to determine the loss due to a sudden enlargement.

The large tube was 2·15 inches diameter, and the small tubes were 0·65 inches diameter. The flat end plate was connected to eight pressure gauges, and pressures were inserted at 2, 4, and 7 from the sudden enlargement. Only the first two gauges were used, and the results were inconsistent. On fitting the third, consistent results were obtained.

If p be the pressure in the small tube past the sudden enlargement, p' the pressure on the flat plate, P the pressure at the third gauge point, v the velocity tube, and V the velocity large tube, then experiment the following results, which may be compared with those calculated by the usual formula. The agreement is very close, the difference being greater at the higher speeds, as might have been anticipated.

Pressure in feet of water.			Velocity in feet per second.		$\frac{p}{\sigma}+\frac{v^2}{2g}$	$\frac{P}{\sigma}+\frac{V^2}{2g}$	Actual loss of head.	Calculated loss $\frac{(v-V)^2}{2g}$
p	p'	P	v	V				
2·078	2·070	2·120	4·195	0·382	2·351	2·122	0·229	0·226
3·50	3·46	3·645	9·174	0·8365	4·869	3·656	1·153	1·580
0·98	0·871	1·290	13·22	1·207	3·700	1·317	2·383	2·250
0·885	0·702	1·420	18·18	1·657	6·015	1·468	4·547	4·250
1·29	0·995	2·14	23·54	2·150	9·900	2·212	7·688	7·10
1·41	1·00	2·35	24·8	2·861	10·96	2·430	8·55	7·90

CHAPTER III

HYDRAULIC-PRESSURE MACHINES

§ 63. **Relative Advantages of using Steam and Water.**—A water-pressure engine acts in a similar way to a steam engine. The piston reciprocates to and fro, and water may be admitted by either a slide-valve or a rotary valve.

The relative advantages and disadvantages of using steam and water as the working agent are—

(1) If the demand for power is intermittent, water has an advantage over steam, in that, in the latter case, condensation takes place in the pipes. In addition, with water no previous warming up is necessary, the parts are always cool, and can be immediately handled for repairs, etc., and the machine is easily controlled. With a high pressure, the machine may be kept small and light. It is specially adapted for the recoil of guns.

(2) A disadvantage is that frictional losses may become excessive. The loss of head in pipes varies as $\frac{lv^2}{d}$, and the loss of pressure as $\frac{\sigma lv^2}{d}$ that is to say, for a given length and diameter [1] of pipe. At 100 pounds pressure per square inch, the weight per

[1] This assumes the coefficients of friction of steam and water are the same:

For water, $f = 0{\cdot}005\left(1 + \frac{1}{12d}\right)$; for 2-inch, 4-inch, 6-inch pipes, $f = 0{\cdot}0075$, $0{\cdot}00625$, $0{\cdot}00583$.

For steam, $f = 0{\cdot}0027\left(1 + \frac{3}{10d}\right)$; for 2-inch, 4-inch, 6-inch pipes, $f = 0{\cdot}00756$, $0{\cdot}00514$, $0{\cdot}00431$.

cubic foot of steam is 0·23 pounds, and one cubic foot of water weighs 62·5 pounds; thus, with the same velocity, the loss of pressure with water is 270 times as great as with steam. In order, therefore, to prevent excessive losses, the velocity with which water is allowed to flow along a pipe, and the speed at which a hydraulic engine runs, is very much less than the corresponding speeds when steam is the fluid used. Steam flows along a steam pipe with a velocity of 100 feet per second—water along a water pipe with a velocity of 3 to 6 feet per second. A steam engine has a piston speed of 400 to 1200 feet per minute—a water engine a piston speed of 80 feet per minute. Unless, therefore, pressures are used many times greater than in steam practice, the water engine would be much larger and more cumbrous than a steam engine. In this connection, it must be remembered that increasing the water pressure does not in any way affect the frictional losses.

(3) A disadvantage of water engines is that since water is non-expansible, the same amount of water is used per stroke, whether the resistance is large or small. In a steam engine, reduced work is obtained by varying the cut-off, or by throttling. In a water engine the only way, if the accumulator pressure be constant, is to throttle the water. This simply causes prejudicial resistances, so that, unless special devices are adopted, a water-pressure engine becomes very uneconomical at low loads.

(4) Water is incompressible, consequently excessive shocks may be caused. For example, if the load on a crane is being rapidly lowered, and the exhaust valve is rapidly closed, the momentum of the moving parts, resisted by the unyielding water, is likely to cause severe shocks unless some provision is made for the water to escape when the pressure reaches a certain amount. Usually relief valves are fitted.

(5) In water, forces are required to cause the alternate acceleration and retardation of the large mass of water in the pipes. If the speed is uniform, those accelerating forces are zero; but, if the velocity vary from moment to moment, they may become considerable.

§ 64. **Uniform Motion.**—Imagine that the piston of the engine,

and therefore the water in the supply pipe, is moving at a steady uniform speed. To obtain the effective pressure on the piston, assume that the pressure at the accumulator end of the main is p_0 pounds per square foot, the uniform velocity of the piston v feet per second, and A, a the areas, in square feet, of the piston and pipe; so that the uniform velocity in the pipe is $\frac{A}{a} \cdot v$. The resistances overcome are due to pipe friction and partially closed valves. The total loss of head may be included in one expression

$$F \frac{v^2}{2g}$$

where F is *referred to the piston velocity.* For pipe friction alone

$$F = \frac{4fl}{d}\left(\frac{A}{a}\right)^2$$

(2) The head due to the velocity of flow in the cylinder is $\frac{v^2}{2g}$. This is so small compared to the frictional term that it may be neglected.

(3) The effective pressure on the piston, p, the corresponding head being $\frac{p}{\sigma}$.

Thus, in *steady* motion, the equation is

$$\frac{p}{\sigma} = \frac{p_0}{\sigma} - F \frac{v^2}{2g}$$

where F is a coefficient which can only be determined by experiment. If p, p_0 be observed, then F may be estimated. The diameter of the pipe, or the necessary opening of a valve, may therefore be found from the principles already laid down (§ 37).

It will be noticed that as F increases, the effective driving pressure p gets less. The value of F may be increased to any extent by gradually closing a valve, so that the speed of the machine can be controlled. If the driving pressure p be equal to zero, the limiting speed of the piston will be

$$v^2 = \frac{2g}{F} \cdot \frac{p_0}{\sigma}.$$

§ 65. **Numerical Illustrations.**—As illustrations of these principles, consider one or two examples, in which the motion is steady.

(1) *A ram,* 10 *inches in diameter, has to lift a load of* 20 *tons, with a velocity of* 6 *inches per second, the accumulator pressure being* 750 *pounds per square inch. If the load on the ram be reduced to zero, and the valves be left untouched, find the limiting speed of the ram, and the value of* F *for the system.*

Since the valves are unaltered, the value of the coefficient of hydraulic resistance, F, is the same in both cases. When lifting 20 tons the pressure on the ram is

$$\frac{20 \times 2240}{\frac{\pi}{4} \times \left(\frac{10}{12}\right)^2} = 82{,}100 \text{ pounds per square foot.}$$

The head equivalent to this is

$$\frac{82100}{62 \cdot 5} = 131 \cdot 5$$

and this, therefore, represents the useful head overcome. The accumulator head is

$$\frac{750 \times 144}{62 \cdot 5} = 172 \cdot 5$$

so that the difference, namely 41 feet, is the head spent in overcoming frictional resistance. The value of F, referred to the ram, is therefore given by

$$41 = \mathrm{F}\frac{v^2}{2g} = \mathrm{F} \times \left(\frac{1}{2}\right)^2 \frac{1}{2g}$$

$$\therefore \ \mathrm{F} = 10{,}600, \text{ about.}$$

If the load be removed, the whole head is available for overcoming frictional resistance; hence, since F is unaltered, the speed of the ram with the load removed is given by

$$\frac{1}{2} \times \sqrt{\frac{172 \cdot 5}{41}} = 1 \cdot 025 \text{ feet per second}$$

or double the speed when lifting 20 tons.

(2) As a second illustration :—

A direct-acting lift has a ram 9 *inches diameter, and works under a constant head of* 73 *feet of water, of which* 13 *per cent. is required by ram friction and friction of mechanism. The supply pipe is* 100 *feet long and* 4 *inches diameter, and is full bore to the ram cylinder. Find the speed of steady motion when raising a load of* 1350 *pounds, and also the load it would raise at double the speed. At what speed would the original load be raised if a valve in the pipe were closed* $\frac{3}{4}$ *of the pipe area?*

If v represent the velocity of the ram, the loss of head in pipe friction is

$$f\frac{4l}{d} \cdot \frac{\left(\frac{81}{16}v\right)^2}{2g} = \frac{0{\cdot}0075 \times 400 \times 3 \times 81^2}{64{\cdot}4 \times 16^2}v^2$$
$$= 3{\cdot}58v^2$$

taking a coefficient of 0·0075.

The loss of head at entry to the cylinder is

$$\frac{\left(\frac{81}{16}v - v\right)^2}{2g} = 0{\cdot}256v^2.$$

The total loss of head is

$$3{\cdot}836v^2.$$

The available head is

$$\frac{1350}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2} \times \frac{1}{62{\cdot}5} = 49$$

whence $$49 = 63{\cdot}5 - 3{\cdot}836v^2$$

or $$v = 1{\cdot}943 \text{ feet per second.}$$

If the speed is doubled, the frictional losses are increased fourfold and, therefore, become

$$4(63{\cdot}5 - 49) = 58 \text{ feet.}$$

The head on the ram is then $63{\cdot}5 - 58 = 5{\cdot}5$ feet, and the corresponding load which can be lifted is

$$5{\cdot}5 \times 62{\cdot}5 \times \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 152 \text{ pounds.}$$

If a valve in the pipe be closed $\frac{3}{4}$ of the pipe area, the coefficient of hydraulic resistance is found, from the curve in § 36. to be 0·54 referred to the pipe velocity. Since the pipe velocity is $\left(\frac{9}{4}\right)^2$ times the ram velocity, the coefficient due to the valve referred to the ram velocity is

$$\left(\frac{9}{4}\right)^4 \times 0{\cdot}54 = 13{\cdot}82.$$

Hence the equation becomes

$$49 = 63{\cdot}5 - (13{\cdot}82 + 3{\cdot}836)v^2$$

or $$v = 0{\cdot}905 \text{ foot per second.}$$

(3) *As a third example, take an ammunition lift.*

The weight lifted is 3250 *pounds, the diameter of the ram* $9\frac{3}{8}$ *inches, the velocity of the ram* 6 *inches per second, and the pressure available at the working valve at the engine* 700 *pounds per square inch. The purchase is such that the mechanical advantage between the ram and cage is* 6. *Estimate the total loss of head caused by the valve, and calculate the necessary area of port.*

In this example, a certain additional load must be added to the net load lifted to overcome the mechanical friction of the purchase. Under ordinary conditions the additional load may be taken as 5 per cent. of the load lifted for each moving part of the purchase, that is to say, for each unit of mechanical advantage. In the present case, therefore, since the mechanical advantage is 6, the load on the cage is

$$3250 + 3250 \times \frac{6}{20}$$
$$= 4225$$

so that the load on the ram is

$$4225 \times 6 = 25{,}350 \text{ pounds.}$$

The pressure per square foot on the ram is

$$\frac{25350}{\frac{\pi}{4} \times \frac{9{\cdot}375^2}{144}} = 53{,}000$$

and the corresponding head of water is

$$\frac{53000}{62 \cdot 5} = 848.$$

The head available at the valve is

$$\frac{700 \times 144}{62 \cdot 5} = 1610$$

and the loss of head is

$$1610 - 848 = 762.$$

The value of F referred to the ram is

$$F\frac{1}{2g}\left(\frac{1}{2}\right)^2 = 762$$

and $$\therefore\ F = 196{,}500.$$

The loss of head, represented by 762 feet, is lost in the passage through the port into the cylinder. The size of the valve chest is not stated, but it may be assumed to be large compared to the port area. In passing through the port, therefore, the contracted area may be taken as 0·618 of the port area, and therefore, if v_1 is the velocity through the port, the velocity at the contracted area is $\frac{v_1}{0 \cdot 618} = 1 \cdot 62 v_1$. The velocity in the cylinder is 6 inches per second, and hence the loss of head due to the sudden enlargement following the contraction is the head due to the relative velocity, and is, therefore,

$$\frac{(1 \cdot 62 v_1 - \frac{1}{2})^2}{2g} = 762$$

whence $$v_1 = 137.$$

Thus the port area is to the piston area as $\frac{1}{2}$ is to 137, and therefore, the port area is

$$\frac{1}{2 \times 137} \times \frac{\pi}{4} \times 9 \cdot 375^2 = 0 \cdot 252 \text{ square inch.}$$

The actual area was 0·26 square inch.

If the purchase carry away so that there is no load on the ram, the speed of the ram is 8·7 inches per second.

$$\frac{1785}{2g}\left(\frac{8 \cdot 72}{12}\right)^2 = 14 \cdot 65 \text{ foot-pounds.}$$

§ 66. **Inertia Effect of Pipe Water.**—If the piston be subjected to a varying velocity, so likewise must the water in the supply pipe. The whole mass of water in the supply pipe may be subject to alternate accelerations and retardations; and to cause these accelerations and retardations, certain forces must act.

Consider a pressure engine of plunger area A, the delivery pipe being of area a and length l. At any instant, let the velocity and acceleration of the piston be v and α respectively. Then, if the water keep pace with the piston, the velocity and acceleration of the water in the supply pipe must be $\frac{A}{a}v$ and $\frac{A}{a}\alpha$ respectively. The mass of water in the supply pipe is $\frac{\sigma al}{g}$, and to impress the acceleration $\frac{A}{a}\alpha$ on this mass requires a force, acting over the area a, equal to

$$\frac{\sigma al}{g} \cdot \frac{A}{a}\alpha = \frac{\sigma lA}{g}\alpha.$$

The intensity of pressure necessary is, therefore,

$$\frac{\sigma lA}{ga}\alpha$$

and the equivalent head, in feet of water, is

$$\frac{lA}{ga} \cdot \alpha.$$

If the piston is being accelerated, the head on it is decreased by this amount; if it is being retarded, it is increased by the above amount. This is equivalent to a total force of

$$\frac{\sigma lA^2}{ga}\alpha$$

on the piston; that is, equal to

$$\frac{\sigma al}{g}\left(\frac{A}{a}\right)^2\alpha.$$

This is the force required to cause an acceleration α in a

mass $\frac{\sigma al}{g}\left(\frac{A}{a}\right)^2$; hence the inertia effect of the water in the pipe is the same as if a mass equal to the mass of water in the pipe $\times \left(\frac{A^2}{a}\right)$ were placed at the piston and moved with it. A hydraulic engine may therefore be looked upon as an engine with a very heavy piston.

In addition, the water in the cylinder itself has to be accelerated, and a certain force is required to accelerate it. The quantity of water varies, and the effect is usually so small compared to the mass of the pipe water that it may be neglected.

Again, if F be the coefficient of hydraulic resistance of the whole system, referred to the piston velocity, the loss of head in friction is

$$F\frac{v^2}{2g}$$

hence, if h be the head at the commencement of the pipe, the effective head on the piston at any instant is

$$h - \frac{lA}{ga}a - F\frac{v^2}{2g}.$$

This equation is true whatever the law of variation of velocity and acceleration may be.

§ 67. Simple Harmonic Motion. Curve of Effective Pressure.—Consider a simple case (Fig. 76). Suppose the piston drives a crank of radius r at a uniform speed of ω radians per second. The motion of the piston may be taken to be simple harmonic, and its acceleration when the crank arm has turned through an angle θ measured from the line of stroke may be taken to be

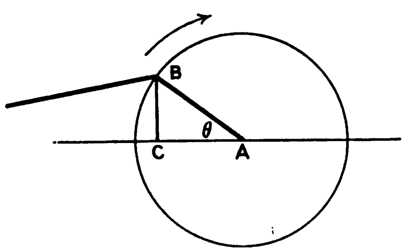


FIG. 76.

$$\omega^2 r \cos\theta = \omega^2 x$$

where x = distance of piston from the point of mid-stroke. The

motion of the piston may be taken to be the same as that of the point of projection C in the line of stroke; and the centripetal acceleration of B is $\omega^2 r$, along BA and, therefore, of C, is $\omega^2 r \cos\theta$. Thus, then, $a = \omega^2 x$.

Again, in the position shown, the velocity of the piston is equal to the projection of the velocity of B, and is, therefore,

$$v = \omega r \sin\theta = \omega r \sqrt{1 - \frac{x^2}{r^2}}.$$

Thus, substituting in the previous equation, when a piston moves in a simple harmonic manner, the effective head on it is

$$h - \frac{lA\omega^2}{ga} x - \frac{F}{2g}\omega^2 r^2\left(1 - \frac{x^2}{r^2}\right)$$

$$= h - \frac{lAu^2}{gar} x - F\frac{u^2}{2g}\left(1 - \frac{x^2}{r^2}\right)$$

in which u represents the crank pin velocity and is equal to ωr.

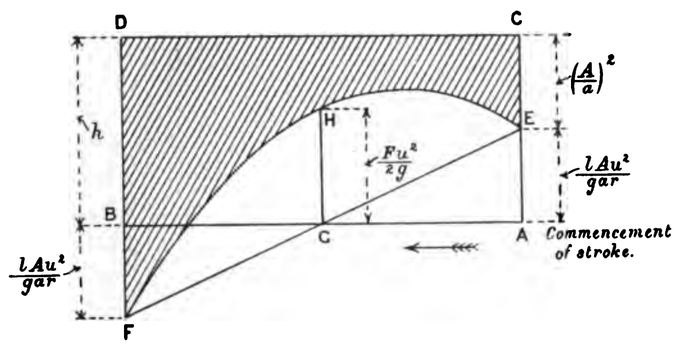


FIG. 77.

The head, therefore, instead of being uniform at each point of the stroke, varies. A curve showing the variation is shown in Fig. 77. In that figure AB is a datum line and CD is parallel to AB and at a distance h from it. At the commencement of the stroke the piston is accelerated, and the head necessary to give it the acceleration is $\frac{lAu^2}{gar}$ At the end of the stroke the water is

being retarded, and the additional head brought on the piston is $\frac{lAu^2}{gar}$. At the centre of the stroke the acceleration is zero, and therefore produces no effect. Thus, to correct for inertia of the pipe water, take an inclined straight line, EF, as datum, and measure heads from it to CD. The line is straight, because the acceleration varies as the distance from the centre. Again, to correct for frictional losses, it may be noticed that at the ends of the stroke the velocities of the piston and water are both zero, and consequently there is no head lost in friction at these points. At the centre of the stroke the velocity is maximum and equal to u. At any other point the loss of head follows the parabolic law. Hence, to correct for friction, erect, at the centre of the stroke an ordinate GH equal to $F\left(\frac{A}{a}\right)^2\frac{u}{2g}$, F is referred to the velocity of the piston; and through the points E, H, and F, draw a parabola. The effective head of the piston in any position is the vertical ordinate of the shaded area. The effective head is therefore decreased at the beginning of the stroke and increased towards the end. Moreover the shaded area is clearly proportional to the work done on the piston, because it may be looked upon as an indicator card for the cylinder. With no losses, the work theoretically possible is $A\sigma h \times$ stroke. The alternate acceleration and retardation of the mass of water does not affect the work done, because the area GEB is equal to the area AGF; and the mean ordinate of the parabola FHE is

$$\tfrac{2}{3}F\left(\frac{A}{a}\right)^2\frac{u^2}{2g}$$

and, therefore, the mean effective head on the piston is

$$h - \tfrac{2}{3}F\left(\frac{A}{a}\right)^2\frac{u^2}{2g}$$

The efficiency is

$$1 - \tfrac{2}{3}F\left(\frac{A}{a}\right)^2\frac{u^2}{2gh}$$

and the force per square foot is $\frac{1250}{32 \cdot 2} \times 100 = 3890$, or 27 pounds per square inch, equivalent to a head of 62 feet of water.

§ **68. Separation of the Piston and Water.**—In Fig. 78 the parabola has been drawn entirely below the line CD, indicating that there is always a driving-head on the piston. But if the speed of the engine increase, or if some artificial resistance—such as closing a stop-valve partially—is introduced, the distance AE and GH both become greater, and the parabola may rise above the line CD. The curve of effective head then becomes CDE. Draw above ED a line MN, such that ND represents the barometric head (34 feet of water). From P to Q the pressure is negative; and if this negative pressure exceed a certain amount, the air held in solution will be liberated, and probably water vapour will be formed. There will therefore be a cavity formed between the water and the piston, and the water will lag behind. Towards the end of the stroke the air and vapour will be absorbed, and the water will knock against the piston and cause a severe shock. Thus it is important to prevent this separation. The condition that it should be prevented at the commencement of the stroke is that

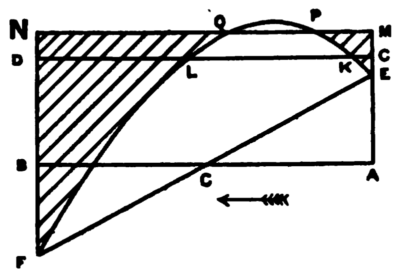


Fig. 78.

$$b + h > \frac{lAu^2}{rag}$$

and, to avoid separation, b should not be greater than 20 feet. Separation may be prevented at the commencement of the strokes, and still take place at a later point (see Fig. 78). To satisfy whether this is so, differentiate the general equation in § 67 with respect to x, which gives

$$x = \frac{lA}{aF}.$$

If $r \gtrless \frac{lA}{aF}$ then separation will, or will not, take place in the stroke.

It is very important to point out that separation can only take place in a pressure-engine, when the engine is provided with

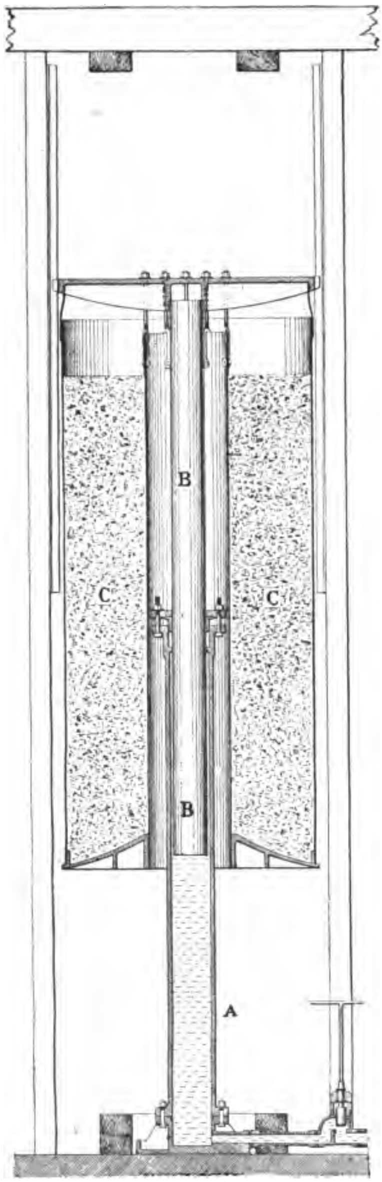


FIG. 79.

a heavy flywheel, or other storage device, so that the engine can be driven by the energy already stored up in the flywheel.

$$\text{When} \quad h = \tfrac{2}{3}\mathrm{F}\left(\frac{\mathrm{A}}{a}\right)^2 \frac{u^2}{2g}$$

all the energy is absorbed in frictional resistances.

§ **69. Hydraulic Accumulator.**[1]—The most important machine in the transmission of power by hydraulic pressure is the accumulator (Fig. 79). It consists of a large cast-iron cylinder, A, fitted with a plunger, B, from which a loaded weight-case, CC, is suspended, to give pressure to the water pumped in by the engine. The accumulator is a reservoir giving pressure by load instead of elevation. The load on the accumulator is such as to produce a pressure equal to that of a column of water 1500 feet high, and the capacity of the cylinder is sufficient to contain the largest quantity of water which can be drawn from it at once by the simultaneous action of all the machines in connection with it. The accumulator also serves as a regulator to the engine, for when the loaded plunger rises to a certain height, it begins to

[1] *Proceedings of the Institute of Mechanical Engineers,* January, 1868.

close a throttle-valve in the steam pipe, or, more conveniently, the inlet valve of the accumulator, so as gradually to lessen the speed of the engine until the descent of the loaded plunger again calls for an increased production of power. The second arrangement is shown in Fig. 80. When the cross-head rises to the stop, A, on the rod, it lifts the rod, B, which lifts the lever, C, on the valve, and this closes the delivery valve, D.

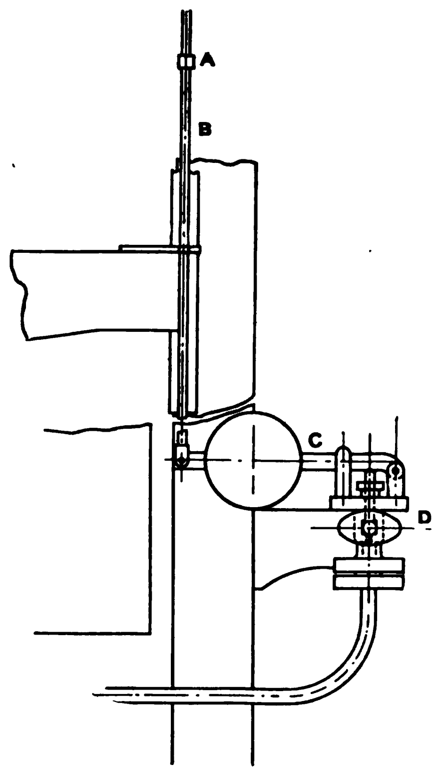


Fig. 80.

§ **70. Capacity of Accumulator.**—If the object is to simply produce a required pressure, the accumulator may be dispensed with, and the pumps pump water directly into the main. But an accumulator, besides giving the required constant pressure, acts, as already pointed out, as a reservoir, and does not necessitate such powerful engines and pumps as would otherwise be the case. In the majority of machines, the work is taken off intermittently, the periods of work being separated by periods of rest required for manipulation, etc. If such a machine were driven by the direct action of a pump, the pump would have to be sufficiently powerful to perform the required work in the short time during which the machine works, whilst in intervals of rest the engine and pumps remain idle. When an accumulator is used the engines and pumps can be employed during the whole of the time to pump water into the accumulator, the

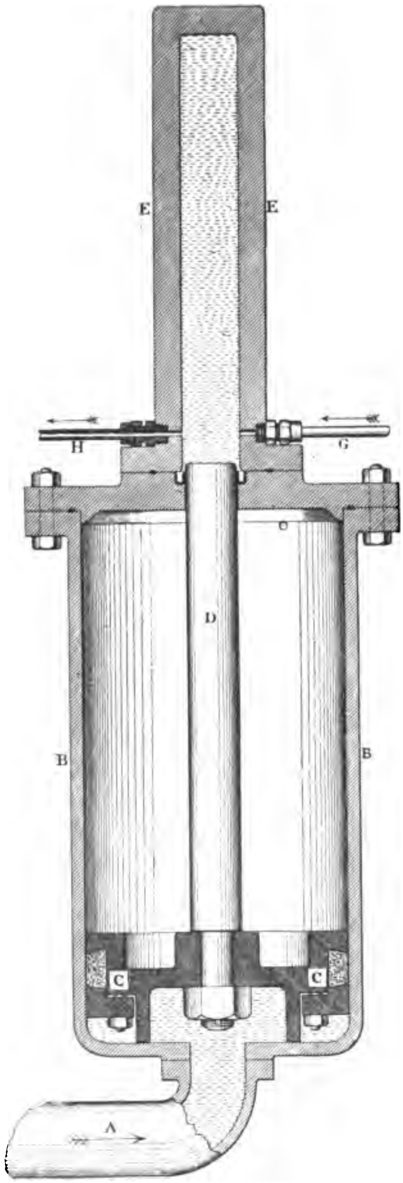


FIG. 81.

water-pressure engine, in the mean time, drawing on the accumulator.

To find the capacity of the accumulator, let V be the volume of water required when the machine is working, t the time of working, and T the time of a whole operation. If an accumulator is not used, the pump must be large enough to supply $\frac{V}{t}$ cubic feet of water per second; whilst, with an accumulator, it need only be $\frac{V}{T}$ cubic feet per second. In t seconds the pump would therefore supply $t \cdot \frac{V}{T}$ cubic feet, and the accumulator would have to supply $V\left(1 - \frac{t}{T}\right)$. This, therefore, represents the *capacity* of the accumulator.

Another disadvantage of pumping direct into the mains is that, in the case of a number of machines working intermittently, the demand on the pumps may be very irregular, and consequently the speed of the pumps and the pressure in the mains vary considerably. On board ship it is impossible to carry an accumulator, so that a governor,

whose action depends on the pressure in the mains has to be used (§ 106).

§ 71. **Intensifying Accumulator.**—Instead of loading the case with weights, an *intensifier accumulator* may be used (Fig. 81).[1] The intensifier consists of a large cylinder, B, with its supply pipe, A, containing a piston, C, to which is attached a rod, D. The rod, D, works in a smaller cylinder, E, which is rigidly attached to the large cylinder, B. The cylinder, B, is open at the top. The cylinder, E, is supplied with water by a pump, the inlet pipe being G, and the outlet pipe to the tool or machine being H.

The pressure in E magnifies that in B in the ratio of the area of the piston to the area of the rod. Thus, if the cylinder is 19 inches diameter and the rod $3\frac{1}{2}$ inches, the ratio of areas is 30, about, the pressure in the ram cylinder is 1750 pounds per square inch, so that the pressure in the cylinder is about 60 pounds per square inch. If it be required for a testing machine having a maximum pull of 100 load, the area of the straining ram will have to be $8\frac{1}{2}$ inches nearly.

§ 72. **Pressure-intensifying Apparatus.**[2]—A pressure-intensifying apparatus in connection with an ordinary hydraulic press is shown in Fig. 82. Its object is to give the most intense pressure at a certain point of stroke, and is usually for pressing bales of cotton. A and B are two steam cylinders fitted with pistons, E and F, and having at their front ends the hydraulic cylinders C and D, in which work the plungers G and H; the steam cylinders are of equal size, but the plungers G and H are of different area, their respective areas as shown in the diagram being in the proportion of 5 to 1; and the combined cubic capacity of these plungers is assumed to be at least equal to that required for a full stroke of the press ram, a certain margin being added to make up for leakage. The lifting ram of the press worked by this pumping apparatus will rise five-sixths of its lift with one stroke of the plunger G, and one stroke of the plunger H will complete the remaining sixth. The diameters of the steam

[1] By Mr. Ralph Tweddell. *Proceedings of the Institute of Mechanical Engineers*, August, 1874.

[2] *Proceedings of the Institute of Mechanical Engineers*, January, 1878.

pistons E and F are equal, and are subjected to the same steam pressure. When the piston E is the driving piston, F is forcing the water into the press at five times the pressure.

Assuming that the pistons E and F are at the back ends of the cylinders, if the pipes, R, and the hydraulic cylinders, C and D, are filled with water, practically incompressible, any movements of the pistons E and F will move the press ram; and again, supposing this ram, either by gravity or other means, is caused to return to the bottom of its cylinder, the pistons E and F will also return to their former positions; therefore no inlet or outlet valves whatever are required, the same water

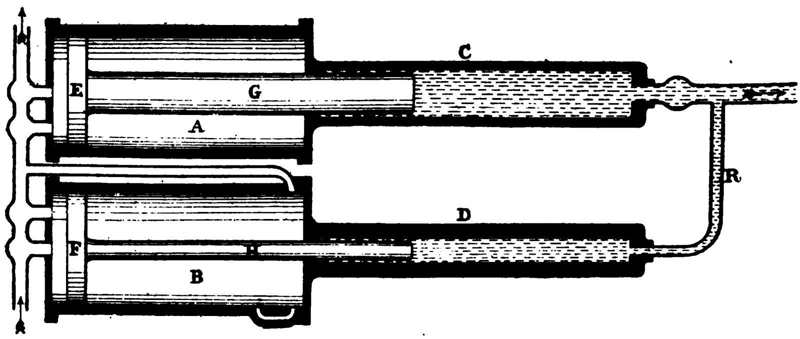


FIG. 82.

being forced backwards and forwards between the press ram and the plungers G and H. The admission of steam to the pistons on the one hand, and the weight of the press ram on the other, when the steam is exhausted, acting through the interposed water as a connection, produce the same effect as if the pistons and press ram were mechanically connected by a rigid coupling.

§ 73. **Differential Accumulator.**—Mr. Tweddell designed a differential accumulator [1] in order to obtain a very high pressure required in hydraulic riveting and other shop operations. In Fig. 83, A is a central rod firmly supported in a bed plate, and acts as a guide. It is held at its top end by a bracket

[1] *Proceedings of the Institute of Mechanical Engineers*, Liverpool meeting, 1872.

attached to a wall or other fixed object. Surrounding the rod is a bush, the shoulder of which is at D. This works through a cup leather gland, E. The cylinder which carries the weights, F, has an internal diameter only slightly greater than the external diameter of the bush; and the weights, to any required number, are placed on the cylinder. The rod is fixed, and the weighted cylinder moves. The inlet port is represented by C, and the high-pressure water runs up a small channel in the central line of the fixed rod, and through small passages at the top. Thus the whole weight is taken by the bush. This enables a very high pressure of water to be attained. The capacity of the accumulator is very small, and unless the pumps work during the withdrawal of the water, the casing will descend with great rapidity. The arrangement of the link-work which closes the throttle-valve is the same as in the ordinary accumulator.

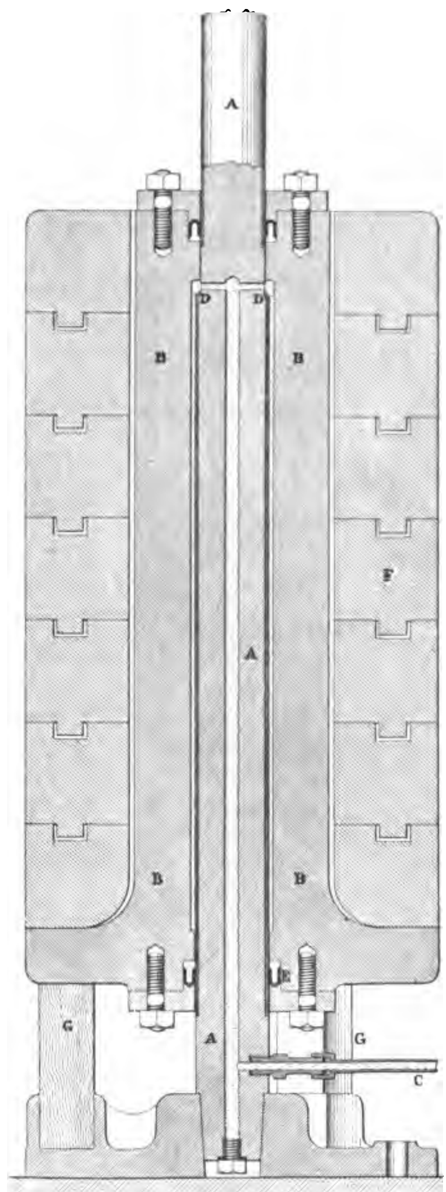


FIG. 83.

In the example illustrated, the bush is $\frac{1}{2}$-inch diameter, the rod is 5 inches diameter, and the stroke is 50 inches. The pressure was 1000 pounds per square inch. The total load is 9800 pounds.

Mr. Hick showed that the friction of the leather collars was

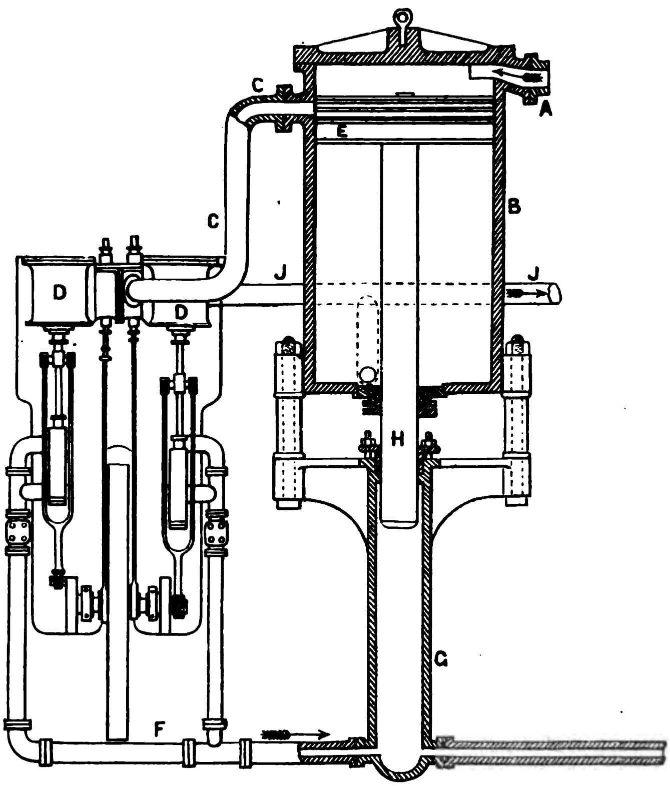


FIG. 84.

only 1 per cent. for rods 4 inches diameter, and $\frac{1}{2}$ per cent. for rods 8 inches diameter.

§ **74. Steam Accumulator.**—In ship use an accumulator has been devised in which the dead weight is replaced by a steam piston.

The arrangement is shown in Fig. 84. The steam enters the top of the cylinder, B, by the pipe, A, and passes through the pipe, C, to the cylinder, D, of the pumping engines, and thus the steam pressure acts on the piston, E. The pumping engine delivers water into the pressure pipe, F, which communicates with the hydraulic cylinder, G, consequently the pump has to force the ram, H, up against the pressure of steam on the piston, E, and since the steam always has access to the top of this piston, there is a constant load on the ram, A. The piston, E, automatically closes the steam pipe, C, when it reaches the top of the stroke, and so stops the pumping engine; but as soon as the ram begins to descend, the passage through C is re-opened and the pump re-started. The lower end of the cylinder is open to the exhaust steam-pipe, J.

This form of accumulator is available for use on shore and ship. It is used in the French Navy, where the steam piston and ram are in the ratio of 13 to 1, which gives a pressure of 780 pounds per square inch. This pressure will be exceeded by taking the weight of the steam and ram into account, and, after allowing for friction, the pressure is about 800 pounds per square inch. A separate throttle-valve is used in the steam-pipe, worked by the accumulator in such a way that, when the pressure reaches 800 pounds per square inch, the pumping engine is automatically stopped; but, re-starts on a fall of pressure. The practice of allowing the pumping engine to be stopped requires constant attention to see that the cylinders are kept clear of water. The steam accumulators are useful in maintaining a constant pressure, but cannot be looked upon as storing a large amount of energy.

§ **75. Hydraulic Riveting.**—The differential accumulator is invariably used in connection with hydraulic riveting. The principle on which the riveter acts is as follows:—The riveter cylinder is coupled through a pipe to a differential accumulator. The capacity of the accumulator is very small, with the result that when water is drawn off when the operation of riveting takes place, the accumulator casing, with its weights, descends slowly in the first part on account of the rivets being squeezed

into position, but when the final pressure is being exerted, the retardation of the water in the pipe and the whole mass on the accumulator becomes very great, with the result that a very great pressure is exerted on the rivet. This is the operation wanted in hydraulic riveting.

As an illustration of the enormous pressure which may be exerted, consider a particular case. The diameter of the fixed spindle in the case just quoted, was 8 inches, the bush was ½-inch thick, and the stroke was 4 feet. The capacity of the accumulator was therefore—

$$\frac{\pi}{4}(6^2 - 5^2) \times 4 = 34{\cdot}5 \text{ cubic inches, or } 0{\cdot}24 \text{ cubic feet.}$$

The maximum pressure was 2000 pounds per square inch, so that the weight on the accumulator was—

$$2000 \times \frac{\pi}{4}(6^2 - 5^2) = 17300 \text{ pounds} = 7{\cdot}75 \text{ tons.}$$

The weight of water in the supply pipe—the pipe was 30 feet long and 1 inch diameter—was 10·2 pounds. Now, it has been shown that the "equivalent mass" at the riveter ram is equal to the mass at accumulator in the ratio of the squares of the respective areas (§ 66). If the diameter of the riveter ram be 8 inches, then the ratio of the areas of the ram and accumulator bush is 64 to 11, that is to say, 5·85; and the ratio of the areas of the ram and pipe is 64 to 1. Thus, the equivalent weight on the ram is—

$$5{\cdot}85^2 \times 7{\cdot}75 + \frac{64^2 \times 10{\cdot}2}{2240}$$

$$265 + 18{\cdot}6 = 283{\cdot}6 \text{ tons}$$

say, 300 tons or 6 tons per square inch. The *force* with which the rivet is closed can hardly be estimated. It would be necessary to know the time of the final squeeze and the amount of compression.

These results may be represented by diagrams (Figs. 85 and

86).[1] The actual curve of pressure on the rivet is shown in Fig. 85. With an accumulator pressure of 1400 pounds per square inch, the maximum pressure was 2100 pounds per square inch. It

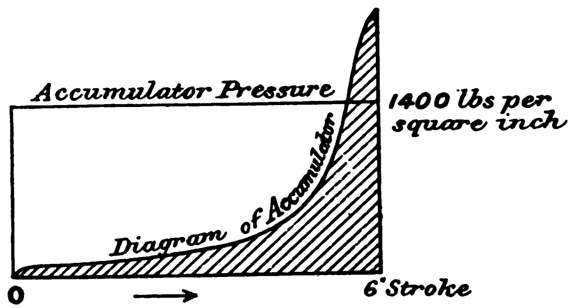


FIG. 85.

will be noticed that the effect of inertia is to greatly diminish the pressure in the beginning of the stroke, and to increase it above the accumulator pressure at the end of the stroke. This is

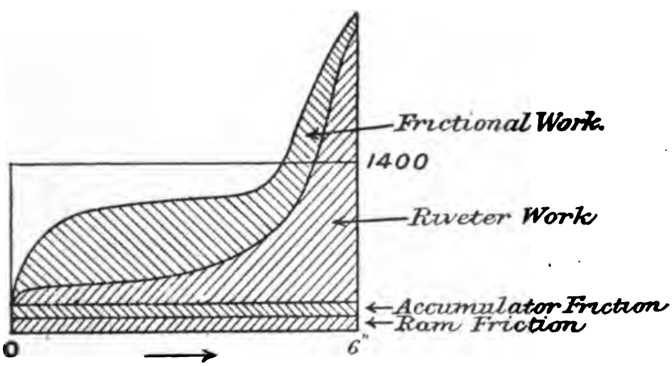


FIG. 86.

advantageous in closing the rivet. The full analysis of the action includes (1) the effect of accumulator friction, (2) fluid friction, (3) friction of riveter. These are shown in Fig. 86.

[1] Lecture by Prof. Unwin, at the Institute of Civil Engineers, March 5, 1885.

The areas represent the energy stored due to each cause.

The speed of the machine is kept down by its own resistance to a limit of about one foot per second.

§ **76. Hydraulic Gun Brake.**—When a gun is fired, the gun recoils, and some device must be adopted to absorb the energy of recoil. This can best be done by using some arrangement by which the energy is dissipated in hydraulic loss.

When a gun, such as the large guns now used in the Navy, is ready for firing, its position in the mounting is known as "The run-out position," or "firing position," and, when fired, two things have to be accomplished:

(1) The energy of recoil must be absorbed in order to bring the gun to rest in a reasonable distance.

(2) When the gun is brought to rest, it must immediately be returned to its original position in the mounting so as to be again ready for loading and firing.

Hydraulic gun brakes act by absorbing energy in hydraulic loss, but there are two ways in which it can be effected.

The first method is to have a cylinder provided with a stout rod and piston, the rod being connected to the gun. The cylinder is filled with water, and the piston has a large number of perforations through it. When the gun is fired, the water is forced from one end of the cylinder to the other, and thus a great resistance is created. This gradually brings the piston to rest.

If, at any instant, the speed of recoil be v, and if A be the piston area, n the ratio of the piston area to the combined area of orifices, the resistance will be—

$$\sigma A(n - 1)^2 \frac{v^2}{2g}.$$

The resistance will therefore increase from the beginning to the end of recoil, and the time taken in recoil will depend on the mass of the gun.

A second method, and one adopted in the more recent guns, is to dispense with orifices and to use a piston so arranged that as

the recoil takes place, the area gets gradually less, becoming practically zero at the end of recoil.

§ 77. "**Royal Sovereign" Class.**[1]—The following brake is of the type fitted in the *Royal Sovereign* class. In Fig. 87—

A = the recoil valves, eight in number, loaded to 2705 pounds per square inch, with a lift of 0·058 inches.

B is the piston valve, loaded to 2580 pounds per square inch, with a lift of 0·07 inches.

C is the disconnecting valve.

D is the "running in and out" cut-off valve.

E is the front end relief valve.

F is the steadying-plate.

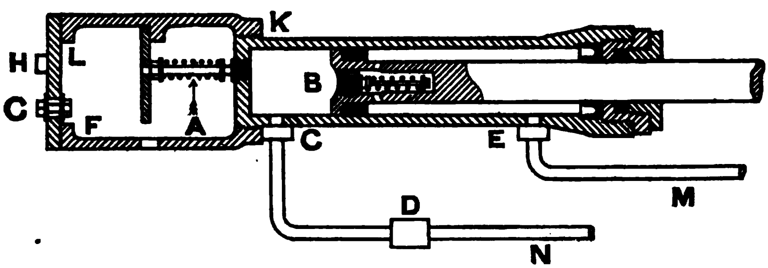


FIG. 87.

G is a relief disc (three in number), yielding at 30 pounds per square inch.

H is a "snift" valve.

K is an air-cock from the rear end of the cylinder.

L is a flange in the air-pipe. This breaks joint when the cover is backed three-quarters of an inch.

M, N, pipes connected to swivels at the pivot-pin.

The principle of action is the first method just described, in which orifices are placed in the piston. The orifices are replaced by loaded relief valves.

[1] Permission to publish the details of the hydraulic gun was kindly given by the Lords of the Admiralty. Captain Emdin, R.N., Chief Mechanical Engineer at Devonport, very kindly supplied me with most valuable information of the details and sections.

§ 78. **Running "In and Out" Cut-off Valve.**—This valve, denoted by D in Fig. 87, is placed in the pipe attached to the rear end of the recoil cylinder. It is fitted to slew up the gun at each end of the stroke, and acts by throttling the pressure just before the end of the run-out stroke, and the exhaust before the end of the run-in stroke. The springs, C_1, C_2, are compressed on to the respective valves just before the ends of the respective strokes. The spring, C_1, compresses 1000 pounds per square inch, and the spring, C_2, 205 pounds per square inch. Pin-holes, A, B, in the valves allow the stroke to be completed slowly. The springs do not touch the valve spindles until compressed upon them. The carriage strikes the stops, D_1 D_2, just before the

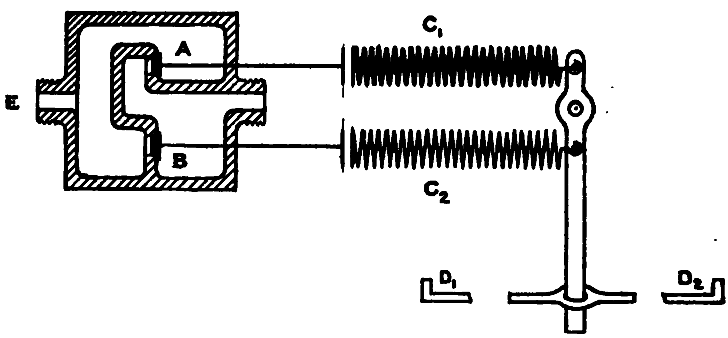


FIG. 88.

end of stroke, putting compression on the springs, C_1, C_2, to an amount 0·7 inch in each case. The run-in is to the left; the run-out to the right. The hole E leads to the rear end of the recoil cylinder, and F to the swivel-joint at the pivot-pin.

§ 79. **Disconnecting Valve.**—This valve, referred to as C in Fig. 87, is shown in Fig. 89. It is fitted to prevent the recoil pressure being felt outside the recoil cylinder at B. The valve is bolted to the rear side of the recoil cylinder at C, and the pipe from the swivel-joint is connected at A. The action is, on running out the pressure at the rear end opens the valve D, and passes to the cylinder at B, at the same time forcing the ram back. When the gun is right out, the flow of water through the valve ceases, but

the pressure remains, and the weak spring, C, forces the valve back. This is the state of affairs when the gun is fired. Running in, as soon as the rear end is put to exhaust, the strong spring, E

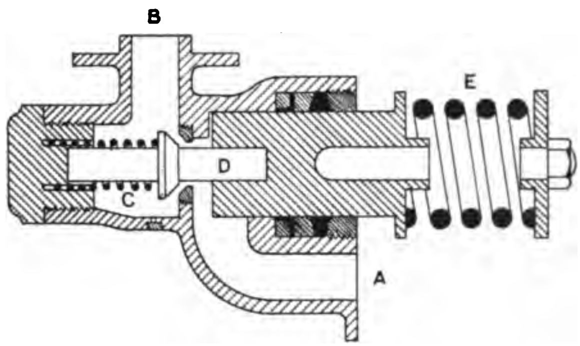


FIG. 89.

(500 pounds per square inch), forces the ram in and opens valves. The reasons for the strong spring are (1) that when the gun is run

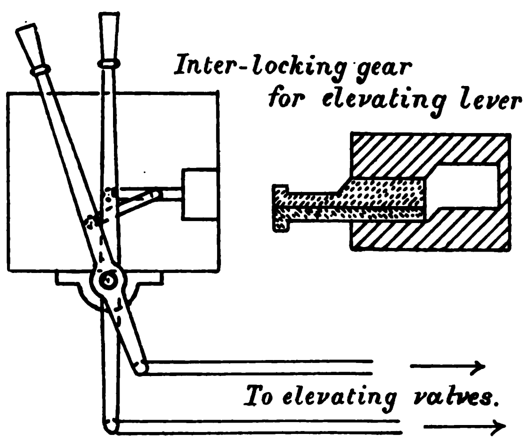


FIG. 90.

out, should the cut-off gear fail to act, it may strike heavily against the front stops and rebound, thus setting up a pressure in rear slightly above the working-pressure. The spring must be strong

enough to overcome this pressure if it is desired to run the gun "in" again. Otherwise, there would be no outlet for the exhaust from the rear. (2) When holding the gun on the slide at high inclination, the pressure in rear of press, due to weight, may be as much as 400 pounds per square inch.

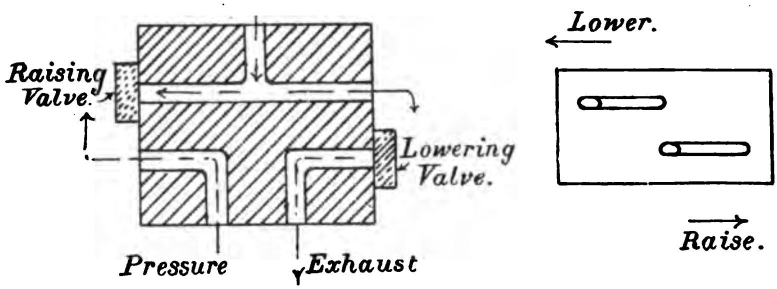


FIG. 91.

§ 80. **Diagram of Elevating Valves and Levers.**—To elevate the gun, two elevating cylinders are placed under each slide girder. The cylinders are put to pressure for lifting, and to exhaust for lowering (Fig. 90).

The two valves are in one valve-box (Fig. 91)—one for lower-

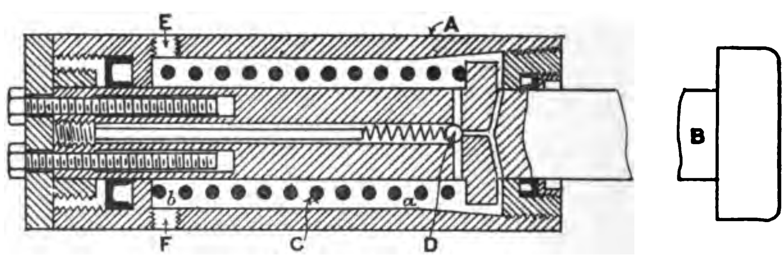


FIG. 92.

ing, the other for raising. The pressure in the elevating cylinder acts under the raising-valve, and on the top of the lowering-valve. There is an interlocking gear to prevent both the valves opening at once (Fig. 90).

§ 81. **Hydraulic Buffer.**—The hydraulic buffer, A, is filled with

mineral oil. There are two buffers, B (Fig. 92), which are attached to the turn-table, and the buffer-stops to the ship. When the buffer comes into action, a pressure is exerted on the oil, which is forced past the piston B, until the piston reaches the parallel part of the cylinder. The spring C is not fitted for buffer action, but returns the buffer to the "out" position. The ball-valve D, fitted in the case-piston, is driven so far as the parallel portion of the cylinder (shown by AB), when there would be no clearance, to allow of the pressure of oil to the rear. The filling hole is shown at E, the drain at F.

§ **82. "Hindustan" Class.**—The recoil cylinder fitted in the *Hindustan* is shown in Fig. 93. It is of the type where the area of discharge gets less as the recoil takes place for 9·2 inch guns. The first condition laid down in § 76 is accomplished principally by means of the recoil cylinder, A, and partly by steel springs. The second condition is performed by these springs.

The energy of recoil is absorbed as follows: When the gun recoils, it moves from right to left, in the direction of the arrow C (Fig. 93), and consequently the fluid in the annular space around the piston-rod is compressed, and can only escape through two channels—one through the small opening, D, between the slot in the piston and the valve-key, and also through the ball-valve in the piston into the central hollow space of the rod, the space increasing in volume as the gun recoils. The ball-valve is not shown in the figure, but it has been illustrated in Fig. 92.

The area of the opening, D, is greatest at section (*cd*), and decreases after that, until at (*ef*) it is practically zero. It is this varying opening which, restricting the free passage of the fluid from one side of the piston to the other, and therefore compressing the fluid which forms the brake.

The gun as it recoils pulls two rods (not shown in the figure) in the direction of the arrow, C, and these rods compress the springs (as in the hydraulic buffer, Fig. 92), and therefore offer a certain resistance to the recoil of the gun; immediately the gun is brought to rest, or, in other words, when the recoil is completed, the compressed springs commence to return the gun to the "firing position"; but, in order that the speed of the gun while running

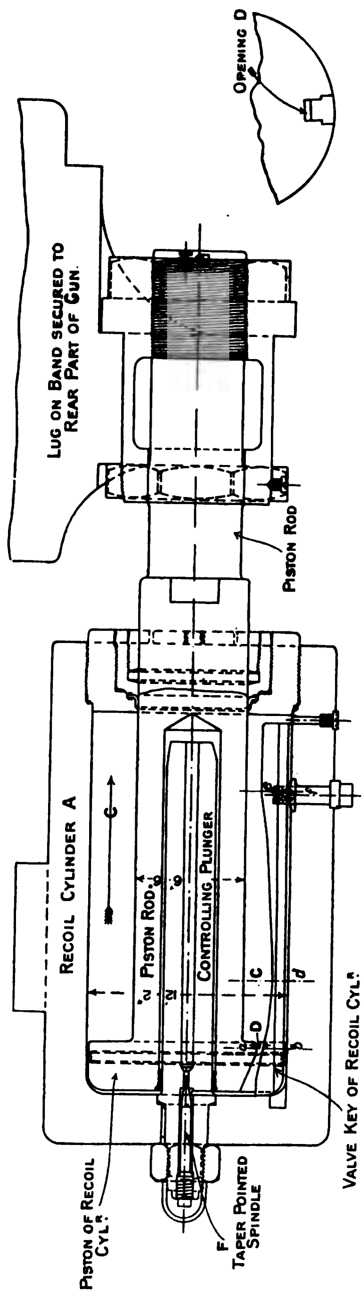


Fig. 93.

out shall not be excessive, and that the gun shall be brought to rest without shock, a "run-out control" is fitted, consisting of a controlling plunger, E, with pointed spindle (Fig. 93), and the ball-valve in the piston (not shown but similar to the ball-valve in Fig. 92).

When the springs push the gun, and the recoil piston goes back to the firing position, the fluid in the hollow space of the piston-rod is compressed, and closes the ball-valve, and can only escape to the other side of the piston through a small space around the taper point of the spindle; and the amount of this opening controls the speed of running-out; for, as the escape of the fluid is throttled in this manner, it is compressed, and therefore offers a resistance to the running out, thus preventing, as before mentioned, an excessive run-out speed.

Fig. 93A shows the

variation of pressure in the recoil cylinder during recoil. They were taken with a Crosby ordnance indicator. The maximum pressure is 5700 pounds per square inch, the length

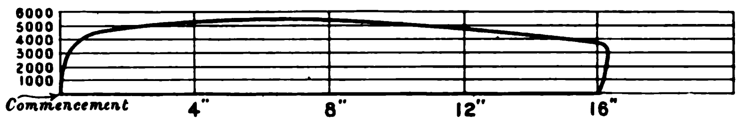


FIG. 93A.

of recoil is 16·1 inches, and the mean pressure per square inch 4745 pounds.

§ 83. **Hydraulic Riveter.**[1]—This machine is shown in Fig. 94. The water from the accumulator is admitted to the cylinder and exhausted from it through the same aperture, A, by means of a

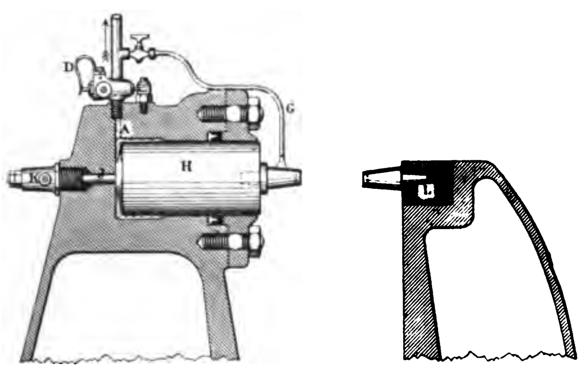


FIG. 94.

simple hydraulic valve of ordinary construction, shown in sectional plan, Fig. 95. The water enters at B, which tends to keep the inlet valve, C, shut, the spring, D, also doing this until the pressure from the accumulator begins to act. When the water has been admitted to the cylinder, the valve, C, is opened by the

[1] *Proceedings of the Institute of Mechanical Engineers*, Liverpool Meeting, 1872.

hand lever, E, and is kept open by hand until the rivet is closed as it is wished to stop the ram at any portion of its stroke. The exhaust valve, F, is kept shut by the pressure of the water entering the cylinder, and at other times by the spring D. When it is desired to draw the ram back, the exhaust valve, F, is opened by pushing the hand lever over the reverse way to that for opening the inlet valve, C; this allows the exhaust water to flow back upon the die to cool it, through the pipe, G (Fig. 94). The ram, H (Fig. 96), is drawn back by means of the small cylinder, J, which is arranged within the ram itself and is in constant communication with the accumulator through an inlet at K. The handle, E, unships readily, and is taken away by the man whenever absent from the machine. By this plan of valves, in conbination with the drawback, the greatest possible control is obtained over the machine, the ram being motionless as soon as the hand lever is released, or removed from the valve.

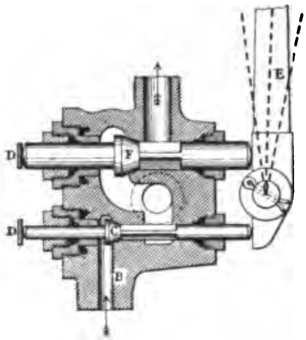


Fig. 95.

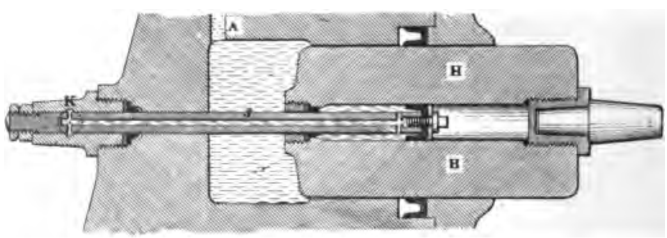


Fig. 96.

§ 84. **Hydraulic Lift.**[1]—This hydraulic balance lift is shown in Fig. 97. The lift cylinder is in hydraulic connection with a second and shorter cylinder, B, below which is a cylinder, C, of larger diameter. There is a piston in each, connected

[1] *Proceedings of the Institute of Mechanical Engineers*, Mr. A. A. Langley, January, 1882.

by the ram, D. The capacity of the annular space, JJ, below the upper piston is equal to the maximum displacement of the lift ram, A. The annular area, E, of the lower piston, C, is sufficient, when subjected to the working pressure, to overcome friction and lift the net load; and the full area, B, of the upper piston is sufficient, subjected to the same pressure, to balance with a small amount the weight of the ram and cage at the bottom. When the parts of the apparatus are properly proportioned, the lift ram and the balance pistons are in equilibrium in every position; or, in other words, the displacement of the ram of the lift cylinder is automatically balanced.

The mode of action of the lift is as follows. Assuming the cage to be at the bottom of its stroke, the valve is opened from the cage by means of a rope or system of levers, and pressure is thereby admitted to the annular area of the lower piston at E. The top of the upper piston is always subjected to the same pressure. The pressures on the two pistons thus act in the same sense on water in the annular space J, below the upper piston; and the intensified pressure of this water is transmitted through the pipe, H, to the lifting ram, A, which thereupon ascends. As it ascends, the ram increases in apparent weight, but at the same time the pistons B and C descend, and are thereby subjected to an increasing head of water, which increased head, acting upon the large area of the pistons, exactly balances the increase of

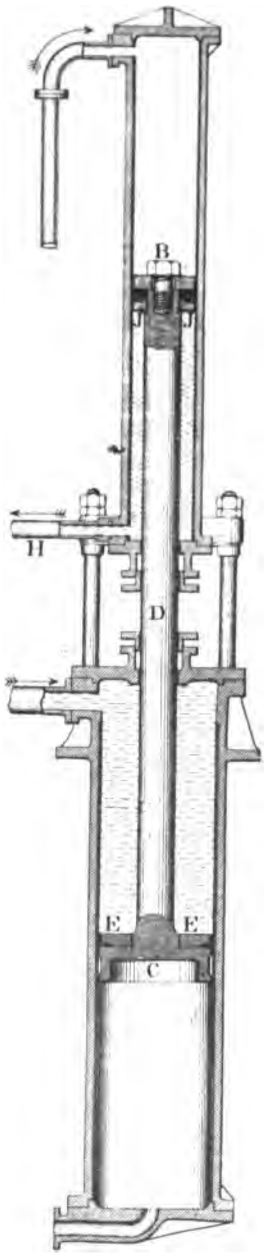


FIG. 97.

weight of the ram, or—to state the case more accurately—compensates for the loss of effective head in the lift cylinder. When the ram reaches the top of its stroke, the valve is closed, and the lift stops. On opening the valve to the exhaust, the pressure is relieved from the space above the piston C, while the piston B remains subjected to the working pressure above it, as in ascending. The lift now descends: the weight of the ram and cage, pressing upon the water in the lift cylinder, transmits the pressure to the annular area at the bottom of the piston B, and overbalances the weight of the pistons and the pressure on the top of the piston B. As the lift ram descends into its cylinder, it displaces the water and loses weight, or, in other words, encounters an increased resistance to its descent. At the same time the two balance pistons ascend, and the pressure above each of them decreases; the decrease in the pressure being in proportion to the increased pressure on the area of the lift ram. The lift ram and the pistons B and C are, as stated, in constant equilibrium. To make good any possible leakage, provision is made for admitting the working pressure through a cock under the piston C, and so raising it, while the cage is at the bottom; this relieves the pressure in the annular intensifying chamber, J, and allows water from above the upper piston, B, to flow down past the packing leather of that piston and replenish the space J. As a general rule, the part of the lower cylinder underneath the piston C is not filled with water in the regular working of the lift, but is open to the atmosphere. If, however, the cock controlling the admission is closed, during the descent of the cage, the rising of the piston C creates a vacuum beneath it, which becomes available as lifting power for the next ascent. In other words, the weight of the descending load is by this means utilized to augment the lifting power in the next ascent of the loaded lift; or, if the lift is being used for the purpose of lowering goods, the vacuum supplies power enough for raising the empty lift without the expenditure of any power at all.

§ 85. **Eiffel Tower Lifts.**[1]—In these lifts, the two ends of the

[1] By Mons. A. Ansaloni, Paris Meeting, *Proceedings of the Institute of Mechanical Engineers*, July, 1889.

hydraulic cylinder (Fig. 98) are connected by a circulating pipe, C, of 9 inches bore, at the bottom of which is placed the water distributor, D. For lifting, the water pressure is admitted into the top end of the cylinder, while at the same time the discharge from the bottom is opened. For lowering, communication is opened between the top and bottom of the cylinder, so that the pressure has access to both sides of the piston, and the water simply passes from the upper to the under side of the piston through the circulating pipe.

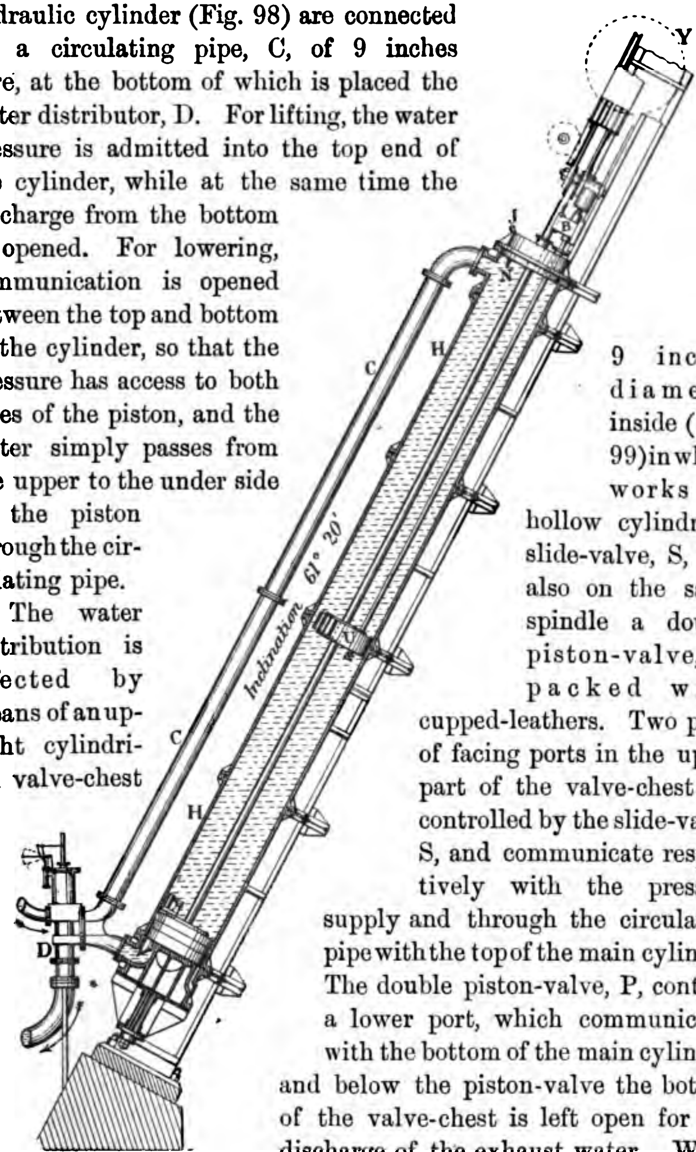


FIG. 98.

The water distribution is effected by means of an upright cylindrical valve-chest 9 inches diameter inside (Fig. 99) in which works a hollow cylindrical slide-valve, S, and also on the same spindle a double piston-valve, P, packed with cupped-leathers. Two pairs of facing ports in the upper part of the valve-chest are controlled by the slide-valve, S, and communicate respectively with the pressure supply and through the circulating pipe with the top of the main cylinder. The double piston-valve, P, controls a lower port, which communicates with the bottom of the main cylinder; and below the piston-valve the bottom of the valve-chest is left open for the discharge of the exhaust water. When the double piston-valve entirely covers the lower port, the slide-valve at the same time covers the upper

ports; the water cannot circulate, and the lift is stopped from moving. When the valve is raised, discharge takes place from the bottom of the hydraulic cylinder, while pressure is admitted to the top, and the cabin rises. When the valve is lowered, the discharge from the bottom is stopped, but the water can circulate more or less freely from the top to the bottom of the hydraulic cylinder through the interior of the hollow slide-valve, S; and the descent of the cabin is effected by its weight, which raises the movable-pulley truck as well as the main piston.

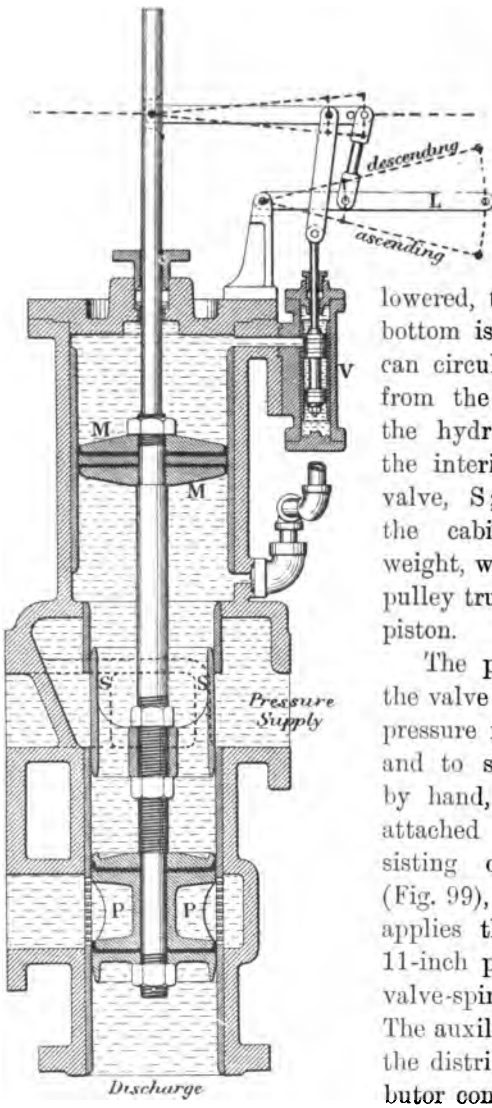


FIG. 99.

The power required to work the valve of the distributor under pressure is about 8800 pounds; and to save having to do this by hand, an auxiliary motor is attached to the distributor, consisting of a piston-valve, V (Fig. 99), $1\frac{3}{4}$ inch diameter, which applies the water-pressure to an 11-inch piston, M, fixed on the valve-spindle of the distributor. The auxiliary motor thus controls the distributor, just as the distributor controls the main hydraulic cylinder.

At each end of its journey the cabin is stopped automatically

by means of an ear or lip, E (Fig. 100), fixed on the piston of the main hydraulic cylinder, whereby the aperture of the port in either end of the cylinder is throttled just as the piston is reaching the end of the stroke. On the top of the piston is a small air valve, A (Fig. 101), which is opened by the pin, N, depressing its tail when the piston reaches the top of its stroke; the air so liberated escapes into the small chamber in the cylinder cover, when it is discharged whenever required by opening the air cock, J. To prevent the pair of long piston-rods from sagging, they are made to work through a sliding spider, consisting of a dummy piston, U (Fig. 102), inside the cylinder and a sliding block above, which are coupled together by a piston-rod of half the length of the main piston-rods; the spider thus travels up and down through half the length of stroke of the working piston, being pushed upwards by the latter and downwards by the movable-pulley track.

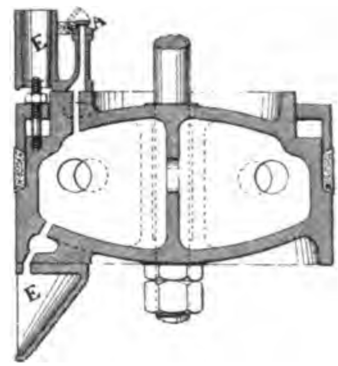


Fig. 100.

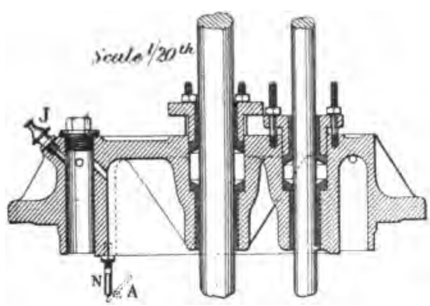


Fig. 101.

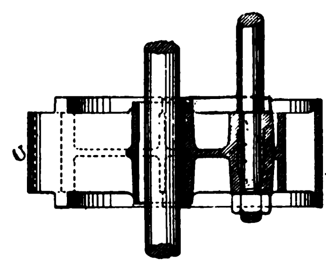


Fig. 102.

§ **86. Hydraulically operated Bulk-head Doors.**[1]—In this mechanism the feature is that the valve which controls the application of hydraulic power for closing each door may be opened automatically by the lifting of a float, owing to the raising of the water-level within the compartment in the event

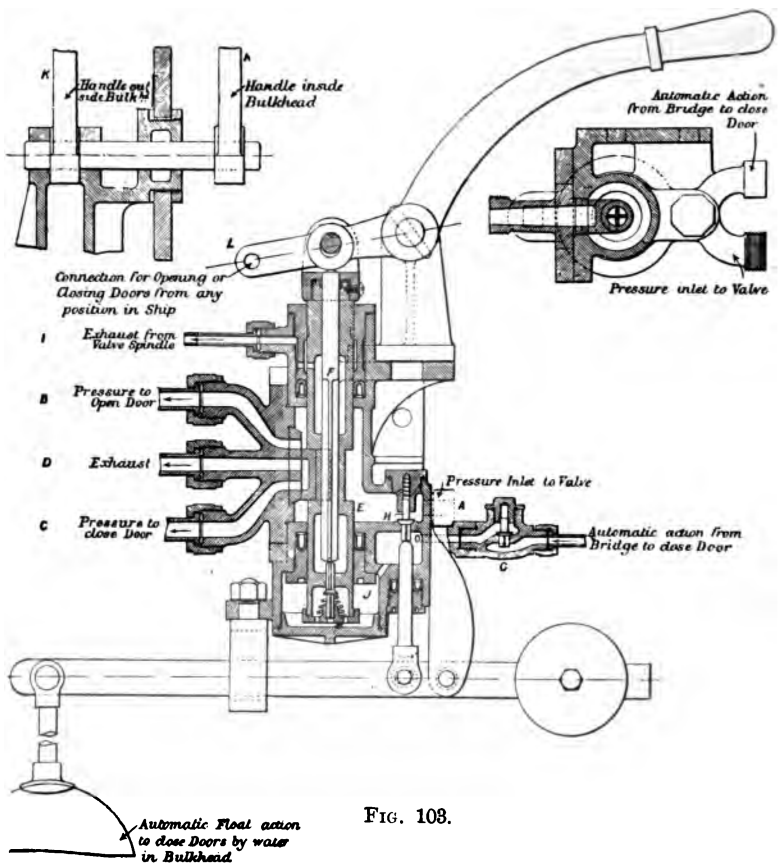


FIG. 103.

of there being accidentally made an ingress for the sea; while at the same time the mechanism may be actuated from hydraulic leads controlled by valves on the bridge and other stations in the ship, or by a hand lever on the upper deck.

[1] *Engineering*, vol. lxxii., July 12, 1901.

The valve (Fig. 103) is of the simple central spindle type, the inlet from the pumps being at A, while B is a connection to one end of a hydraulic cylinder placed about the door, and C to the other end. Within the cylinder is a rod with piston at either end; and, as the pressure is admitted at one end of the cylinder, the piston-rod within the cylinder is driven forward, gearing on the rod engaging in a pinion secured to the vertical shaft. This shaft, thus rotated, closes the door by direct gearing. Admission of water pressure to the opposite end of the cylinder causes the piston and its geared rod to travel in the opposite direction; thus the door is opened or closed.

To operate the valve by hand, the lever, K, being raised depresses the main spindle, E, allowing the water pressure to pass through the port to the connection B to the cylinder for opening the door, the water from the other end of the cylinder escaping through the exhaust, D. This is the position of the valve shown in sections. By depressing the handle, the spindle is raised, the port to C opened, and that to B connected to the exhaust, D; the result being the closing of the bulk-head door.

The action of the float is equally simple. The rise in the water-level in the compartment lifting the float, open the valve, H, admitting pressure water from the pump lead at A through ports to the chamber J, forces the valve spindle upwards, and thus admits the pressure through C for actuating the cylinder to close the bulk-head door. Should it happen that when the door has thus been automatically closed in emergency, some one is left in the compartment, depressing the lever, K, overcomes the effect of the float, thus reopening the door for a sufficient time to ensure escape; but the door will again be automatically closed by the float.

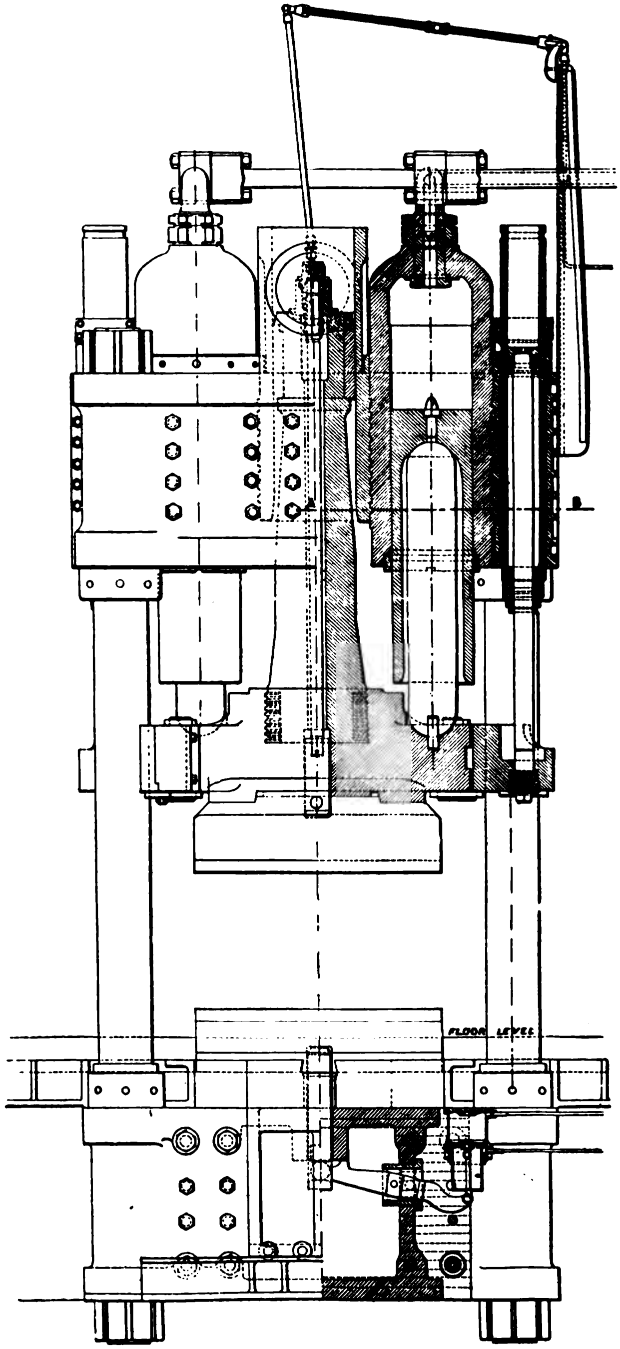
At G there is a non-return valve, which is connected to a small cock on the bridge and other stations of the ship. The opening of the cock results in the valves of all the bulk-head doors being closed in the same way as by the action of the float. The rod to the upper deck, L, it will be seen, is connected to the top of the valve by a lever.

There are thus provided four methods of closing the door by the same hydraulic cylinder, automatically through the float; by the non-return valve through the lead from the bridge and other stations on the ship; alongside the door by a lever for lowering the valve spindle; and by a lever and rod from the upper deck, also connecting with the valve spindle.

§ 87. **8000-ton Hydraulic Press.**[1]—The press consists of two hydraulic cylinders 40 inches in diameter, having a stroke of 10 feet; the body of each cylinder of cast steel is $9\frac{3}{4}$ inches thick, and is supported by cast-steel girders, carried in turn on four forged-steel columns, which are secured at the bottom ends to the cast-steel base plate, on which the anvil rests (Fig. 104). The girders have a boss at each end, through which the columns pass, being screwed at one end and secured by coned bushes and screwed nut. The pressure is transmitted to the cross-head, which is of ⊥ section, by two rods with spherical ends, one end fitted in cup bearings in the ram, and the other abutting against the cross-head, as shown in Fig. 104. The cross-head is a heavy casting of steel, the upright leg of the ⊥ being guided between the two main hydraulic cylinders, while the ends of the horizontal part bear against the columns supporting the cylinder. The cross-head is held up to the rams by lifting cylinders 15 inches diameter, placed alongside the pressing cylinders. These lifting cylinders are each fitted with a plunger of Delta metal, secured at the bottom end to the cross-head. The press is made strong enough to work with a hydraulic pressure of more than 3 tons per square inch in the main cylinders; but for the largest forgings and armour-plates yet attempted, $2\frac{1}{2}$ tons per square inch is found amply sufficient. The pressure on the bottom or the bottom side of the plunger of the lifting cylinder is 2 tons to the square inch.

§ 88. **Brotherhood's Engine.**—In the application of hydraulic power to machinery, however well it may be adapted for tools having a reciprocating motion, hydraulic motors have generally been considered not so suitable or applicable for driving machines with a rotary motion. But in Brotherhood's Three-Cylinder

[1] By Messrs. Vickers, Sheffield. *Engineering*, vol. lxiv. p. 55.



A
B
FLOOR LEVEL

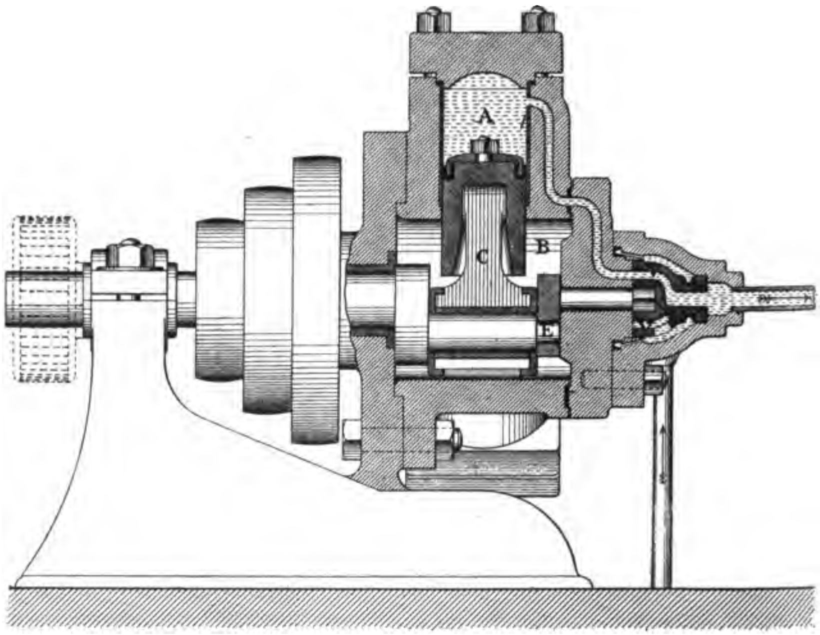


FIG. 105.

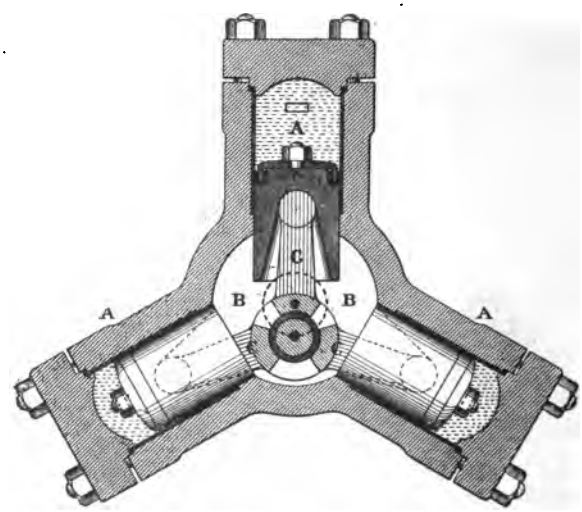


FIG. 106.

Hydraulic Engine,[1] shown in Figs. 105 and 106, all the objections hitherto met with in other attempts have now been successfully overcome. Three cylinders, A, always open at their inner ends, are attached to a central chamber, B, and a single crank-pin receives the pressure on the three pistons acting through the struts, C; the water is admitted and exhausted by means of the

FIG. 107.

valve, V, which is rotated about the eccentric pin, E. A section, taken from a model at the South Kensington Museum, is shown in Fig. 107. There being no dead centre, the engine will start in any position of the crank-pin, and a perfectly uniform motion of the shaft is obtained without using a flywheel. The pressure

[1] Made by the Hydraulic Engineering Co., Chester.

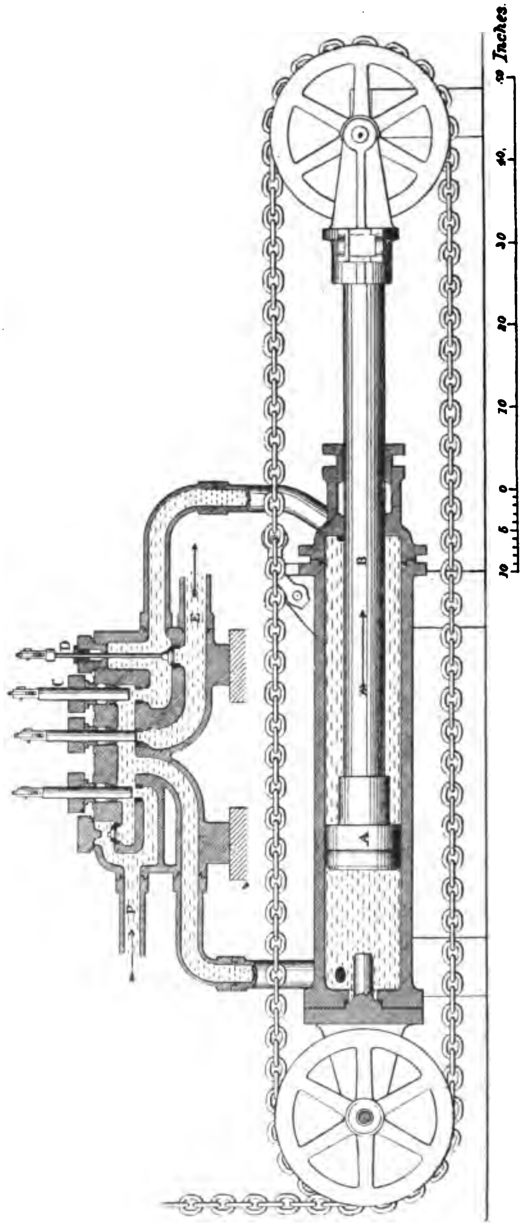


FIG. 108.

being always on the outer end of the pistons, the struts C are always in compression, and take up their own wear. The power can be taken off from the engines either by a belt or by gearing, both plans being shown in the diagram; or, as in the case of a capstan, direct from the shaft. It is particularly applicable for shop tools, because, owing to the great speed at which it can be run, it will not only save the present loss from friction of gearing for transmission, but also reduce the friction in the machine itself by dispensing with gearing for getting up speed.

§ **89. Double-power Crane.**[1]—Fig. 108 represents a double-power crane for lifting, consisting of a bored cylinder with a combined ram and piston. For the lower power, the pressure is admitted upon both sides of the piston A, and, therefore, virtually acts only upon the ram B, which is half the area of the piston. For the higher power, the front side of the piston is open to the exhaust E, leaving the pressure P to act on the back only, and the effect is then proportionate to the area of the piston, which is twice that of the ram. This alternative action is determined by the intervention of an extra valve, valve C (Fig. 108); when this valve is opened, the pressure P has access to both sides of the piston, and the lower power is then obtained; while for the higher power the valve C is closed, and the exhaust valve D is opened, whereby the front side of the piston is kept constantly open to the exhaust E. In cases where three powers are required three simple hydraulic presses are commonly used, which act either singly or in combination; but the same effect may be obtained with a bored cylinder and piston combined with two concentric rams, the external ram acting through a watertight gland in the outer ram. In this case, the lowest power is obtained by admitting the water only on the front side of the piston, when it enters into the interior of the larger ram through a hole near the piston, and forces out the inner or smaller ram. The second power is obtained by admitting the water to both sides of the piston; and the highest power is brought into action by opening the front side of the piston to exhaust, whilst the pressure operates on the back of the piston.

[1] *Proceedings of the Institute of Mechanical Engineers*, January, 1868.

The absence of elasticity in water gives great steadiness and precision to the movements of the machine actuated by water pressure; but, on the other hand, water being incapable of expanding, like steam in a cylinder, the quantity expended is not proportionate to load. Thus a machine propelled by hydraulic pressure measures off the same quantity of water whatever may be the resistance overcome; and, therefore, when the machine is inadequately loaded, the expenditure is more than equivalent to the effect produced. This loss of power may in great measure be obviated by making the machines with variable powers; but the simplicity of single-power machines renders them preferable in many cases, notwithstanding their greater expenditure of power. In fact, for the purposes of which water pressure is most usually applied, safety, simplicity, and general convenience are more to be considered than economy of power, because, owing to the intermittent character of the work, the required quantity of water is not large in the aggregate.

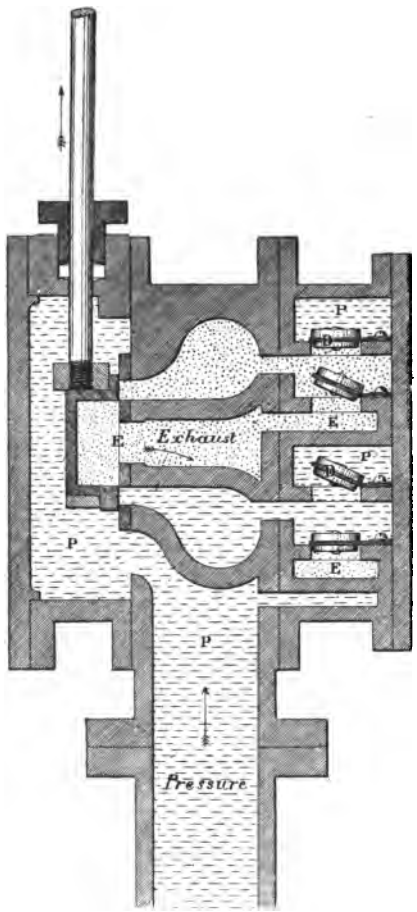


Fig. 109.

In connection with the non-elastic character of water, it will be observed that its compressibility in the cylinder of the machine would, if not provided against, cause very injurious shocks and

strains in the machinery by suddenly arresting the momentum of the moving parts on the closing of the outlet passages. To obviate this liability, nearly all the varieties of water-pressure machines adapted for rapid action require the introduction of what are termed "relief" valves. In Fig. 109, DD are small clacks opening against the pressure P in the supply pipe, and yielding to any back pressure on the pistons that exceeds the accumulator pressure; PP represents the pressure, and EE the exhaust passage.

CHAPTER IV

RECIPROCATING PUMPS

RECIPROCATING pumps may be broadly divided into three divisions.

(1) A bucket pump (Fig. 110).

(2) A single-acting plunger pump (Fig. 111).

(3) A double-acting plunger pump (Fig. 112).

§ **90. Bucket Pump.**—Fig. 110 shows a diagrammatic sketch of a bucket pump, *i.e.* a pump with a hollow valved bucket. A is the suction tank, B the suction pipe, C the suction valve, D the pump cylinder, E the bucket valve, F the plunger rod, G the delivery pipe, and H the delivery tank. Let the suction and delivery heads be h_1 and h_2 respectively. As the piston moves from left to right, the bucket-valve, E, will be closed, and the suction-valve, C, open. Consequently water will be discharged up the delivery pipe, G, and water will be pumped from A into the cylinder. During the return stroke, when the piston moves from right to left, the suction-valve, C, will be closed, and the bucket-valve, E, will be open. Thus, if the rod be of small diameter no discharge takes place during the stroke from right to left. Actually, the discharge from left to right is the cylinder volume less the volume of rod ; and during the stroke from left to right the discharge is the volume of the rod. Thus by making the area of the rod half the area of the plunger, the discharge per stroke may be made the same. But this proportion of areas will not in general equalize the *work* per stroke. For on the stroke from left to right, the head on one side is h_2, and on the other h_1 ; thus the net driving head is

$$h_2 \text{ (plunger area } - \text{ rod area) } + h_1 \text{ (plunger area).}$$

In moving from right to left, the bucket-valve is open, and the pressure on both sides is h_2. Thus the net force is rod area $\times$ h_2. If the plunger area is twice the rod area, the ratio is

$$\frac{h_2 + 2h_1}{h_2}$$

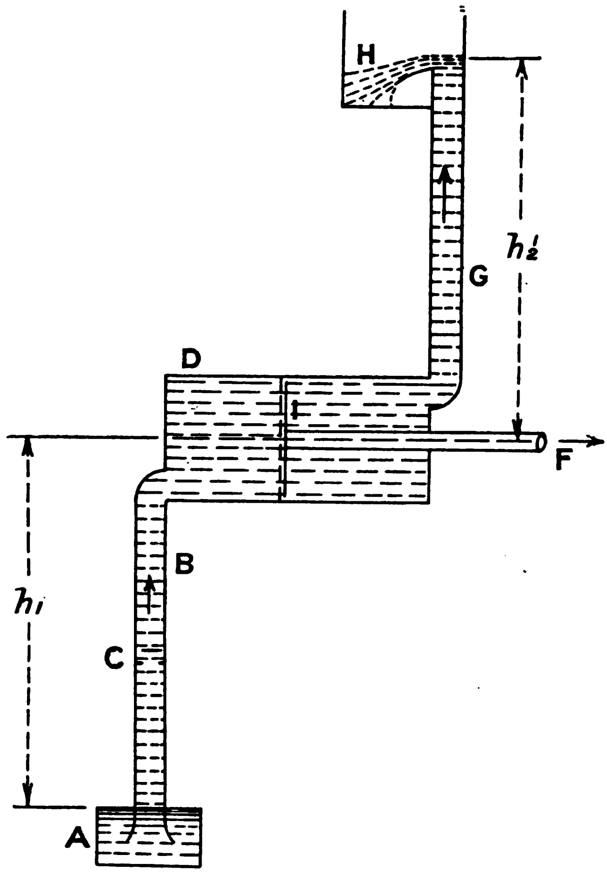


FIG. 110.

which can only be unity provided $h_1 = 0$, that is to say, provided the pump is a simple force pump and has no suction.

§ 91. **Single-acting Plunger Pump.**—A section of a single-acting plunger is shown in Fig. 111. The notation is as before, and K represents the delivery valve. The stroke is from right to left, so that water is being delivered. The valve K is open, and the valve C closed, and this takes place during the whole of the stroke. On the return stroke, the valve K closes, and the valve C opens. Thus water is pumped up into the cylinder D. This is discharged during the next stroke.

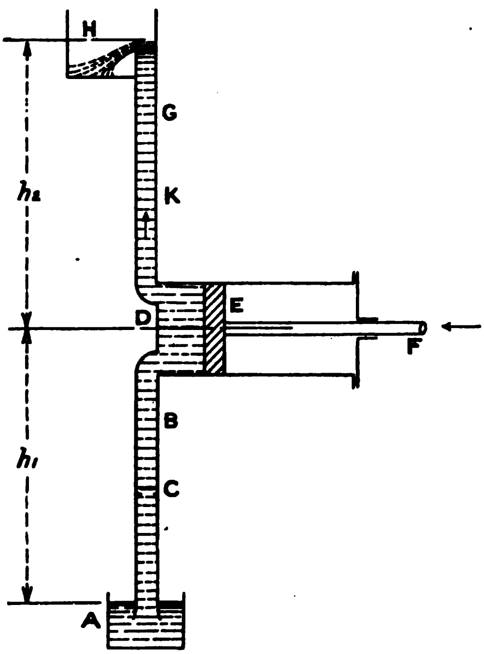


Fig. 111.

It is impossible, in a single-acting pump, to make either the discharge or the force equalize in the two strokes.

§ 92. **Double-acting Plunger Pump.**—Fig. 112 represents a double-acting plunger pump. The two suction valves are C_1 C_2; and the delivery valves are K_1 K_2. The plunger is supposed moving from right to left. The water therefore flows from the suction tank through B and the valve C, and fills the right-hand side of the pump barrel; at the same time the water is delivered through the valve K_1 into the delivery pipe G. This operation takes place throughout the stroke. During the stroke from left to right, the operations are repeated, the valves C_1 and K_2 being opened, and C_2 and K_1 closed.

In this arrangement the discharges are not the same on account of the rod, neither are the forces equalized in the two strokes. But if the area of the rod is made half the area of the plunger,

the discharge is equal for the two strokes. This is adopted in the Admiralty Pump (§ 110).

THEORETICAL CONSIDERATIONS AFFECTING PUMPS.

§ 93. **"Slip" in a Pump.**—When a reciprocating pump is used for pumping water, the actual discharge is almost invariably found to be less than the actual. The reasons for this are—(1) leakage past glands and the plunger; (2) leakage due to a gradually closing valve. The latter is by far the most important, as one side of the valve is subjected to the delivery head, and the other side to the suction side.

The first loss can only be reduced by good workmanship, and constant attention to the packing.

The second loss, due to the valve gradually closing, may be calculated as follows:—

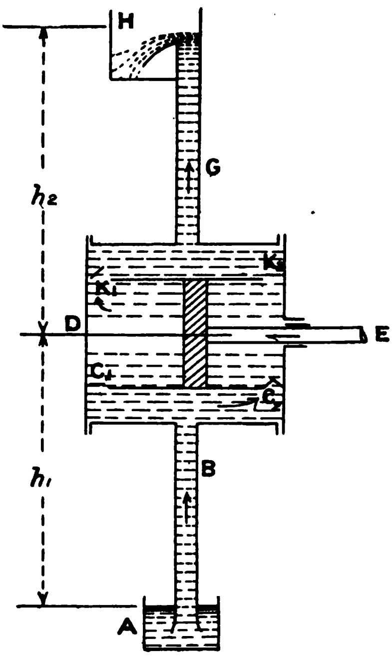


FIG. 112.

Let A be the area of the plunger, a the area of maximum opening of valve, s the maximum lift, P the perimeter of the valve, h_1 and h_2 the suction and delivery heads respectively, and v_m the mean velocity of the piston. The loss per second by leakage

$$\propto a\sqrt{2g(h_1 + h_2)}$$

the theoretical discharge per second

$$\propto Av$$

so that the ratio of the leakage to the theoretical discharge is

$$\frac{a\sqrt{h_1 + h_2}}{Av_m}.$$

The greater the mean speed of the plunger, the less the leakage. When the valve falls, the leakage will depend on the time taken, and the perimeter and lift of the valve. Since the valve descends in water, the acceleration is

$$\frac{\text{weight of the valve}}{\text{weight in water}} = a, \text{ say}$$

so that
$$t = \sqrt{\frac{2l}{a}}.$$

The average opening is two-thirds the maximum, so that the flow back in cubic feet per stroke is

$$\tfrac{2}{3}Pl\sqrt{\frac{2l}{a}}.$$

The "slip" is proportional to the full valve area, and to the square root of the lift. With four valves, the perimeter being the same, the lift is one quarter of a single valve, and the leakage of each valve is one-eighth; the total leakage being one-half the single valve. The leakage could be reduced, in addition, by increasing the perimeter. For efficient working, large low lift valves must be used.

Frequently the valves are held down by springs. This enables them to close rapidly, but more work has to be spent, on both delivery and suction valves, in opening them; and this work is not returned during the down-stroke. As a matter of practice, mechanically controlled pump valves are seldom used in Germany, but they are used by many firms in this country. Some loaded valves, seen by the author, required a very great pressure to close them.

Another important point connected with reciprocating pumps is that the major part of the resistance is due to the valves themselves. The water escapes, from both the delivery and suction

valves, with considerable velocity, and eddies are freely developed. This is unavoidable, and the pipe-friction is a small proportion of the total. Thus, in a pump working under a low lift, the efficiency must necessarily be very small; and the higher the lift, the more efficient the pump.

§ 94. Curve of Effective Head on Plunger of a Single-acting Plunger Pump.—The simplest case to consider is a single-acting plunger pump (Fig. 113). Let h_1 h_2 be the suction and delivery

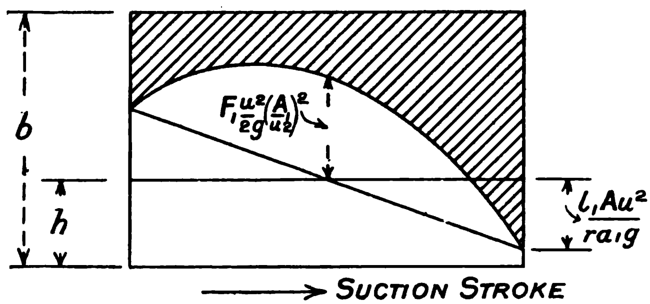


FIG. 113.

heads; a_1 a_2, the sectional areas; and l_1, l_2, the lengths of the suction and delivery pipes respectively; v_1, v_2, the velocities in the suction and delivery pipes; and F_1, F_2, their coefficients of hydraulic resistance referred to the plunger velocity.

Suction Stroke.—Consider the suction stroke when (Fig. 113) the plunger is moving from left to right. At rest, the pressure on the plunger is

$$b - h_1.$$

When the plunger moves to the right, the pressure is reduced, and the suction valve opens, and water flows into the pump, the valve K being closed. The water in the suction stroke has to be accelerated; towards the end of the strokes the water is retarded.

As in pressure engines (§ 67), assume the piston moves with simple harmonic motion. The static pressure is

$$b - h_1.$$

The head due to inertia of the water in the suction pipe at the two ends of the stroke is

$$\frac{l_1 A u^2}{g a_1 r}.$$

The frictional head at the middle of the stroke is

$$F_1 \frac{u^2}{2g}\left(\frac{A}{a}\right)^2.$$

The effective head therefore when the plunger is at a distance x from the centre is (§ 67)

$$b - h_1 - \frac{l_1 A u^2}{r a_1 g} - \frac{F_1}{2g} h^2\left(1 - \frac{x^2}{r^2}\right)\left(\frac{A}{a}\right)^2.$$

The curve of effective head is shown in Fig. 113.

In Fig. 113 the pressure head is everywhere positive. With long pipes and excessive frictional resistance, the curve may rise above the atmospheric line, as shown in Fig. 114. In that case, the

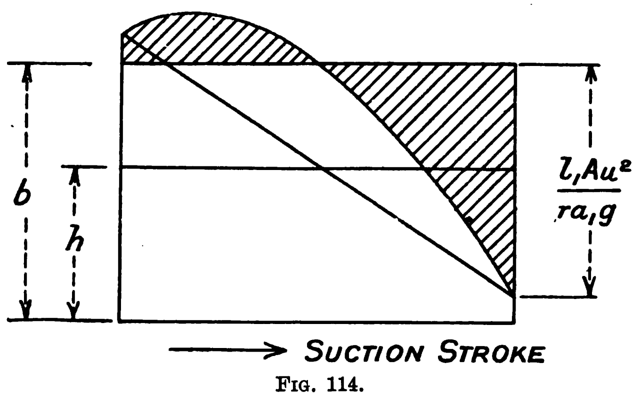


Fig. 114.

curve towards the early part of the suction stroke lies about the b-line, and the pressure head becomes negative. A cavity is at once formed, and the water lags behind the plunger towards the end of the stroke, and the water catches up to the plunger and shock takes place.

The condition necessary to prevent separation is, at the beginning of the stroke,

$$b - h_1 > \frac{l_1 A u^2}{r a_1 g}.$$

This condition may be satisfied, but the crest of the curve may be in the line of stroke. If $\frac{a_1 l}{F_1 A} > r$, the condition is that first stated; but if $\frac{a_1 l}{F_1 A} < r$, then

$$b - h_1 > \frac{F_1 u^2}{2g}\left(\frac{A}{a}\right)^2\left(\frac{a_1^2 l_1^2}{F_1^2 A^2 r^2} + 1\right). \qquad (\S\ 68)$$

§ 95. **Separation.**—The absolute head is therefore much reduced during the commencement of the suction stroke and increased towards the end of the stroke. As long as the parabola lies below the b-line, the motion assumed is theoretically possible. But if the parabola cross the b-line, the motion assumed involves a pressure less than absolute zero on the plunger, which is impossible. Separation, therefore, takes place. The acceleration of the piston is so great that the available head is not sufficient to cause the necessary acceleration of the water in the suction pipe. The water does not keep pace with the plunger, but a cavity is formed between the two. Later on in the stroke, the plunger is retarded and the water again catches up with the plunger, so that a shock takes place. This separation of the water and the plunger is very much more liable to happen in a pump than in an engine. In an engine it can only take place by the flywheel acting as a motor when the engine is working at a load much less than the maximum. But in a pump, the plunger is driven from a steam engine, and therefore separation may very well take place even when the pump is working under normal conditions.

The actual separation between the water and the plunger, here explained, represents a theoretical limit. Before the absolute zero of pressure is reached, the air held in solution will be liberated and water vapour would also be formed. There would, therefore be a cavity between the water and plunger filled with an expansive

mixture which would be automatically reabsorbed as the pressure increased. To a certain extent, therefore, the prejudicial effect

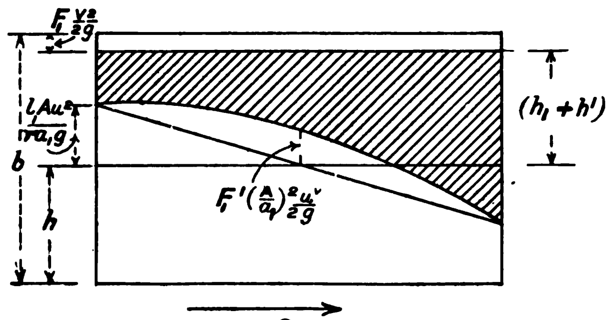


FIG. 115.

of shock might be modified, but it ought, if possible, to be prevented.

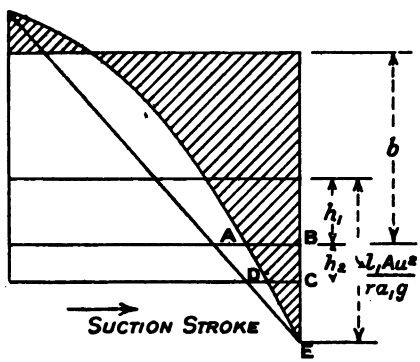


FIG. 116.

§ 96. Air-chamber on Suction Pipe.—The tendency to separation increases as the length of pipe increases, and may be partially remedied by using a suction pipe of large diameter. If this size is prohibitive, an air vessel may be fitted on the suction pipe, near to the pump. The action of the air-chamber is discussed in §§ 101, 102. The inertia effect is practically confined to the short length of pipe between the chamber and plunger cylinder; the flow in the suction pipe between the suction tank and air-chamber being sensibly uniform (Fig. 115).

§ 97. Suction Valve Inoperative.—Again, considering Fig. 116, at the point A the pressure head in the plunger cylinder is b. If the pump were merely a suction pump, this would be the head on the delivery valve. But the delivery head is h_2, hence set down

the distance BC equal to h_2, and draw a line CD. The head $(b + h_2)$ is the head on the delivery valve; but from D to C, the head in the cylinder is greater than $b + h_2$. Thus towards the end of the suction stroke, the delivery valve opens, and water flows up the delivery pipe. To start the water in the delivery pipe in motion will take some interval of time, and it will continue after the plunger reaches the end of the stroke. At the beginning of the return stroke, the suction, normally, would immediately close, but so long as the ascending motion takes place, so long will the valve remain open. In other words, the suction valve will remain open during some portion of the return stroke.

The analysis of the problem is difficult. The area EDC represents a certain amount of energy which is not returned to the crank, and which is expended in forcing the extra supply of water up the delivery pipe. These results are considerably modified by friction. Frictional losses in the suction pipe delay the premature opening of the delivery valve and decrease the work available for causing flow. When extra flow commences, the velocities are increased and extra work must be expended. The water in the delivery does not affect the energy, but it affects the time during which the operation takes place. Thus the pressure on the suction valve is greater than the inertia and frictional heads, and will probably be forcibly closed.

§ **98. Bucket-Pump.**—This type tends to a more accurate solution than the single-acting pump. Referring to Fig. 110 the absolute head on the delivery side is

$$b - h_1 - \frac{l_1 A u^2}{r a_1 g} \cos a - F_1 \frac{A^2 u^2}{a_1^2 2g} \sin^2 a$$

a being the crank angle, and on the driving side the head

$$b + h_2 + \frac{l_2 A_2 u^2}{r a_2 g} \cos a + F_2 \frac{A^2 u^2}{a_2^2 2g} \sin^2 a$$

so that effective head which has to be overcome is

$$h_1 + h_2 + \frac{Au^2}{2g}\left(\frac{l_1}{a_1} + \frac{l_2}{a_2}\right) \cos a + \frac{A^2 u^2}{2g}\left(\frac{F_1}{a_1^2} + \frac{F_2}{a_2^2}\right) \sin^2 a.$$

When this is zero the delivery valve opens prematurely. In

this case, the water in the delivery pipe is in motion, as well as that in the suction pipe, and, therefore, has not to be started from rest. Notwithstanding, the extra quantity of water causes an increased velocity in both pipes. But in the return stroke no work is done, and the suction valve can remain open as long as it naturally chooses.

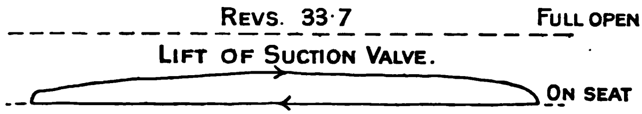


FIG. 117.

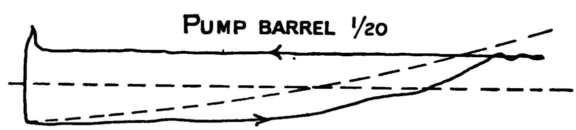


FIG. 118.

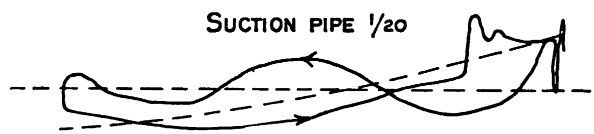


FIG. 119.

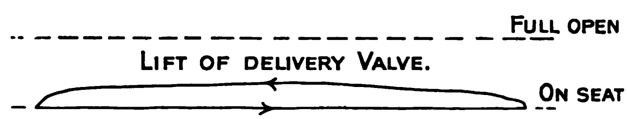


FIG. 120.

A double-acting pump is intermediate between the two cases.

§ 99. **Experiments on "Slip" in Pumps.**[1]—Professor Goodman has made experiments on "slip" and separation in a small pump. The diameter of the ram was 2 inches, the stroke 24 inches, diameter of suction pipe 2 inches, diameter of delivery pipe $1\frac{1}{2}$ inch. The length of suction pipe was 63 feet.

[1] *Proceedings of the Institute of Mechanical Engineers*, 1903, i.

Professor Goodman took indicator cards in the pump cylinder and suction pipe, and he also recorded the lift of the suction and delivery valves. The curves are very interesting. The critical speed by calculation was 60·5, and the experimental between 56 and 62.

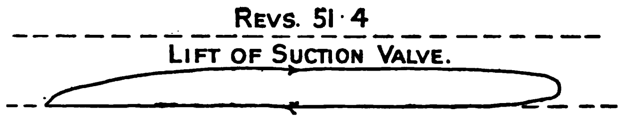


FIG. 121.

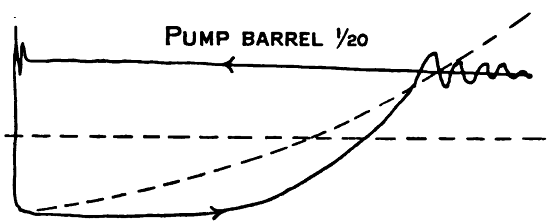


FIG. 122.

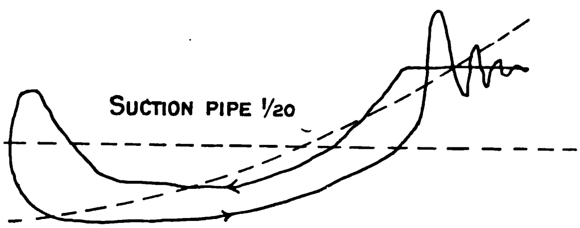


FIG. 123.

FIG. 124.

When the revolutions were 33·7 per minute, the results are shown in Figs. 117–120, which give the lift of the suction valve, the pump barrel indicator card, the indicator card for the suction pipe, and the lift of the delivery valve. Figs. 121–124 refer to a speed of 51·4 revolutions per minute.

Professor Goodman tried the effect of an air-chamber on the suction pipe of capacity 0·3 cubic feet, and the results are shown in Figs. 125–128, the diagrams showing the corresponding result.

In the cases previously considered, the obliquity of the rod

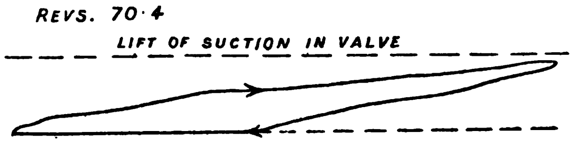


FIG. 125.

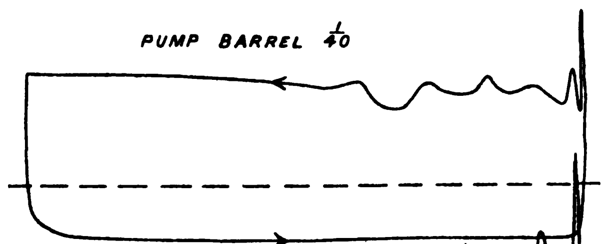


FIG 126.

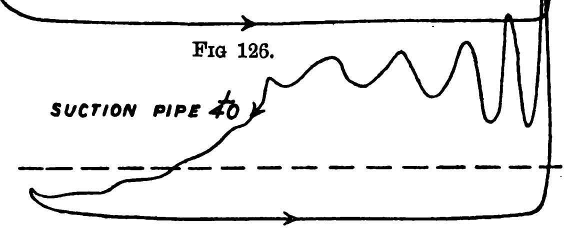


FIG. 127.

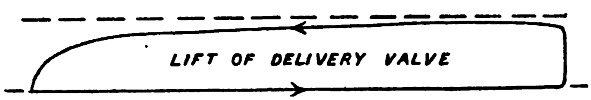


FIG. 128.

was neglected. In these experiments it has been taken into account, as shown by the dotted line.

§ 100. **Alternative Solution to Separation.**—A letter appeared in *Engineering,* April 10th, 1903, on the question of "slip." The writer takes a concrete case from Professor Goodman's

paper, and the results are plotted on a time base. Referring to Fig. 129, the stroke commences at O, its time of 0·43 seconds expires at A. The water in the suction pipe commences to move 0·0156 second after O, that is to say at C. The curve FG shows the acceleration, CE the velocity of the suction water, and the curve CD the distance passed over by the suction water. If, now, the curve of distance passed over by the plunger is plotted (the broken line OHJ, which commences at O), it is found that the water only follows at C, reaches the plunger for a short time, but very soon separates again, and only reaches the plunger again on the return stroke, at the point K, and at a velocity of 10·2 inches per second, as shown by the line MBL, representing the distance of the velocity curves of water and plunger —by the dotted line—at the line OB. Fig. 129 shows the point C_1 when the suction water reaches the plunger on the return stroke.

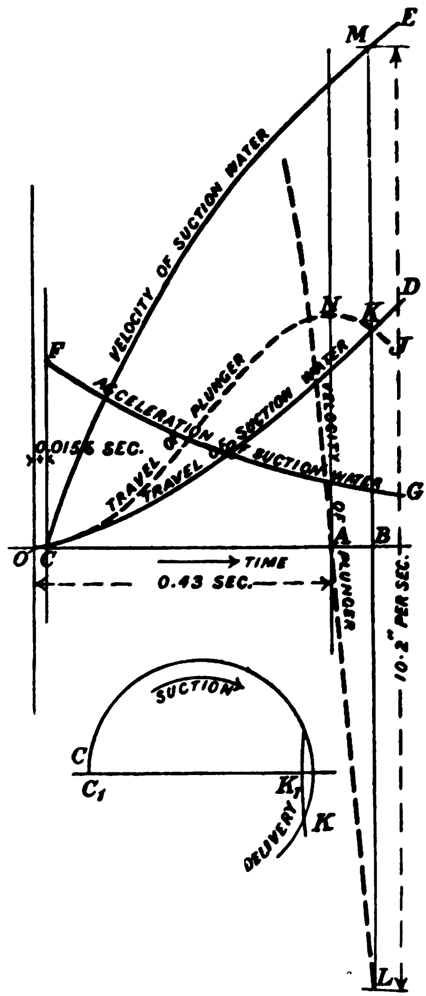


FIG. 129.

Apparently this diagram did not agree with the actual pump diagram, in which the water catches up to the plunger nearly exactly at the end of the stroke. The writer of the letter points out that in the

pump referred to the plunger *must* have leaked, on account of the low vacuum.

§ 101. **Air-chamber.**—It has been pointed out that separation is much more likely to occur in a pump. The less the diameter of the suction pipe, the greater its length; and the less the suction head, the greater the tendency to separation.

The evils may be entirely remedied by placing what is called an air-chamber on the suction pipe. This, in Fig. 130, C is the suction pipe, and D the cylinder. Placed as near to the cylinder as possible is a branch pipe, L, leading to air-chamber, M. This chamber is partially filled with water and partly with air under pressure. During the suction stroke, the water will flow more readily from the air-chamber to the pump barrel than from the suction tank, on account of the short length of pipe between the air-chamber and cylinder, but also on account of less frictional resistances. This happens during the first part of the suction stroke, when the piston is being accelerated; but in the latter part of the suction stroke, when the piston is coming to rest, the water will commence to flow into the air-chamber, M. Thus the air-chamber is subjected to a fluctuation of pressure being greatest from the suction pipe, C, when the plunger is near the left, and least when the plunger is near the right. But, assuming the fluctuation of pressure in the air-chamber to be small, the water in the suction pipe between the air-chamber and cylinder is practically moving with uniform velocity; whilst that in the suction pipe is moving in an irregular manner.

§ 102. **Theory of Air-chamber** (Fig. 130).—Let h' be the height of the level of the water in the air-chamber above the centre of the pump cylinder, and let h'' be the head in the air-chamber. These will be reckoned absolute. If the volume of the air-chamber be assumed large compared with the discharge per second, the variation of pressure in it may be neglected. Also assuming that no separation of water and plunger takes place at the end of the stroke, the air-chamber will have supplied the volume $A \cdot 2r$ cubic feet to the cylinder. In the mean time, a certain quantity of water has reached the air-chamber from the suction tank, though during the first stroke this quantity will be smaller than

that which has flowed out of the air-chamber; because, at the start, the head available for flow in the suction pipe is zero, and it only assumes a definite value when the pressure in the air-

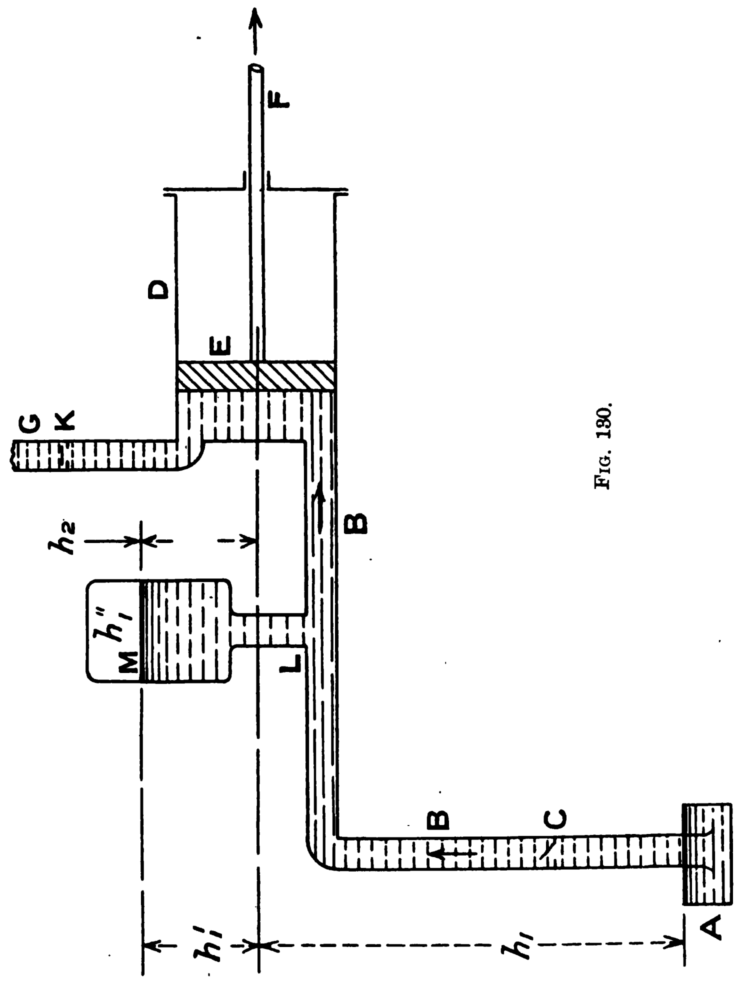


FIG. 130.

chamber becomes smaller than $h_1' + h_1''$, that is to say, when more water is drawn from the air-chamber by the plunger than is supplied by the suction tank. Thus the motion in the suction

M

pipe is practically continuous—the variation in h'' being neglected—whilst that between the air-chamber and plunger cylinder varies.

First Approximation.—In order to analyse more closely the action of the air-chamber, let it be assumed that a permanent *régime* has been established. Then the effective head causing flow from the suction tank is

$$b - h_1 - h' - h''.$$

If v_1 is the velocity in the suction pipe, F_1 the coefficient of hydraulic resistance between the suction tank and air-chamber, including loss at entrance to the air-chamber—then

$$b - h_1 - h' - h'' = F_1 \frac{v_1^2}{2g}.$$

But $a_1 v_1 = \frac{Au}{\pi}$ or $\frac{2Au}{\pi}$ according as to whether the pump is single or double acting; and, therefore, h'' may be calculated from the equation

$$a_1\sqrt{\frac{2g}{F_1}(b - h_1 - h_1' - h_1'')}\ \frac{Au}{\pi} \text{ or } \frac{2Au}{\pi}.$$

The water between the air-chamber and pump moves in a simple harmonic manner. Let dashed letters refer to the pipe between the air-chamber and pump. Let F_1' be the coefficient of hydraulic resistance, which includes losses in air-chamber, in connecting pipe, and in getting into the pump cylinder. The diagram is shown in Fig. 115. If the water level in the air-chamber is below the level of the cylinder, h'' is negative.

Second Approximation. — Suppose when the water in the suction pipe is moving uniformly, the average head in the air-chamber, absolute, is h'', the velocity being v. The available head $b - h_1 - h_1' - h_1''$, being expended in overcoming frictional resistance, and not in giving acceleration to the water. Let h_1'' and h_2'' be the maximum and minimum pressure heads in a cycle, h_1'' being greater than h'', and h_2'' less than h_1''. When the pressure head in the chamber is greater than h'', the water in the suction pipe is being retarded; when it is greater, it is being accelerated. In the

regular work of the pump, the work spent in retarding must be equal to the work spent in accelerating. Since the air is in contact with water, the operations will be isothermal, and therefore the compression and expansion of the air will follow the isothermal law.

Thus $$\log_e \frac{h_1''}{h''} = \log_e \frac{h''}{h_2''}$$

or $$h_1'' h_2'' = h''^2.$$

Suppose, moreover, that the total fluctuation of volume in the air-chamber is mV, where V is the piston displacement per stroke; and let V_a = volume of air in chamber when the pressure head is h''. Then, the greatest and least volumes are

$$\frac{h''V_a}{h_2''} \text{ and } \frac{h''V_a}{h_1''}$$

and, consequently, $$h'V_a\left(\frac{1}{h_2''} - \frac{1}{h''}\right) = \nu V.$$

Thus $$h_1'' = h''\left\{\frac{mV}{2V_a} + \sqrt{1 + \left(\frac{mV}{2V_a}\right)^2}\right\}$$

$$h_2'' = h''\left\{-\frac{mV}{2V_a} + \sqrt{1 + \left(\frac{mV}{2V_a}\right)^2}\right\}.$$

Call these angles a_1 a_2. The first angle corresponds to a minimum volume, and the second to a maximum volume, that is to say that the maximum pressure head is h_1 and the minimum h_1. The total fluctuation is

$$dq = Av\int_{a_1}^{a_2}\left(\sin a - \frac{1}{\pi}\right)da, \text{ since } u = r\frac{da}{dt}$$

$$= Ar\left(\cos a_1 - \cos a_2 - \frac{a_2 - a_1}{\pi}\right)$$

for a single-acting pump.

$$= Av\left(\cos a_1 - \cos a_2 - 2\frac{a_2 - a}{\pi}\right)$$

for a double acting pump.

By substitution, these become

$$1{\cdot}10 Av = 0{\cdot}55 V, \text{ for a single-acting pump.}$$
$$0{\cdot}42 Av = 0{\cdot}21 V, \text{ for a double-acting pump.}$$

For two double-acting engines working on cranks at 90°, the coefficient is 0·042. Thus h_1'' and h_2'' can be calculated. The total volume of the air-chamber if the pipes remain full is

$$\frac{h''V_2}{h_1 - h_1''}.$$

Thus, since h'' is known from the previous equation, the values of V and V may be taken to be known, and m is determined in the following manner :—

At the instant that the crank-angle is a, the flow in time dt from the air-chamber to the pump is Au sin a dt, and along the suction pipe to the air-chamber is $a\ v\ dt$ (neglecting any variation in velocity). The air-chamber is, therefore, emptying at the rate of

$$Au \sin \theta - av) \text{ cubic feet per second}$$
$$= Au\left(\sin \theta \frac{1}{\pi}\right) \text{in a single-acting pump}$$
$$= Au\left(\sin \theta \frac{2}{\pi}\right) \text{in a double-acting pump.}$$

This is zero, that is to say, the supply is equal to the demand, when

$$a = 18° \ 35' \text{ and } 160° \ 25' \text{ for a single-acting pump.}$$
$$a = 39° \ 35' \text{ and } 140° \ 25' \text{ for a double-acting pump.}$$

§ 103. **Delivery Stroke of a Pump.**—Next, consider the delivery stroke. The plunger moves from right to left, the suction valve C closes, and the delivery valve opens, water being delivered through the pipe G to the tank H (Fig. 111). The static head on the plunger is $b + h_2$ (Fig. 131). During the first part of the stroke, the water in the delivery pipe has to be accelerated, and so the effective driving head on the plunger is increased. During the latter part, the water is being retarded, and the effective head is reduced. If

suffix 2 refer to the delivery pipe, then the expression for pressure head corresponding to the angle a of the crank is

$$b + h_2 + \frac{l_2 A u^2}{r a_2 g} \cos a + F_2 \frac{A^2 u^2}{a^2 2g} \sin^2 a$$

and the curve is given in Fig. 131.

The initial pressure is limited by no condition; but if

$$\frac{l_2 A u^2}{r a_2 g} > b + h_2$$

then the resultant line will fall below the datum line, as shown in Fig. 132.

At the point A, the pressure becomes negative, and separation takes place. The water in the delivery pipe moves ahead of the plunger, a cavity is formed, and later the water falls back and shock ensues. A similar action takes place in the delivery as happened in the suction stroke. If a line be set down from the datum line at a distance h_1, then at the point B the suction pressure becomes greater than the pressure in the plunger cylinder, and the suction valve opens. Thus a discharge through the suction pipe takes place whilst water is being delivered.

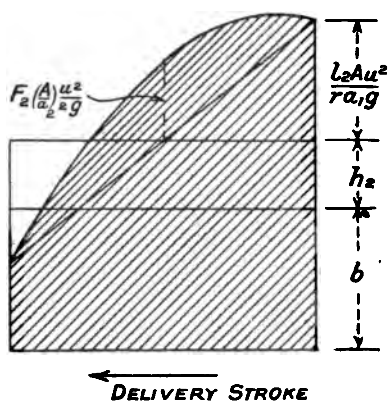


FIG. 131.

The discharge is therefore increased due to this cause. Moreover, as in the previous case, time is required to set the additional water in motion, and time is required to stop it; thus, even during the early part of the suction stroke, the suction valve will remain inoperative and water will be delivered up the delivery valve.

To prevent this additional delivery

$$b + h_1 + h_2 > \frac{l_2 A u^2}{r a_2 g}.$$

Thus, during the suction stroke, the delivery valve prematurely

opens, and the suction valve remains open during part of the delivery stroke; during the delivery stroke, the suction valve opens prematurely and the delivery valve remains open during the early part of the suction.

Is it possible to design a pump without a suction valve? The device shown in Fig. 133, permits this to be done. It consists of a pipe open at the top and bottom, and provided at the lower end with a delivery valve. By rapidly moving this pipe up and down water can be made to rise and discharge from the spout. During

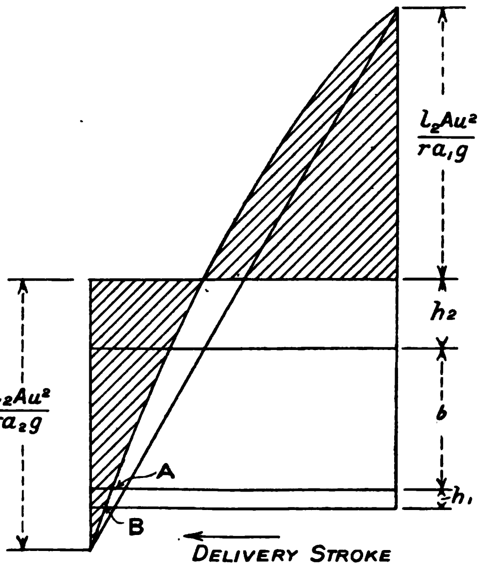


FIG. 132.

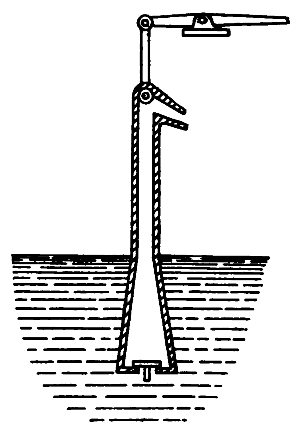


FIG. 133.

the upward motion the valve is kept closed, and on the downward stroke it is opened by the inertia of the water ascending in the pipe.

The question arises whether, if separation be prevented at the pump, it may take place at some other point in the delivery pipe. To prevent separation at the pump, the condition that has to be satisfied is

$$\frac{b + h_2}{l_2} > \frac{Au^2}{ra_2g}.$$

At any other section, depth below delivery tank h and length to delivery tank l, the condition must be

$$\frac{b+h}{l} > \frac{Au^2}{ra_2g}$$

Thus, if separation is first prevented at the commencement of the stroke, to ensure separation does not take place at any other point the condition

$$\frac{b+h}{l} > \frac{b+h_2}{l_2}$$

must be satisfied.

As an illustration, take the arrangement shown in Fig. 134. Here $h = h_2$ and

$$\therefore \text{ if } \frac{b+h}{l} > \frac{b+h_2}{l_2}$$

and $$h = h_2$$

$$\therefore l_2 > l$$

and the condition is satisfied.

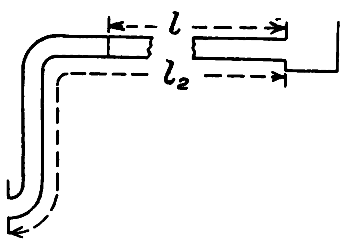


Fig. 134.

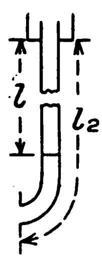


Fig. 135.

In Fig. 135, $l_2 = h_2$ nearly

$h = l$ nearly

$\therefore l_2 > l$

again satisfying the condition.

If (Fig. 135A) $h = o$

$l_2 = l + h_2$

$$\frac{b}{l} > \frac{b+h_2}{l+h_2}$$

$b > l.$

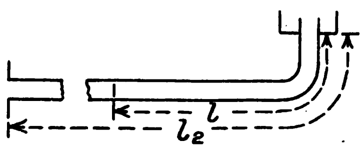


Fig. 135A.

Therefore, if l be more than 34 feet, separation will take place at some section in the horizontal pipe.

§ 104. **Air-chamber and Delivery Pipe.**—To prevent any chance of separation and premature opening of the valves, and in particular to make the driving effort more uniform, an air-chamber may be fitted on the delivery pipe as near to the pump as possible (Fig. 136). If h_2'' be the absolute pressure head in

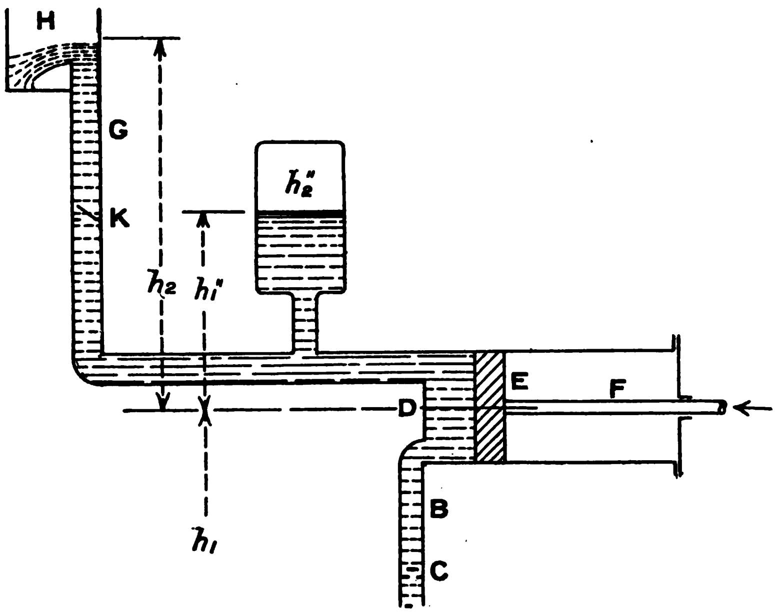


Fig. 136.

the air-chamber, h_2'' the difference of level between the centre line of the pump and water-level in the air-chamber, l_2 the length of pipe between air-chamber and delivery tank, and h_2'' that between pump and air-chamber, F_2' coefficient of hydraulic resistances for lengths l_2 and l_2' (Fig. 137). The theory and action of the air-chamber have been fully described in § 101. Neglecting any variation of pressure in the air-chamber, the velocity v_2 in the delivery pipe is given by

$$a_2\sqrt{\frac{2g}{F_2}}\,(b + h_2'' + h'' - h_2) = \frac{Au}{2\pi} \text{ or } \frac{2Au}{\pi}$$

according as the pump is single or double acting. The curve of effective pressure heads is shown in Fig. 137.

The more accurate determination of variation of pressure has already been treated in § 102.

§ **105. Worthington Direct-acting Pump.**—An interesting problem in inertia effect is a Worthington direct-acting pump. In this, the steam piston works tandem with water plunger. The plunger has its velocity gradually increased from rest, reaching a maximum, and coming to rest at the end of the stroke. Thus the plunger is alternately accelerated and retarded, and so likewise is the water in the delivery pipe.

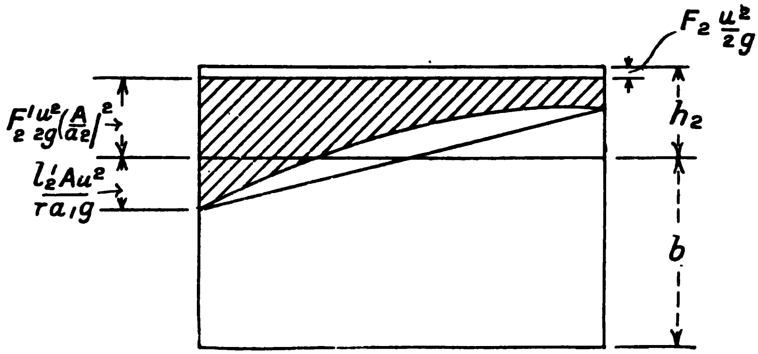


FIG. 137.

Let v be the velocity after the plunger has moved through a distance x from the commencement of the stroke, t the time taken, v_0 the speed of *steady* motion, that is to say, at the instant when there is no acceleration or retardation, and F the coefficient of hydraulic resistance referred to the plunger. Assume also that the pump works against a constant head, h. Then (§ 66) the actual head on plunger

$$= h - \frac{l\mathrm{A}a}{ga} - \mathrm{F}\frac{v^2}{2g}.$$

If p be the accumulator pressures, p_0 the resistance at the instant

of no acceleration or retardation, that is, when the velocity is a maximum, then

$$\frac{p}{\sigma} = \frac{p_0}{\sigma} + \mathrm{F}\frac{v_0^2}{2g}$$

At any other instant

$$\frac{p_0}{\sigma} = \frac{p}{\sigma} - \mathrm{F}\frac{v^2}{2g} - \frac{l\mathrm{A}}{ga}\,a.$$

Writing $$a = v\frac{dv}{dx} = \frac{1}{2}\frac{dv^2}{dx}$$

the equation, combining with the preceding equation, becomes

$$v^2 + \frac{\pi l\mathrm{A}}{ga\mathrm{F}} \cdot \frac{dv^2}{dx} v_0^2$$

whence $$v^2 = v_0^2 + \mathrm{C}\epsilon^{-\frac{\mathrm{F}ga}{l\mathrm{A}}x}.$$

When $$v = 0,\ x = 0,\ \therefore \mathrm{C} = -v_0^2$$

$$= \frac{2g}{\mathrm{F}}\left(\frac{p - p_0}{\sigma}\right)\left(1 - \epsilon^{-\frac{\mathrm{F}ag}{l\mathrm{A}}x}\right)$$

$$\therefore v^2 = v_0^2\left(1 - \epsilon^{-\frac{\mathrm{F}ag}{l\mathrm{A}}x}\right)$$

To find the time required to move over a distance x, taking logarithm of the last equation

$$x = -\frac{l\mathrm{A}}{ga\mathrm{F}}\log_\epsilon\frac{v_0^2 - v^2}{v_0^2}$$

$$x = \frac{dv}{dt} = v = \frac{l\mathrm{A}}{ga\mathrm{F}} \cdot 2v\frac{dv}{dt}$$

$$dt = \frac{2l\mathrm{A}}{ga\mathrm{F}}\frac{dv}{v_0^2 - v^2}$$

$$= \frac{l\mathrm{A}}{ga\mathrm{F}v_0}\left(\frac{1}{v_0 + v} + \frac{1}{v_0 - v}\right)$$

$$t = \frac{l\mathrm{A}}{ga\mathrm{F}v_0}\log_\epsilon\frac{v_0 + v}{v_0 - v} + \mathrm{C}.$$

When $$t = 0,\ v = 0,\ \therefore \mathrm{C} = 0, \text{ and}$$

$$\therefore t = \frac{l\mathrm{A}}{ga\mathrm{F}} \cdot \frac{1}{v_0}\log\frac{v_0 + v}{v_0 - v}$$

Take a numerical example,

Suppose $h = l = 100$

$$d = 4 \text{ inches}$$

$$\text{diameter of ram} = 4 \text{ inches}$$

$$\text{diameter of steam piston} = 4 \text{ inches}$$

$$f = 0{\cdot}015$$

$$\text{difference of pressure} = 45 \text{ pounds.}$$

Then
$$\mathrm{F} = \frac{4fl}{d} = 4 \times 0{\cdot}015 \times 100 \times 3$$
$$= 18 \text{ referred to pipe or ram}$$
$$\frac{p}{\sigma} = \frac{45 \times 144}{62{\cdot}5} = 103{\cdot}8$$
$$\frac{p_0}{\sigma} = 100{\cdot}0$$
$$\frac{2g}{\mathrm{F}} = \frac{64{\cdot}4}{18} = 3{\cdot}58$$
$$\frac{\mathrm{F}ag}{l\mathrm{A}} = \frac{\mathrm{F}g}{l} = 5{\cdot}8$$
$$\therefore \mathrm{V}^2 = \frac{2g}{\mathrm{F}}\left(\frac{p - p_0}{\sigma}\right)\left(1 - \varepsilon^{-\frac{\mathrm{F}ag}{l\mathrm{A}}x}\right)$$
$$= 3{\cdot}58 \times 3{\cdot}8\,(1 - \varepsilon^{-5{\cdot}8x})$$
$$= 13{\cdot}6\,(1 - \varepsilon^{-5{\cdot}8x})$$
$$\mathrm{V}_0^2 = \frac{2g}{\mathrm{F}}\left(\frac{p}{\sigma} - \frac{p_0}{\sigma}\right)$$
$$= 3{\cdot}58 \times 3{\cdot}8$$
$$= 13{\cdot}6$$
$$\mathrm{V}_0 = 3{\cdot}7$$

also
$$t = \frac{l\mathrm{A}}{ga\mathrm{F}}\frac{1}{v_0}\log_e\frac{v_0 + v}{v_0 - v}$$
$$= \frac{1}{5{\cdot}8} + \frac{1}{3{\cdot}7}\log_e\frac{3{\cdot}7 + v}{3{\cdot}7 - v}$$
$$= \frac{1}{21{\cdot}4}\log_e\frac{3{\cdot}7 + v}{3{\cdot}7 - v}.$$

By taking different values of x, the values of v and t are obtained. Thus

x in feet.	v in feet per second.	t in seconds.
0·025	1·35	0·0358
0·05	1·88	0·0455
0·0625	2·07	0·0552
0·125	2·64	0·0778
0·5	3·7	∞

The curves showing this variation, plotted on a speed base are given in Fig. 138. The scale of t had to be made large, so that the

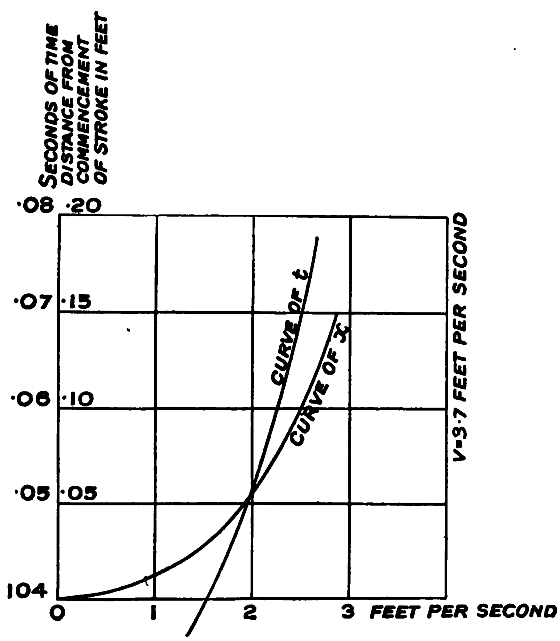


FIG. 138.

zero is four divisions below the horizontal axis, and the curve of t would touch this line in the vertical axis.

The v curve touches the horizontal at o, and would become asymtotic to the line $v = 3·7$. This represents the speed of steady motion. The corresponding time would be infinite.

§ **106. Hydraulic Governor.**[1] — Very frequently the varying motion of the piston of the steam engine causes a variation of pressure in the mains. This has to be prevented, if possible, automatically. Two descriptions of governors are appended below. The governing is brought about by the pressure in the delivery main, rising beyond a certain pressure.

For large pumps, however, the variations of water pressure would be too great, and some means of regulating the steam pressure is required. The necessary variations of speed are too sudden and frequent to admit of hand control, and an automatic pressure governor is generally used. A section through one of these governors is shown in Fig. 139. A small hydraulic cylinder, B, is connected to the delivery pipe of the pumps. A plunger, A, which works in it is pressed down by springs, which are so proportioned that the plunger cannot rise until the water pressure reaches 950 pounds per square inch and will have risen through its full travel when the pressure reaches 1150 pounds. Leakage between the ram and cylinder is prevented by the cup leather, C, shown unshaded in the figure. The upper end of the plunger is connected to a throttle valve in the steam pipe, and when the plunger is at the top of its stroke this valve is nearly closed, only allowing sufficient steam to pass to keep the engines moving slowly at 4 to 6 revolutions

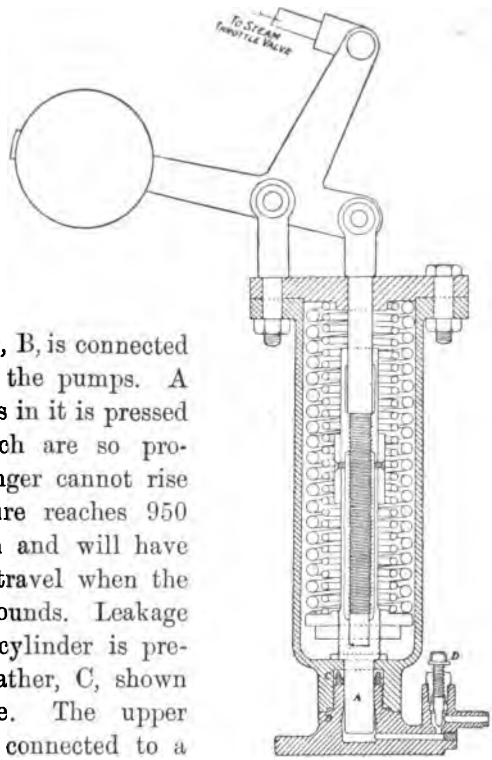


FIG. 139.

[1] From Sennett and Oram's "Marine Steam Engine," p. 383.

per minute, the water delivered from the pumps being returned to the suction tank through the relief valve shown in Fig. 139, loaded to about 1100 pounds.

When one or more hydraulic machines are started, the pressure in the delivery pipe falls, and the springs force the plunger down and open the throttle valve until the speed of the engine increases sufficiently to bring the pressure back to the normal. To prevent a too sudden variation in speed a small valve, D, is fitted between the governor cylinder and the delivery pipe. This is regulated to only allow a slow motion of the plunger. To prevent the dangerous racing which would result if a pipe burst or the suction tank became empty, and which would cause the plunger of the hydraulic governor to fall to the bottom of its cylinder and so admit full steam pressure to the engines, a centrifugal speed governor is fitted (Fig. 139). The water used in the various machines is returned to the suction tanks. To reduce the friction of slide valves, etc., and to prevent corrosion of internal steel parts, soft soap and mineral oil are mixed with the water.

§ **107. Automatic Regulator for Pumping Engine.**[1]—The apparatus consists of a piston valve, D (Fig. 140), through which all the steam has to pass. The valve is connected by a rod to a water piston, A, which is acted upon by the water in the discharge pipe of the pump entering through E. If this pressure rises, the piston is pushed forward in the water cylinder, gradually closing the steam valve and stopping the engine. As soon as the pressure diminishes, the spring, C, returns the water piston to its normal working position, restarting the engine in doing so. The piston, A, rests upon a sleeve moving freely within the valve closed to the right, the engine cannot start until the piston, A, is moved back to its normal working position. To accomplish this a by-pass valve is fitted, which makes communication between the two parts of the piston valve, and will pass a sufficient supply of steam to the engine to restart it. This part of the apparatus is not automatic.

§ **108. Automatic Regulator for Admission of Air to Air-chamber.**—The arrangement shown in Fig. 141 is by Messrs.

[1] *Engineering*, January 31, 1902.

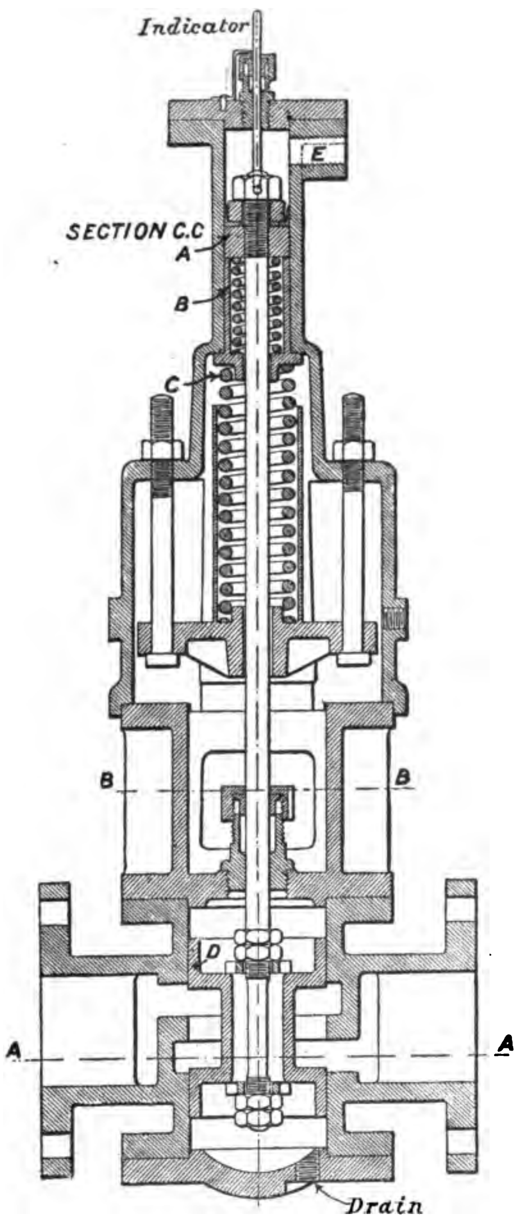


FIG. 140.

Wipperman & Lewis. The suction and delivery valves, and the pump plunger are shown at the bottom of the diagram. G is the air receiver, B is a small pipe communicating with a small air-chamber, A, having a gauge-glass, F. E is a pipe leading from a valve-box into G. The valve contains two automatic valves, one for admitting air into A from the atmosphere, and the other for discharging air into the chamber G. If the cock in the pipe B be opened, and the plunger is moving to the right, that is, the suction stroke is taking place, then the water in the pipe B will be sucked down, and air will enter from the atmosphere on account of reduced pressure in A. In the return stroke the suction valve closes, the water will be forced along the pipe B into the chamber, the first valve will be closed, and the second valve will be open, and so admit air into the air-chamber G.

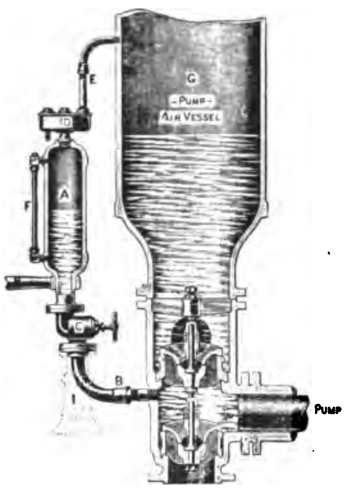


FIG. 141.

§ 109. **Replenishing the Air in the Air-chamber.**—The air in the chamber is gradually reduced on account of its absorption by the water—the quantity of air absorbed by a given volume of water being proportional to the pressure—consequently means must be taken for replenishing it. Air-chambers on the suction side never lack the necessary quantity of air, since the water, being saturated with air at atmospheric pressure, gives up a portion of it when subjected to the smaller pressure in the air-chamber. In order to keep the chamber or the delivery pipe filled with air, it is customary to place a small air-cock on the suction pipe, through which a small quantity of air may be sucked or shifted in by the plunge and allowed to rise to the chamber, or after the latter has been filled to pass off with the water. By this means a certain elasticity is also given to the water, the air contained in the latter serving to moderate shocks that may arise.

§ 110. **Admiralty Pump.**—In this pump the action is as follows (Fig. 142). During the out-stroke of the ram, water flows into the pump through the suction valve, A, and fills the barrel. At the same time the water in the annular space, B, in front of the piston is driven out through the delivery valve C. While the ram is moving into the barrel during the return or in-stroke, the water in the cylinder is forced out through the intermediate valve, D, half passing into the annular space around the ram, and the other half being forced through the delivery valve. The pump therefore delivers water during both strokes, but only draws water through the suction valve during one stroke. The delivery of water from the two pumps is very regular, and the arrangement has the advantage of keeping a constant pressure on the packing of the ram, which reduces leakage and prevents the entrance of air during the in-stroke. This is very important, as the presence of air in hydraulic machinery causes irregularity of working, and may seriously increase the stresses on the parts.

As the water is used for working a considerable number of machines which are used intermittently, the demand on the pumps may be very irregular, and the speed of the engines must frequently and rapidly change. If friction and the inertia of the moving parts were negligible, it would be necessary only to provide constant steam pressures on the cylinders high enough to balance the required pressure in the pumps, and the engine would automatically vary its speed so as to maintain a nearly uniform water pressure.

§ 111. **The Hydraulic Ram.**—The hydraulic ram acts, not by direct pressure or by impulse, but on the inertia effect of a column of water. By this apparatus, which was invented by Montgolfier in 1796, a portion of the driving water is forced to an elevation which is greater than that from which it has fallen. The arrangement is essentially as follows:—The reservoir, A (Fig. 143), from which the water is supplied, connects by means of a pipe BCD with the air-chamber E, from which the delivery pipe F leads to the tank G, where the lifted water is received. At D, where the supply pipe enters the air-chamber, a

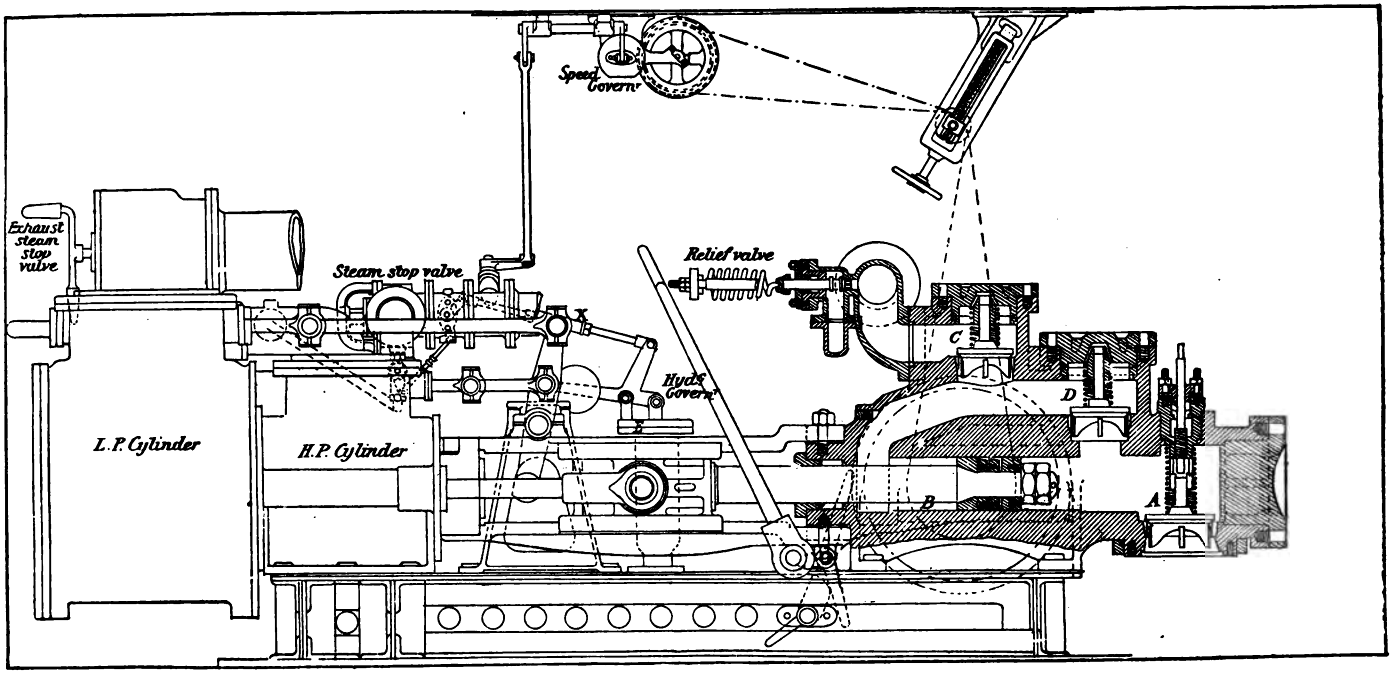

Speed Govern'
Exhaust steam stop valve
Steam stop valve
Relief valve
Hyd^c Govern'
C
D
A
B
L.P. Cylinder
H.P. Cylinder

FIG. 142.

delivery pipe H, opening upwards, is placed, and in the short branch CI is located a so-called *waste valve* which opens downwards.

In order to explain the action of the machine, imagine the two valves H and I to be closed, and that the two pipes BCD and CI are full of water, the air-chamber, E, being at the same time partly filled with water and partly with air. Now, if the valve, I, be depressed, water will discharge through the branch pipe and a fresh supply will flow from the reservoir, A, into the pipe BC. Since the hydraulic pressure on the upper surface of the waste valve is less than that on the lower surface, on account of the greater velocity of the water passing through the valve opening, the valve will close again as the velocity of efflux has become sufficient to cause an upward excess of pressure greater than the weight of the valve. This closing of the waste valve

Fig. 143.

causes the water in AC to open the delivery valve, and consequently water is forced into the air-chamber, E, till the "living" force of the water in the supply pipe is wholly destroyed by the operation. The incoming water causes a compression of the air, and compresses the air in the chamber, thus causing the discharge into the tank, G, through the delivery pipe, F. After the water has been brought to rest in BC it gradually begins to move in the opposite direction from C to B under the influence of the greater pressure in the air-chamber. As this flow soon closes the delivery valve, H, water ceases to flow from H, and the atmospheric pressure now becoming greater than the water at C, pushes down the waste valve, thus automatically causing a new series of operations to begin.

§ 112. **Theory of the Hydraulic Ram.**—The exact theory of the hydraulic ram is very complicated. An approximate theory, given by Weisbach, is as follows:—

Let A denote the sectional area and l the length of the supply pipe, h the fall, and h_1 the head of the delivery tank,—then the mass of water in the supply which has to be set in motion when the waste valve opens by the force σAh will be $\frac{\sigma Al}{g}$, and consequently the initial acceleration is $a = \frac{h}{l}g$. Owing to this constant acceleration, the velocity acquired after t seconds is

$$v = at = \frac{h}{l}gt$$

and if the waste opening has the same area A as the supply pipe, the volume of water which flows out during the time t will be

$$\text{volume} = V = A\frac{v}{2}t = \frac{Ah}{l} \cdot \frac{gt^2}{2}.$$

Now, if the volume of water in the delivery pipe is small in comparison with that in the supply pipe, the retardation of the latter mass caused by the back pressure σAh_1, when the waste valve closes and the delivery valve opens, will be given by

$$a_1 = \frac{\sigma Ah_1}{\sigma Al}g = \frac{h_1}{l}g$$

and consequently the velocity of the water in the supply pipe will, when t_1 seconds have elapsed after the opening of the delivery valve, amount to

$$v_1 = v - a_1t_1 = v - \frac{h_1}{l}gt_1.$$

The time t_1 required to bring the whole volume of water to rest will now be obtained by placing $v_1 = o$, which gives

$$t_1 = \frac{l}{h_1} \cdot \frac{v}{g} = \frac{h}{h_1}t$$

and determines the volume of water which, in the mean time, has entered the air-chamber to

$$V_1 = A\frac{v}{2}t_1 = \frac{Al}{h_1} \cdot \frac{v^2}{2g} = \frac{Ah^2}{h_1 l} \cdot \frac{gt^2}{2}.$$

Assuming the delivery valve to remain open for a short interval t_2 after the subsequent return flow into the supply pipe has commenced, the driving force σAh_1 will give the volume σAl a velocity

$$v_2 = \frac{h_1}{l}gt_2$$

and therefore the volume returned from the air-chamber will be

$$V_2 = \frac{Ah_1}{l} \cdot \frac{gt_2^2}{2}.$$

Finally, when the delivery valve closes and the waste valve opens, the mass of water σAl will move with a retardation

$$a_5 = \frac{h}{l}g$$

and accordingly, after the time t_3, it has attained a velocity

$$v_3 = v_2 - a_3 t_3 = v_2 - \frac{h}{l}gt_3.$$

The volume σAl is now again brought to rest, that is, v_3 becomes zero, and a new stroke begins after the time

$$t_3 = \frac{l}{h} \cdot \frac{v^2}{g} = \frac{h_1}{h}t_2$$

during which a volume

$$V_3 = \frac{Av_2 t_3}{2} = \frac{Av_2}{2} \cdot \frac{h_1}{h}t_2 = \frac{Ah_1^2}{hl} \cdot \frac{gt_2^2}{2}$$

returns, and an equal amount of air or water flows in through the waste valve. The driving or waste water required by the ram per minute is

$$Q = \frac{V - V_3}{t + t_1 + t_2 + t_3}$$

or, approximately, when V_3 t_2 and t_3 owing to their small values, are neglected

$$Q = \frac{V}{t + t_1} = \frac{V}{t\left(1 + \frac{h}{h_1}\right)} = \frac{h_1}{h + h_1} \cdot \frac{Av}{2} = \frac{h_1}{h + h_1}\frac{h}{l}A\frac{gt}{2}.$$

Further, the volume of water lifted per second will be

$$Q = \frac{V_1 - V_2}{t + t_1 + t_2 + t_3}$$

or, approximately,

$$Q_1 = \frac{V_1}{t + t_1} = \frac{h_1}{h + h_1}\frac{V_1}{t} = \frac{h}{h + h_1} \quad \frac{Av}{2} = \frac{h}{h + h_1}\frac{h}{l}A\frac{gt}{2}$$

and consequently the ratio of the lifted water to the driving or waste water

$$\frac{Q_1}{Q} = \frac{h}{h_1}.$$

The total volume of water consumed will be

$$Q + Q_1 = \left(\frac{h_1}{h + h_1} + \frac{h}{h + h_2}\right)\frac{Av}{2} = \frac{Av}{2}$$

which shows that the sectional area of the pipe should be proportional to the quantity $Q + Q_1$ of water consumed. The efficiency will be

$$\eta = \frac{(V_1 - V_2)h_1}{Vh} = \frac{h^2t^2 - h_1^2t_2^2}{h^2t^2} = 1 - \left(\frac{h_1}{h}\right)^2\left(\frac{t_2}{t}\right)^2$$

that is to say, that, as has also been proved by experience, it will approach more closely to unity the smaller the ratio $\frac{h_1}{h}$ of delivery height to fall, and the shorter the time during which the delivery valve remains open after the return flow commences.

The above investigation is, as already pointed out, only approximate. *Eytelwein* made over 1100 experiments on large and small rams, and from these experiments he deduced the formula—

$$\eta = 1{\cdot}12 - 0{\cdot}2\sqrt{\frac{h_1}{h}}$$

whence the following table:—

$\frac{h_1}{h} =$	1	3	5	8	10	15	20
η –	0·920	0·774	0·673	0·555	0·488	0·345	0·226

This shows that for a given fall h the efficiency decreases with an increased lift h_1, and Eytelwein suggests that, for great lifts, instead of a single ram several should be employed, the first supplying water to the second.

§ 113. **Test of a Hydraulic Ram.**—Messrs. Bailey of Manchester have kindly supplied me with a section and test (Fig. 144) of their patent "Decœur" hydraulic ram. This ram gives a greater efficiency than the ordinary type, and efficiencies of over 90 per cent. on low lifts have been obtained. It will force as much as one tenth of the supply to a height equal to eight times the fall. The following are particulars of a test made with one of these rams:—

Fall of water, 121·6 inches.

Drive-pipe 6 inches diameter, 45 feet long.

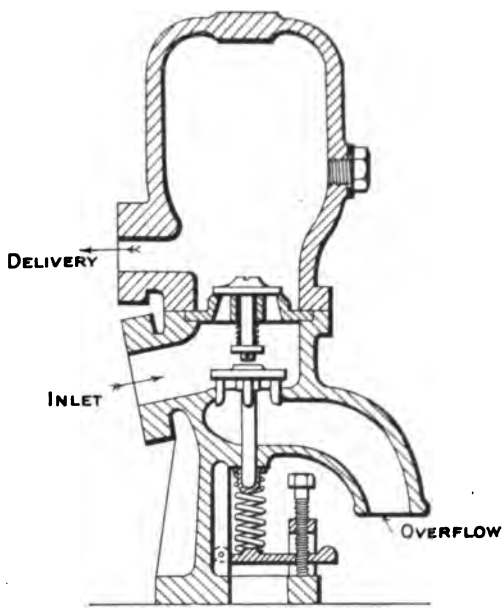


FIG. 144.

Vertical height of delivery, 65 feet through 1280 feet of 1½-inch pipe. The friction in the pipes increased the head 100 feet, and during the test the ram delivered 360 gallons of water per hour, and used 60 gallons per minute. The efficiency therefore was 80 per cent.

FIG. 145.

The principal advantage of this ram is that besides its high efficiency, it can be regulated so as to suit varying conditions. The moving parts are light in weight, as they are controlled by springs, so that the stroke, or the number of beats, can be varied. The escape or waste outlet should be covered with water, so as to give a suction effect, which automatically keeps the ram supplied with air, and thus takes advantage of every inch of fall available. The noise is much reduced, and there is consequently less wear and tear.

§ 114. **Borehole Pumps.**[1]—A double-acting deep-lift pump for raising water from boreholes is illustrated in Fig. 145.

The working barrel, fitted with a foot valve is placed well below the lowest water level in the borehole. The foot valve shown in Fig. 145, which represents an enlarged view of working barrel, bucket, and clack valve, has four seatings, this design permitting a very small lift of the valve, and so reducing the concussion and wear to a minimum.

The double-acting pump has two buckets moving in opposite directions, the pump buckets being similar valves to the pump valve. The driving mechanism is placed at the top of the shaft, the pump being driven by an electric motor.

The pump buckets are connected by vertical rods to cross-heads working in guides in the well at the top of the

[1] The information contained in this article was kindly supplied by Messrs. Sir Wm. Mather and Platt, Salford.

boreholes. A reciprocating motion is imparted to the buckets and rods by two long connecting rods, the upper ends of which surround pins on a spur-wheel. This is driven by gearing and belting from an electric motor, a flywheel being fitted to the first shaft.

A large air-vessel in the well serves to eliminate shock, and to produce a steadier delivery of water through the pipes to the surface.

Single-acting pumps are also made, but a double-acting pump has the advantage of giving a steadier and much larger delivery of water than a single-acting pump, for a given diameter of bucket. As the two buckets move in opposite directions, the reciprocating parts are also better balanced.

§ **115. Reidler "Express" Pump.**[1]—The pump has two barrels, the plungers being $5\frac{1}{4}$ inches by 7 inches stroke, and is capable of pumping 200 gallons per minute to a head of 1500 feet, when running at a speed of 200 revolutions per minute. This may be taken as its normal or constant duty; but the maximum duty far exceeds this, since the pump is capable of pumping for a number of hours at a stretch 300 gallons per minute to the same head, running then at 300 revolutions per minute. Particular care was taken in the construction and design of the crank shaft, its bearings, and the connecting-rod; the surfaces allowed for bearing being greater than for ordinary slow-running pumps. A sectional view of the pump and engine is shown in Fig. 146. The suction chamber, A (Figs. 146 and 147), is of cast iron, bolted to the bed-plate and guide section. This suction chamber is a rectangular box extending right across the two barrels, and a suction air-vessel, B, is mounted in the centre of the same. To this suction chamber is bolted the pump barrel, C—which for heavy pressures is made of cast steel—also the cover, D, bolted to its end. Each of these barrels C is bored and turned to fit the faces on the suction chamber. The suction chamber, C, is also bored to receive the combined suction valve seat and plunger case, E. This plunger case, E, is made of gun-metal fitted with a cast-iron gland, gun-metal bushed, and a lantern ring at the bottom of the stuffing-box to ensure having lubricant right round the plunger.

[1] *Engineering*, lxxiii. 310. Built by Messrs. Fraser and Chalmers.

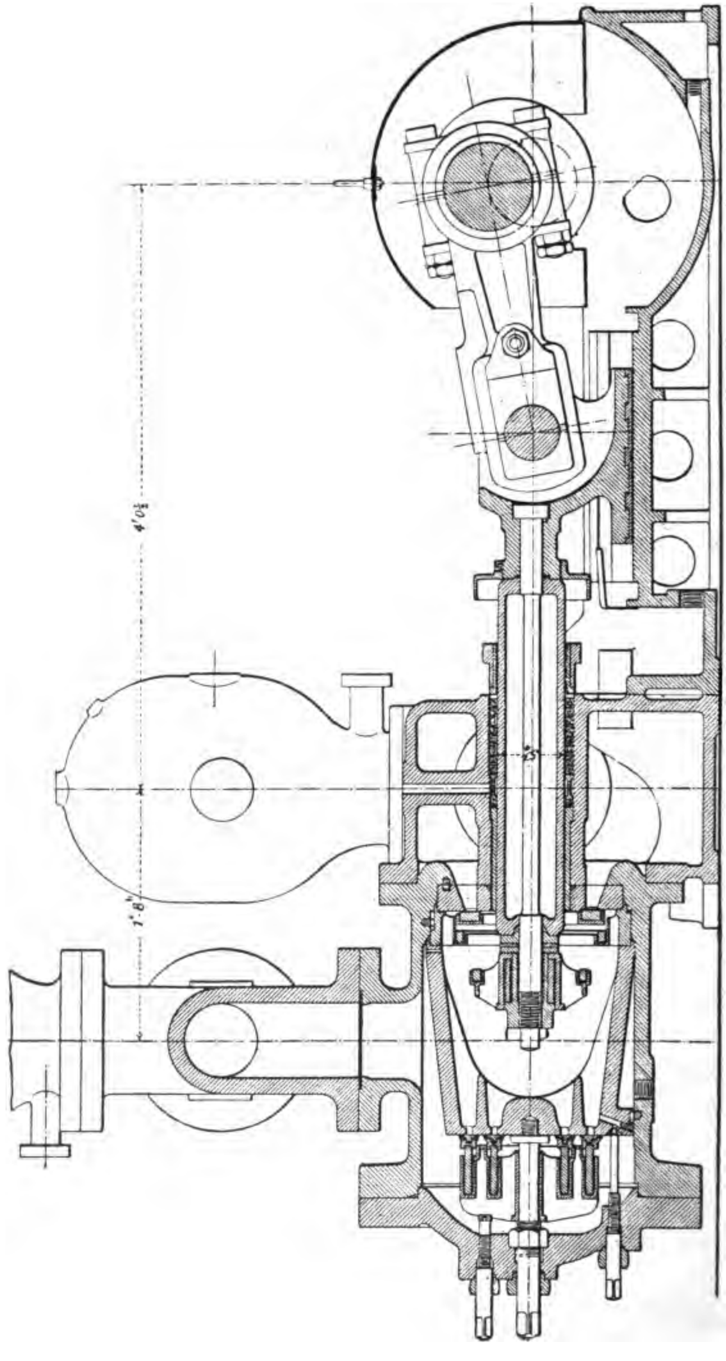


FIG. 146.

This plunger case, E, is secured in the seat, bored and turned out to receive it in barrel C, by means of a cover, F, which is fitted to the front of the suction chamber, and so draws the plunger case, E, towards it by the nuts G hard against the seat in pump-barrel. In the barrel C there is an inner barrel which is made of cast iron, and forms also the seat for the delivery valves. This is turned, and bored and bedded on to one joint, which may be of leather or any other suitable material. This barrel is forced down on to this joint by a screw, J, and so makes perfectly tight all connections between the suction and delivery side of the pump. In the barrel

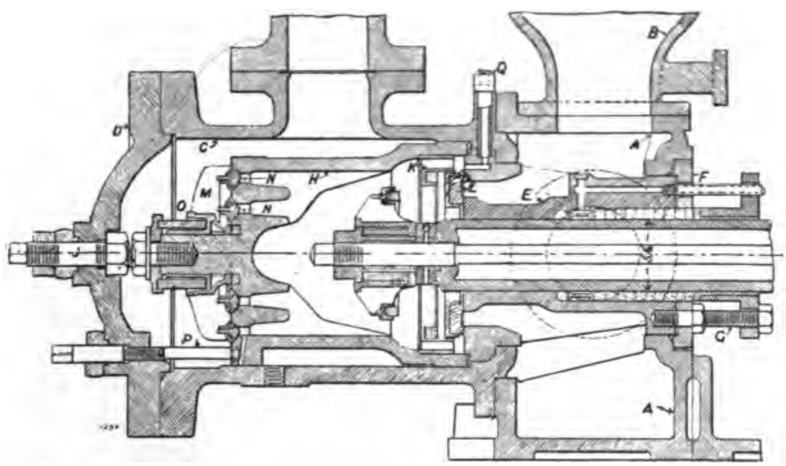


FIG. 147.

H is fitted a gun-metal ring K, which ring holds in position the suction valve L, and, being fitted with a rubber buffer, prevents this valve slamming open. The suction valve L, annular in design, is made of wood, with the end grain forming the bearing surface, and is mounted in a gun-metal case, as shown. The plungers, of cast iron, accurately turned and polished, are secured to the cross-heads, and are fitted with a buffer arrangement at the opposite end of the plunger, all being held in position by one main centre bolt. This buffer, fitted with rubber, comes in contact with the suction valve when the plunger has travelled about

nine-tenths of the stroke, closing the valve during the last one-tenth. All shock is prevented by the rubber ring mounted in the buffer, and the additional rubber ring mounted between the nut on the bolt and the buffer.

The delivery valve, also annular in design, consists of a gun-metal casting M, in which are mounted two rings N of a special section which close down on the valve seat H. Between the rings N and the casting M are fitted leather sealing rings, the idea being that whilst the rings M bear down metal to metal on the seat H, and are intended to take the full pressure on the back of the delivery valve, they are not meant to be absolutely water-tight. This is the function of these leather sealing rings, which act as ordinary cup leathers. This delivery valve is spring-loaded by means of a rubber buffer O, as shown. It has ports which are of large area; the lift need therefore only be very small, so that not the slightest shock occurs when the valve closes. Under a head of 300 feet or more the loss of, say, 1 pound or 2 pounds pressure in forcing water through the delivery valve is such a small percentage that it need not be considered. As it is impossible to draw water at a high velocity through small openings, a different plan is needed for the suction valves, which must permit of a high lift to enable these pumps to be run at the comparatively high speeds mentioned above. It will be seen that the main difference between Figs. 146 and 147 is the arrangement of the delivery valves. In Fig. 147 there are no sealing leathers, and the valve seats are flat, instead of inclined; but the rubber buffers limiting the free lift of the valve are common to both designs.

The matter of inspection and that of taking the pump apart quickly have been thoroughly considered in the designing of this pump. It will be noticed that having once taken away the back cover, the inner barrel with its delivery valve can be removed, and then, by taking off the nut at the back of the plunger, the buffer, suction valve, and plunger may be removed. The valves P and Q are provided for priming the pump, and for drawing off air when starting. The two delivery vessels are connected by a branch pipe, on which is mounted the delivery air-vessel.

The suction and delivery air-vessels form essential parts of a

high-speed pump, as it will be seen on consideration that it is impossible to send water through a long rising main with pulsations corresponding to that of the pump. To damp these out is the function of the air-vessel, and this is just as important on the suction side as on the delivery side of the pumps. With all the "Express" pumps an air-charging pump is provided, which is connected and driven directly from some reciprocating part of the pump. For high heads the air is compressed in two stages, and the supply of air to the suction and delivery air-vessels is perfectly automatic in its action. The function of the air-charger being to keep the water in the suction air-vessel at the right height, by abstracting the accumulating air and delivering a sufficient quantity of air to the delivery air-vessels. Both air-vessels are fitted with gauge glasses. An automatic and adjustable change-over gear is provided on the suction side of the air-charging pump. With this gear in operation the pump draws air for, say, one minute from the air-chamber on the suction side of the pump, and for the next five minutes from the outer atmosphere. The relative duration of these periods can be altered if it is found that air is being drawn too quickly or too slowly from the suction air-chamber.

The inner barrel H is a patented feature in the "Express" pump, and serves to divide up the strains which come on the castings. In the ordinary pump the working barrel is subject to a collapsing pressure on the suction stroke of the plunger, and a bursting pressure a little above that, due to the head on the delivery stroke. This perpetual reversal of strains of so severe a nature is often the cause of trouble with the pump chambers. Castings which may have been supplied strong enough to do the work when first started, eventually give out, or show signs of cracking, owing to the perpetual bending, or what is usually known as the "breathing" of the pump. This exists in all pumps, and in some cases is readily perceptible. With the pump illustrated, the inner barrel H is on the suction stroke subjected to a collapsing pressure, which is a pressure that cast iron will thoroughly stand. On the delivery stroke this inner barrel is subjected to only a slight bursting pressure, due to the forcing of

water through the delivery valve, say one or two pounds per square inch. The main pump barrel C is subjected all the time to a bursting pressure, and cannot therefore be troubled with the "breathing" action, and may for this reason be made much thinner than pumps of the same diameter of the ordinary design; or, if of the same thickness, it will be much stronger.

§ **116. High-duty Worthington Pump.**[1]—In Figs. 148 and 149 is represented a general view of this engine, which was designed to pump water to Coolgardie. The problem was to pump 5,600,000 gallons a day against a total estimated head, including friction, of 2700 feet through a pipe 30 inches in diameter, 30 miles, the speed of water through the pipe being 2 feet per second.

The engines are of the "Duplex" type, that is, it consists of two triple-expansion engines placed side by side, each engine operating a double-acting pump. The valve gear is the semi-rotary "Corliss" type, and works on the Worthington principle, namely the right-hand engine working the valves of the left, *vice versâ;* but each engine controls its own cut-off gear. The piston and plungers of each engine are connected rigidly—the high-pressure piston and water-plunger are connected to the cross-head, the intermediate piston is connected to the low-pressure, and the low-pressure, by means of two rods, is also connected to the cross-head. The compensating cylinders are placed on each side of each cross-head, and a high pressure is maintained in the cylinders by connecting with an accumulator. The pressure in the accumulator is either obtained by connecting with the delivery air-vessel, or with an air compressor.

It will be noticed that during the first half of the stroke the engine is forcing the compensating rams against a very heavy pressure, but directly after half-stroke, the plungers are forced out of their cylinders, and assist the engine when the steam pressure has fallen due to expansion. In other words, the compensating cylinders serve the same function as a massive flywheel.

[1] I am indebted to the Worthington Pump Company, Queen Victoria Street, London, for kindly giving permission to publish the two diagrams. They are taken from the *Coolgardie Souvenir*, which gives a complete account of the initiation and carrying out of the scheme.

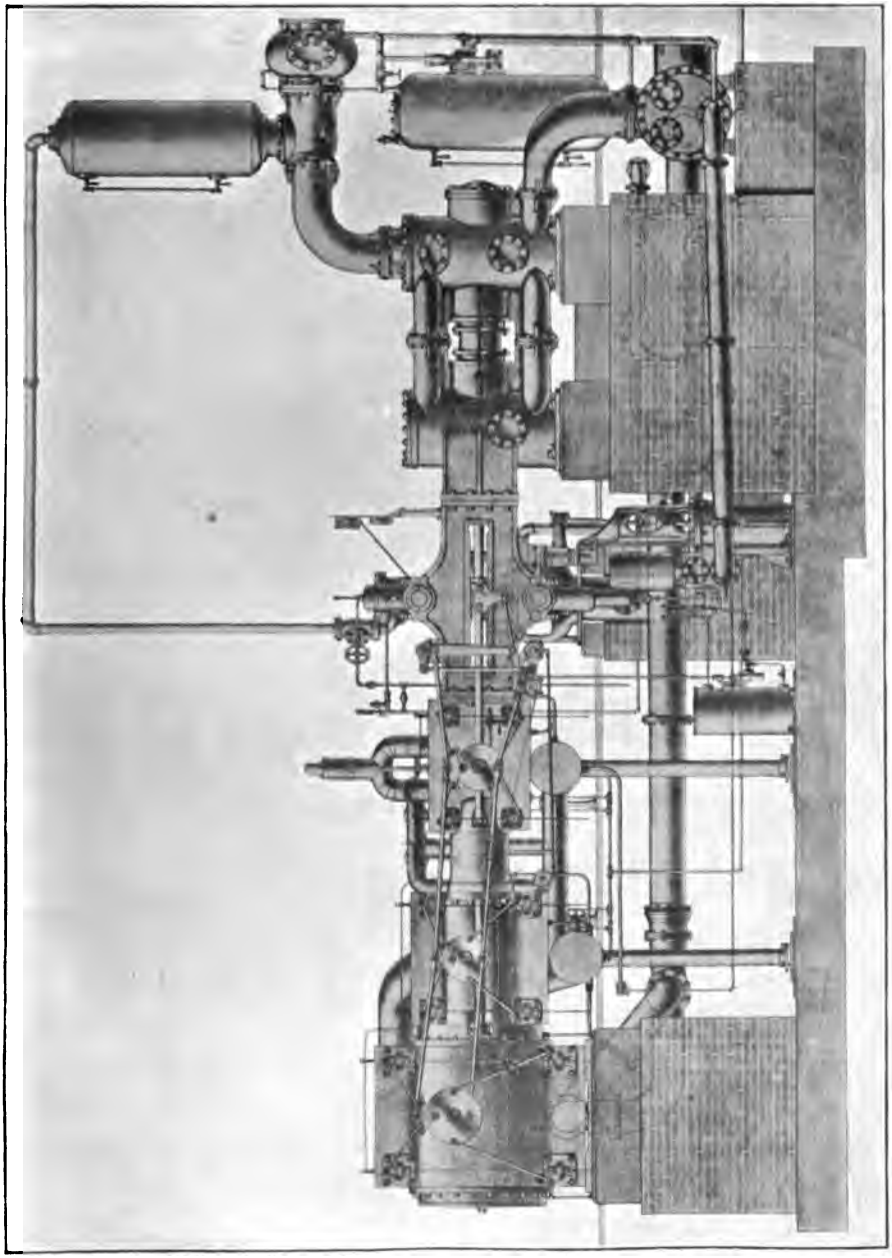


Fig. 148.

By slight variations of the pressure in the accumulator, the stroke of the engine can be regulated to the greatest nicety.

The action of the main pumps will be readily understood from Figs. 148 and 149. Fig. 148 gives a sectional view of engine, tank gear, pump, and air-chamber. The suction valves are placed below, and the delivery valves above the plunger. The valve seats are of gun-metal, and the valves are stamped out of the best manganese bronze. One or other of the pumps is always delivering into the main, with the result that an extremely even flow is maintained, with an entire absence of shock, which is of the greatest importance when pumping through long lengths of main. The running is very silent, and the cost of upkeep small.

§ 117. **Electric Pumping Plant for De Beers Consolidated Mines, Kimberley.**[1]—In this plant the pump has four chambers to lift 1000 gallons of water per minute against a total head of 320 feet.

As will be seen from the curves (Fig. 150), the efficiency is remarkably high, and in consequence of this fact the temperature rise is extremely small, the ventilation provided being ample. This point is of great importance in cases where motors are required to work for long periods, and in situations where the circulation of air is defective. In deep-level mines the conditions in this respect are naturally adverse, and care has, therefore, to be taken to prevent overheating of the plant, a design which gives a small rise of temperature is the only satisfactory way of obviating the difficulty.

Special care is taken with these motors to get a perfect balance of the rotating parts; that this had been obtained was evidenced by the total absence of vibration of the plant, which we saw started and running without holding-down bolts, on a springy foundation. The curves bear out the conclusions already arrived at.

The pump efficiency at 1000 gallons per minute is 75 per cent., and the combined efficiency of pump and motor, 69·5 per cent., and here again the efficiency is maintained throughout a range of

[1] The plant was designed by Messrs. Mather & Platt, and this article is an abstract from *The Electrical Review*, November 13, 1903. By inadvertence this pump has been placed in the present chapter. It ought to be read at the end of Chapter VI.

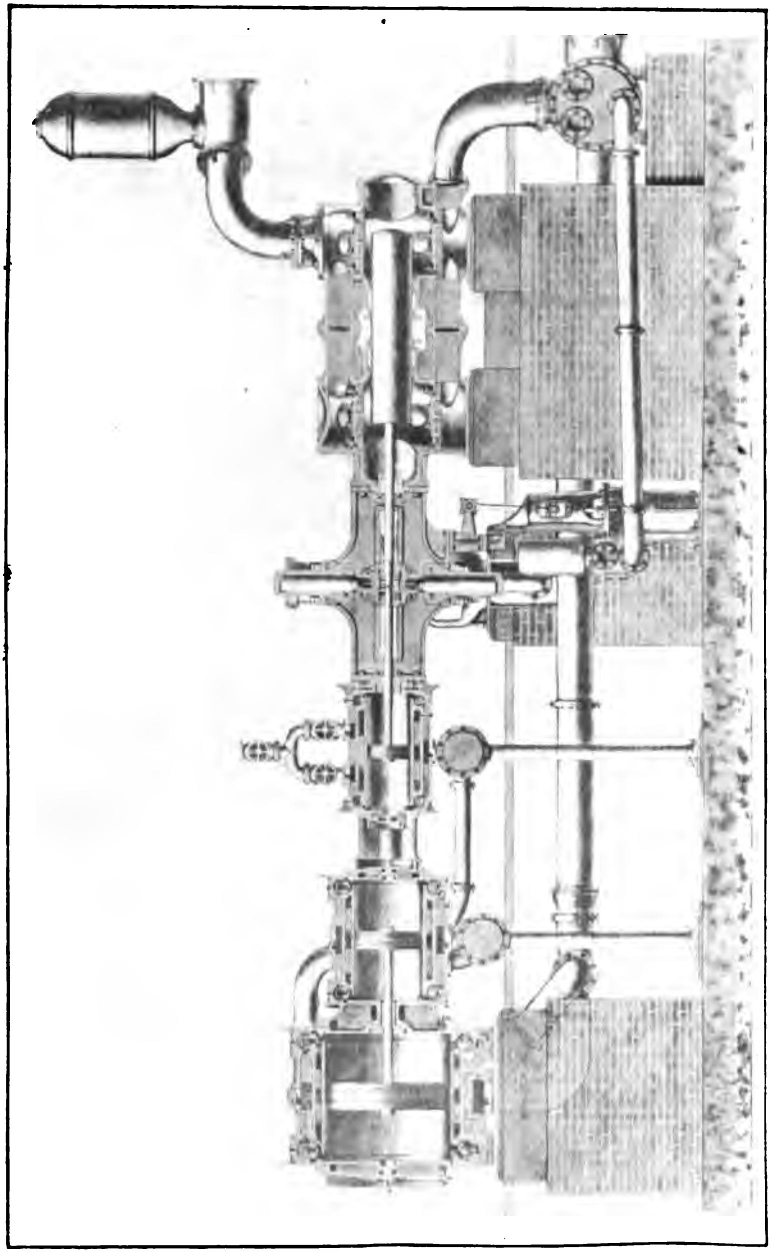


FIG. 149.

25 per cent. under and 25 per cent. overload, being 65 per cent. between these limits.

The figures and curves for the smaller set are equally satisfactory, and the general design of both motors is simple and

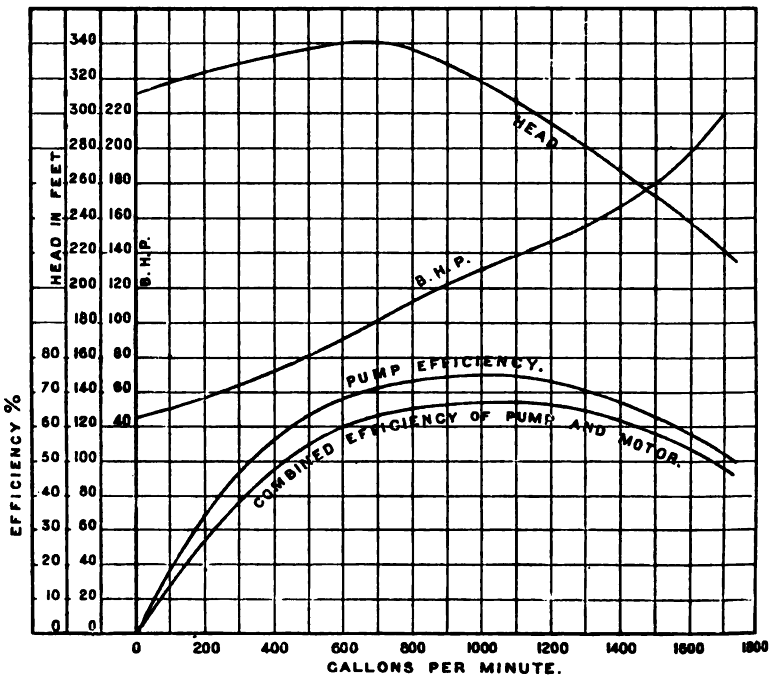


Fig. 150.

strong, making with the pumps a very servicable and compact combination.

In this improved type of pump the water enters the revolving wheel axially, and in the case of single pumps symmetrically on each side of the wheel, thus eliminating axial thrust. The water then traverses the curved internal passages between the vanes, and is discharged tangentially at the periphery into a stationary guide ring; this conveys it to the annular chamber in the body of the pump where the velocity head is converted into pressure head. From this chamber the water is discharged into the pipe line, or,

in the case of multiple pumps, into the second and subsequent chambers.

THE "GUTERMUTH" VALVE.[1]

§ **118. Description.**—This valve is peculiar in this respect—that it is made out of one single piece of sheet metal. Fig. 151 shows the shape of the blank, cut from a piece of steel or gun-

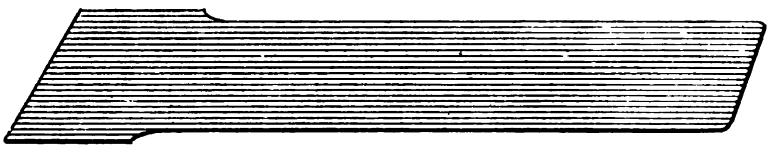


FIG. 151.

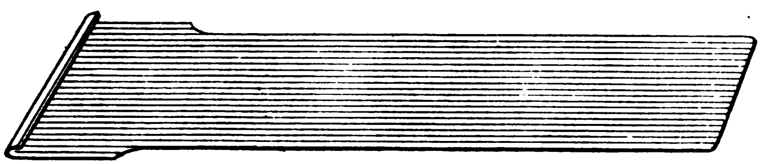


FIG. 152.

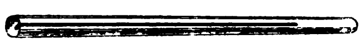


FIG. 153.

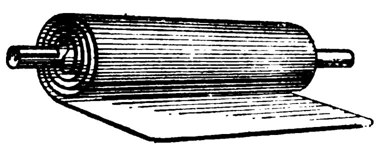


FIG. 154.

metal, of high tensile strength and elasticity. The left-hand edge of the blank is first bent at a suitable angle, as in Fig. 152, and fits into the slot (Fig. 153). The mandril is chucked in a machine of special design and turned round on its own axis, coiling up the blank as indicated in Fig. 154,

[1] The information contained in these following articles was kindly supplied by Messrs. Fraser and Chalmers, Erith, Kent, who are the sole agents in this country of Messrs. Kühne, Darmstadt.

only that part remaining flat which is to serve as the flap of the valve.

§ 119. **Movement of Valve.**—Fig. 155 shows the movement of a "Gutermuth" valve, the thick lines being the position at rest; the thin lines that when the valve lifts. By way of illustration, a spring of two coils is shown; in practice "Gutermuth" valves have generally not less than four coils, but, by multiplying the number of coils, the valves can be opened to any required degree without appreciably straining the material.

Fig. 155 shows that the valve in action lifts bodily from its

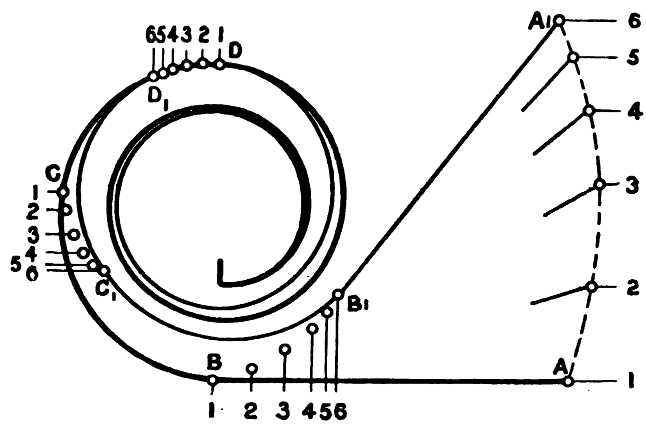


FIG. 155.

seat; the point A travels to A_1, B to B_1, and the points 1, 2, 3, 4, 5, 6 are corresponding points. The whole of the port is uncovered; and becomes, in fact, a hinged valve with elastic pivot. The elasticity of the entire length of the whole coil is pressed into service, with the result that there is no friction; and, without any guides or guards, the action of the valve is positive and certain, as the flap is always bound to return to its original place, however wide it may have been thrown open. The force required to open the valve is very small; to raise a valve 4 inches wide, 3 inches long, and $\frac{1}{32}$ inch thick, having four coils, to an angle of 30° requires only 2 pounds, which,

for adequate port area, is equivalent to a pressure of 0·25 pounds per square inch.

"Gutermuth" valves are always so fitted that they cover the port opening at an angle. This ensures that the port becomes uncovered at a small deflection of the valve, the resistance to flow

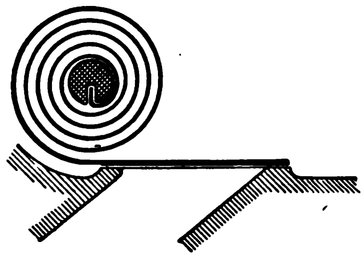


FIG. 156.

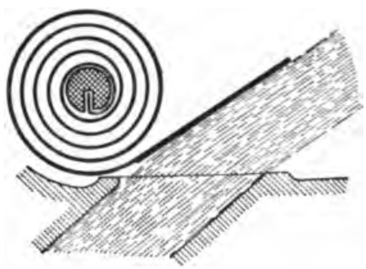


FIG. 157.

is small, and that the valves close promptly immediately the flow ceases (Figs. 156, 157).

§ 120. **Comparison between the Ordinary Valve and the "Gutermuth" Valve.**—To show the great advantage of the "Gutermuth" valve (Fig. 158) over the ordinary mushroom valve (Fig. 159),—with a low lift the escaping water in the mushroom valve is shown in Fig. 160, and in the "Gutermuth" valve in Fig. 161; with high lifts, by large quantities of water being forced through, the mushroom valve is shown in Fig. 162, and the "Gutermuth" valve in Fig. 163; with the valve released, or temporarily held back, the mushroom valve is shown in Fig. 164, and the "Gutermuth" valve in Fig. 165.

The difference in action is—in the mushroom valve, the water strikes the valve plate at right angles, and is, by impact, deflected and dissipated in all directions. When the valve closes there must be considerable dissipation in eddies. With the "Gutermuth" valves the stream is a clear stream without deflection or eddies.

In the ordinary clack valve loaded by springs, a considerable resistance is offered to the opening of the valve, and thus the head

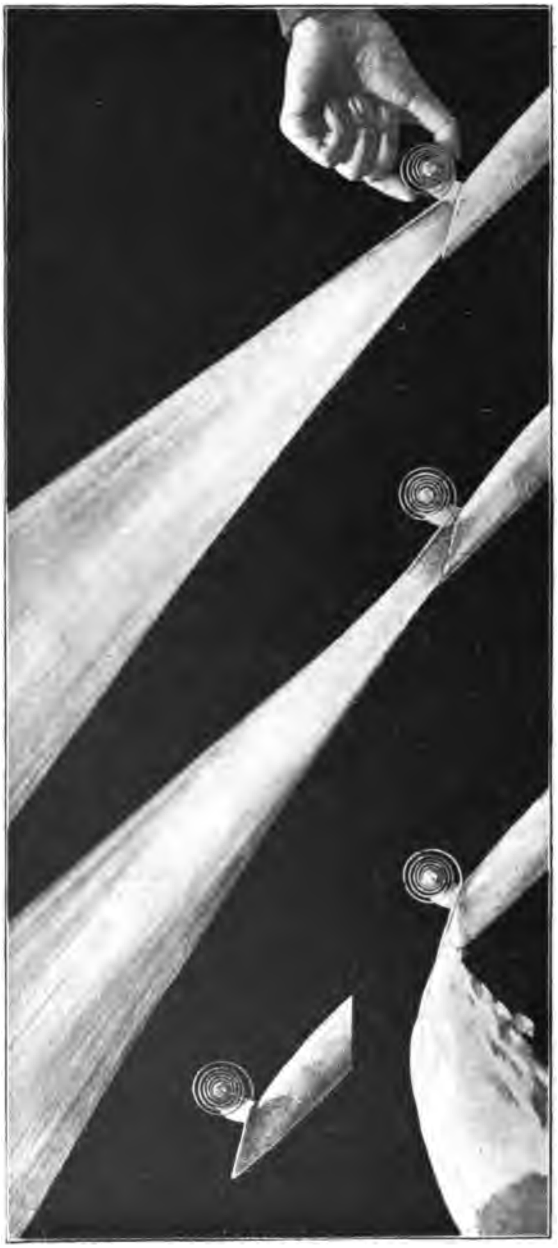


Fig. 158. Fig. 161. Fig. 163. Fig. 165.

FIG. 159.

FIG. 160.

FIG. 162.

FIG. 164.

against which the pump works is very much increased. To take an example: a delivery clack valve (Fig. 166) for a waterworks pump, weighed 113 pounds, the foot valve weighing 71 pounds. It was replaced by a "Gutermuth" valve (Fig. 167), the weight

FIG. 166.

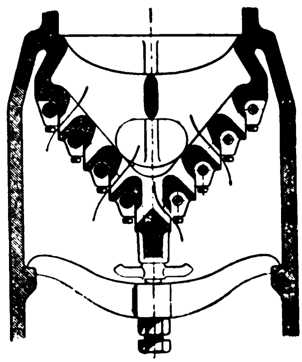


FIG. 167.

of this valve on its seat being $10\frac{1}{4}$ pounds, only one-sixth of which has to be put in motion.

§ 121. **Quantitative Comparison between the Ordinary Valve and the "Gutermuth" Valve.**—In a comparison between the "Gutermuth" valve and the ordinary bucket valve—the pump having been reconstructed—the results were—

	Available port area in sq. inches.	Revolutions per minute at a water speed of		Water pumped per hour in gallons, at	
		500 feet.	825 feet.	500 feet.	825 feet.
Original pump . . .	4	40	—	1585	—
Reconstructed pump .	8·562	85	140	3365	5550

The ordinary valve, at high speeds, produces dangerous hammering; but the "Gutermuth" can be used with safety at a speed of 825 feet per minute, and the water pumped is considerably increased.

§ 122. **Three-plunger High-pressure Pipe-line Pump.**—As an

illustration of a three-plunger high-pressure pipe-line pump,

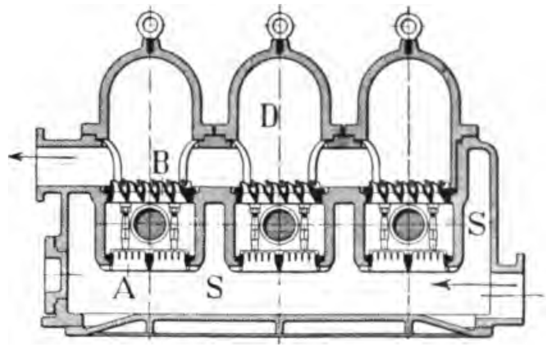


FIG. 168.

Fig. 168 shows a longitudinal section through the engine, A

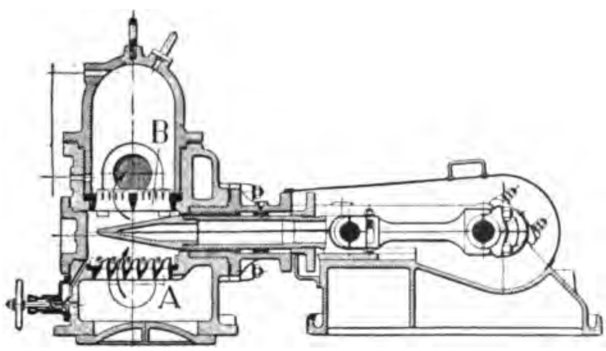


FIG. 169.

being the suction and B the delivery valves, S being the suction and D the delivery chambers. Fig. 169 shows a cross-section, and Fig. 170 shows an enlarged section, of the delivery valve, showing the "Gutermuth" valve.

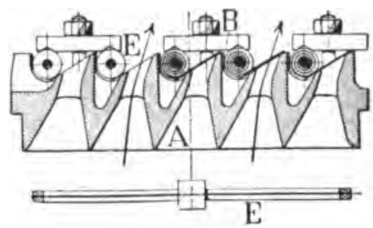


FIG. 170.

§ 123. High-pressure Differential Mining Pump.—This pump was made for the Engineering

Laboratory of the Technical High School, Darmstadt (Figs. 171, 172). It was made especially strong, because it was intended for purposes of scientific research. The main features of the pump are—

(1) The small space occupied, compared with the output. The over-all dimensions are 9 feet 3 inches in length, 3 feet in breadth, and 5 feet 9 inches in height, for a delivery of 7750 gallons per hour at 250 revolutions per minute.

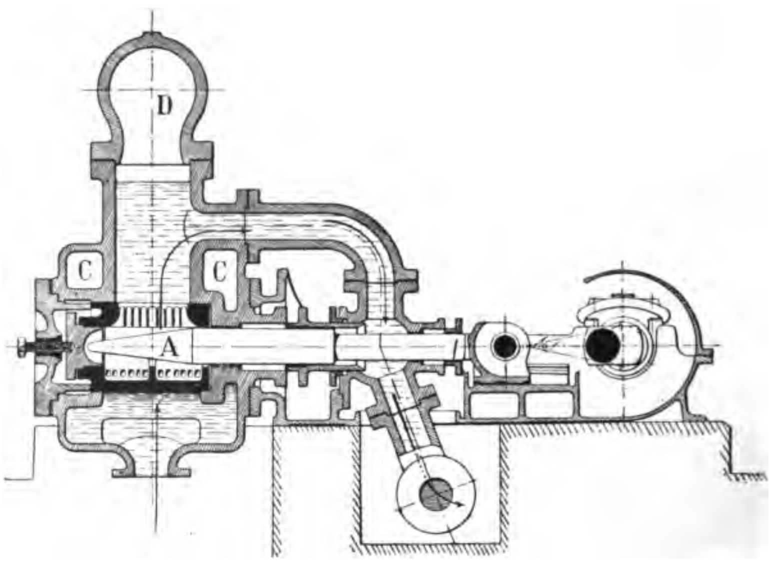


FIG. 171.

(2) The accessibility of the valves, the forty valves being contained in one valve cone measuring 16 inches diameter and 15 inches in length, which can be withdrawn without disturbing any other part of the pump.

(3) The great port area for the passage of the water, which is 19 square inches for both suction and delivery, resulting in a water velocity of only 495 feet per minute in 250 revolutions per minute.

(4) The arrangement of suction air vessel D, with a jacket round the pump barrel C (Figs. 171, 172), whereby a very easy flow of the

water into the latter by gravitation is ensured. The column of water in the suction pipe takes no part in the acceleration caused by the action of the plunger, but maintains a gentle and constant rising motion.

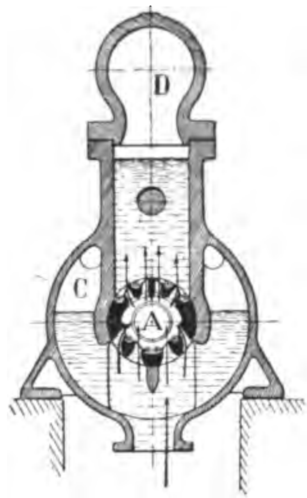


FIG. 172.

(5) The plunger, A, has a long, conical shape projecting right between the valves (Fig. 171); it has therefore a very large suction surface in relation to its displacement, and the water is not required to follow the horizontal motion of the plunger, but merely rises vertically into the space vacated by the same; in a similar manner, on the return stroke, the water is not pounded by a flat surface of the plunger, but is displaced in a vertical direction through the delivery valves. In this way the current does not meet with any obstruction during its course through the pump, and all unnecessary friction or throttling, reciprocating motion, and the formation of eddies is avoided, resulting in the easy working of the pump, the saving of motive power, and the absence of any shocks or hammering. Figs. 173 and 174 are sections through the valve cone, illustrating, on a larger scale, the arrangement of the valve and ports, and, at B, the mode of fixing and locking of the valve spindle.

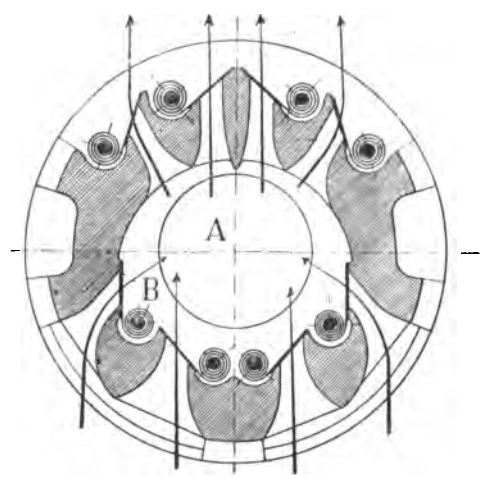


FIG. 173.

(6) The speed must be as high as possible, in order to move large volumes of water. Ordinary ring or mushroom valves must be of large dimensions, as high speeds only admit of low valve lifts. The resulting valve cases often exceed in size that of the pump proper, and as there must be two such cases, which are of circular form and must be built side by side for accessibility, the floor space occupied by such pumps is considerable. The

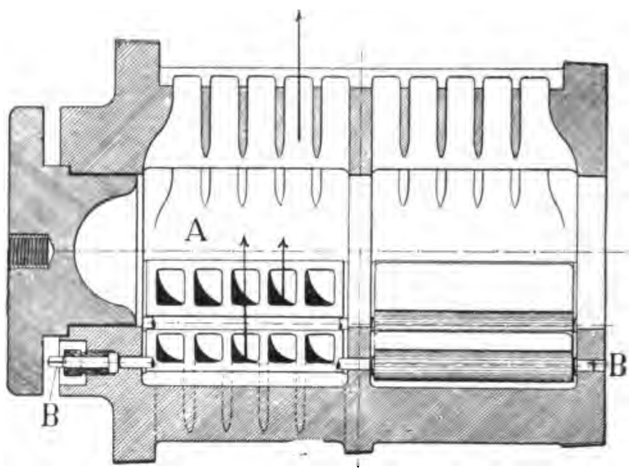


Fig. 174.

pump illustrated is provided with two valve cones, which are built close to the pump barrel, and as they can be drawn sideways, they are readily accessible, without the necessity of disturbing any other part. By a cross-flange the cones are divided into two equal halves, and the valve spindles are of the same pattern, and are interchangeable. The water and port areas are very liberally dimensioned.

CHAPTER V

SIMPLE MACHINES—TURBINES

IMPACT OF WATER ON FIXED AND MOVING VANES

§ **124. Impact on Fixed Plates.**—*Flat Plate.*—In Fig. 175 a jet of water strikes a flat plate, and glances off tangentially in a vertical direction. If Q is the water, in cubic feet per second, impinging on the plate, a the sectional area of the jet in square feet, v the velocity of impact,—the force of impact will be equal to

$$\frac{\sigma Q v}{g} = \frac{\sigma a v^2}{g}.$$

Semi-cylindrical Cup.—If the cup be semi-cylindrical (Fig. 176), the water strikes the cup at the centre, and is deviated

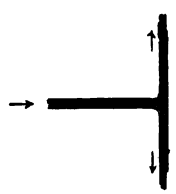


FIG. 175.

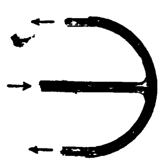


FIG. 176.

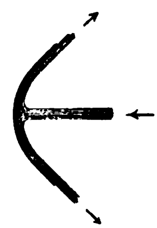


FIG. 177.

round the cup and discharged at the two edges in a direction parallel to the original direction. The velocity is reversed, and the momentum doubled, and so is the force. If, instead of the water issuing in a direction parallel to the original jet, it moves off at an angle, a (Fig. 177), the resultant momentum in the line of motion of the jet is

$$\frac{\sigma w}{g} v(1 - \cos a).$$

Inclined Flat Plate.—If water impinge on an inclined plate (Fig. 178), it will be turned partly up the plate and partly down the plate. The sliding velocities will be each equal to v, in the absence of rebound and friction, but the thickness of the streams will be different. Suppose the acute angle between the jet and plate is a, and the thickness of the jet be denoted by 1 inch, and let x and $1 - x$ be the thicknesses of the streams along the plate, then the momentum along the plate is

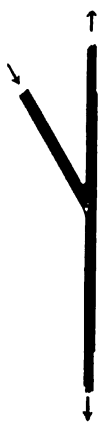


FIG. 178.

$$v \cos a + vx - v(1 - x),$$

and this must be zero; whence

$$x = \tfrac{1}{2}(\cos a - 1)\text{; and } x = \tfrac{1}{2}(\cos a + 1).$$

The pressure is wholly normal, and its magnitude is

$$\frac{\sigma}{g} a v^2 \sin a.$$

Forces in Pipes.—Suppose water is flowing along a pipe of length l and area a, with a velocity v feet per second, and let a valve at the end be closed in time t seconds; the water will be retarded, and the average retardation will be $\frac{v}{t}$. Thus the force on the valve will be $\frac{\sigma a l}{g} \cdot \frac{v}{t}$, and per square foot, $\frac{\sigma l}{g} \cdot \frac{v}{t}$; or, expressed in feet of water, $\frac{lv}{gt}$.

§ **125. Simple Types of Machines.**—The question of pressure-machines has been fully discussed in Chapter III. In machines which act by the motion of water, there are three types:

(1) Those which act by weight, such as overshot water-wheels.

(2) Those in which the available energy is entirely converted into kinetic energy before acting on the machine, and which are called *impulse turbines*.

(3) Those in which only part of the energy is converted into kinetic energy before acting on the machine, the remaining part being pressure energy, which is converted into kinetic energy in the machine itself.

It is not necessary to notice the first type, but a few simple cases of the second and third types may be given.

Undershot Water-wheel.—An undershot water-wheel with radial vanes is shown in Fig. 179. The water impinges with velocity, say, v feet per second, on to radial floats having, say,

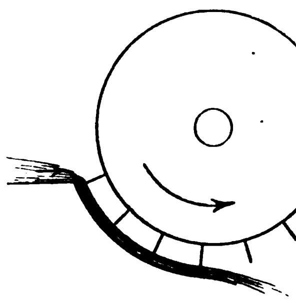


FIG. 179.

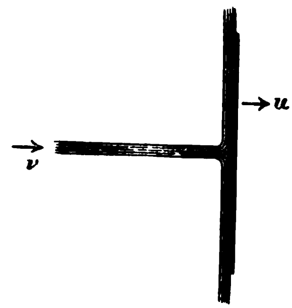


FIG. 180.

a velocity of u feet per second. The problem is equivalent to a flat plate moving with a velocity u, and water impinging with a velocity v (Fig. 180). In the undershot water-wheel a number of floats are successively interposed to the stream, so that

$$\text{the water impinging per second} = \frac{\sigma av}{g}$$

a, as before, being the area of the streams. The change of velocity in the direction of motion is $v - u$, and the force on the float is

$$\frac{\sigma va(v - u)}{g}$$

Hence, the work done per second is

$$\frac{\sigma auv(v - u)}{g}$$

The energy in the stream is

$$\frac{\sigma av}{2g} \cdot v^2$$

and, therefore, the hydraulic efficiency is

$$\frac{2a(v - u)}{v^2}.$$

This is a maximum, and equal to $\frac{1}{2}$ when $a = \frac{v}{2}$. In practice, there are losses due to bad direction, leakage, and the water spreading; and the best efficiency, in ordinary work, is about $\frac{1}{3}$.

Poncelet Water-wheel.—This wheel is of the undershot type, but the vanes, instead of being radial, are curved. Let a jet of water, of velocity v, impinge tangentially in a horizontal direction on a curved vane (Fig. 181), moving with velocity u. The relative velocity at impact is $v - u$, and the height to which the water will rise is $\frac{(v-u)^2}{2g}$. The water will then descend, and at exit will have a velocity backwards relative to the float of $(u - v)$. The change of velocity is $2(u - v)$. The driving force is

FIG. 181.

$$\frac{\sigma a v}{g} \cdot 2(u - v)$$

and the work per second is

$$\frac{2\sigma a v u(u - v)}{g}$$

or twice that of radial floats. The efficiency is

$$\frac{4u(v - u)}{v^2}$$

which is a maximum, and equal to unity, when $u = \frac{v}{2}$.

In the Poncelet water-wheel, the tip of the bucket is invariably inclined to the direction of the jet. Let the direction of u and v be as shown in Fig. 182. For the jet to impinge on the vane in a tangential direction, the direction of the relative velocity (r) must be given by the triangle. If the angle a and the velocity v be given, then u and r are determined. On leaving the wheel, the relative velocity will be the same as at entrance,

and the diagram of velocities is as shown in Fig. 183. If v' be the absolute velocity of discharge, the useful work is

$$\frac{v^2 - v'^2}{2g}$$

and the efficiency is

$$\frac{v^2 - v'^2}{v^2}$$

Pelton Wheel.—The same result may be brought about by

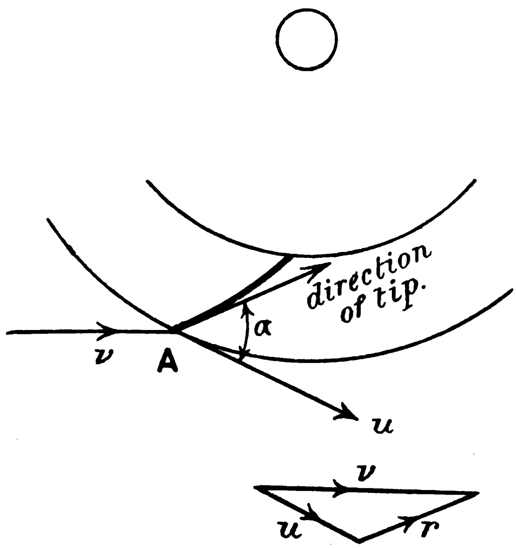


FIG. 182.

making the cups semi-cylindrical, the axis of the cylinder being parallel to the axis of the wheel. A number of these buckets are arranged round the wheel. In practice, the angle between the discharging edges must be less than 180° to allow the water clearing the bucket next below it. The efficiency is high, usually about 75 to 85 per cent. The water is discharged on the wheel through nozzles, which can be regulated to suit the altered conditions of working, and three or more nozzles may be distributed round the wheel. The wheels are small and compact.

Messrs. W. H. Bailey, Salford, make a Pelton wheel under the

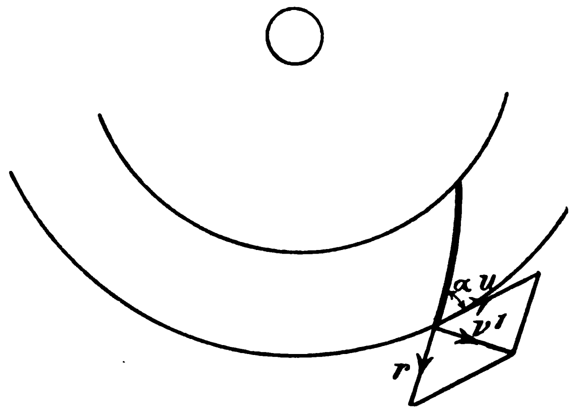


Fig. 183.

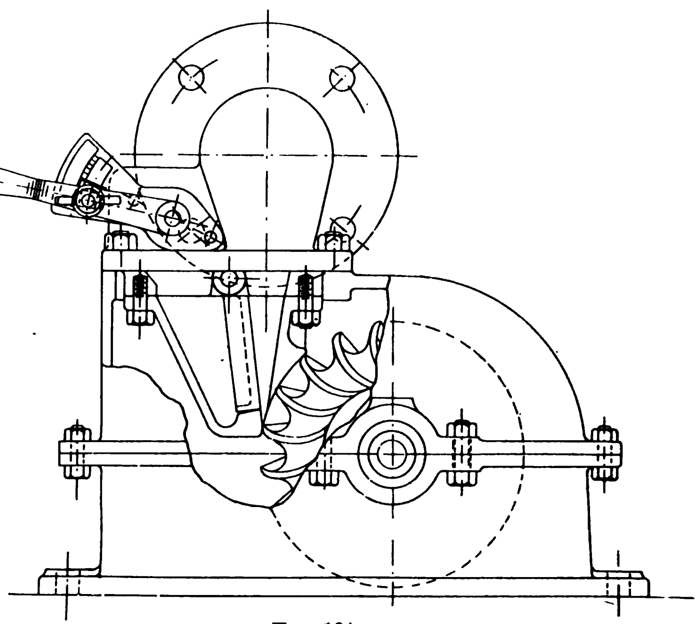


Fig. 184.

name of "Count de Visues" turbine. Tests on these turbines give an efficiency of 75 per cent. It is illustrated in Fig. 184.

§ 126. **Self-regulating Water-wheel.**[1]—Mr. Cassel has invented a self-regulating device suitable for a wheel of the vertical reaction type. The wheel (Fig. 185) consists in principle of two discs or sections, carrying upon their peripheries an equal number of buckets at the same pitch. These discs are free to slide upon the shafts, and bear upon a projection from an arm keyed to the shaft,

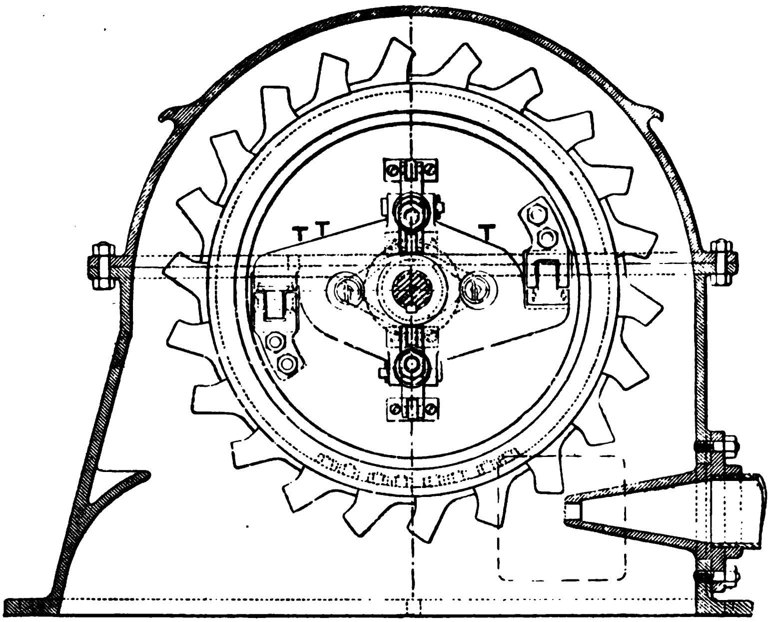


FIG. 185.

whereby the impulsive effort of the jet upon the buckets is transmitted to the shaft. In this particular type of wheel, rollers are introduced at the various bearing points with a view to diminishing friction. Upon each of the arms, which are keyed to the shaft, a double "T" lever is pivoted as shown, one end of the head of the "T" bearing against each of the discs. At the free end of the "T" lever a weight is fixed, which bears some definite proportion to the tension of the springs holding the two discs

[1] Institution of Mechanical Engineers, International Congress, Glasgow. No. 1901.

together. When the wheel is at rest, or running under normal speed, the discs are kept close together in obedience to the action of the spring, the buckets being then close together and forming practically one bucket with a dividing rib down the centre, a form which has been much used in Pelton wheels and similar types. The nozzle is so placed that the jet impinges centrally upon this rib. The action of the governing mechanism is as follows: The weights at the free end of the "T" levers are so proportioned that at a normal speed of the wheel the centrifugal force exerted by them is exactly equal to the tension of the spring holding the discs together. Should any portion of the load be thrown off, the tendency of the wheel is to increase the speed; this, however, also increases the centrifugal effort exerted by the weights, and the tendency to separate the discs becoming greater than the tension of the spring holding them together, the discs, which are free, slide down the shaft, move away from each other, thus forming a gap between the buckets, through which a portion of the jet passes without exerting any impulsive force on the wheel. The relative position of the buckets with regard to the jet is therefore determined by the load, provided, of course, that this load is not in excess of the mechanical equipment of the water delivered by the nozzle. At every change of load between the maximum load for which the wheel is built, downwards, the buckets automatically take up a position relatively to the jet, whereby only so much of the jet is utilized as is necessary to keep the wheel revolving at normal speed. The volume of water in the jet is then adjusted, and the buckets close up again at normal speed under the new load.

In a wheel 18 inches diameter the variation of speed was under 2 per cent. from the normal when the whole load was thrown off, the variation being practically momentary, the normal speed being again registered by the tacheometer within three or four seconds after the change in load had been effected. Again, when the head of water was varied suddenly from about 150 feet to over 1000 feet, the variation in speed did not exceed 1·8 per cent. from the normal. When the head was increased slowly there was no perceptible change.

§ 127. **General Problem of Vane Design.**—In the Poncelet wheel the receiving and discharging points are the same; in the Pelton wheel they are inclined, approximately, at 180° with each other.

Let AB be a moving vane (Fig. 186), and let the receiving edge A move in the direction AC, and the discharging edge in the direction BG. The vane, therefore, is virtually turning about a point, whose position is obtained by drawing a perpendicular through A to AC, to intersect a line through B perpendicular to BG. The lengths of AC, BG represent the linear velocities of A and B. Let a jet of water impinge on the vane in the direction DA, and let DA represent the magnitude and direction of the velocity with which the water leaves the guides. If the vane is properly designed, the water ought to glide on the wheel without shock. To satisfy this condition the relative velocity of the jet and wheel at A must be tangential to the vane. If a parallelogram DECA be drawn, EA will represent the velocity in magnitude and direction of the water as it impinges on the vane, and the lip of the vane at A must be tangential to EA.

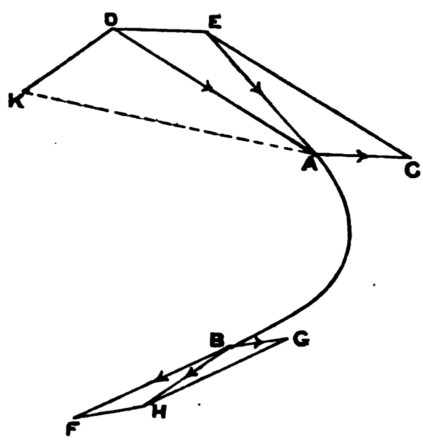


FIG. 186.

The water gliding on the vane is gradually deviated by the vane, the precise curvature being immaterial, and it will leave the vane at B in the direction BF. In the absence of friction and gravity, the relative velocity remains unaltered; hence, if BF be made equal to EA, BF will be the velocity of discharge relative to the wheel. Knowing the velocity of A, the velocity of B is determined; hence, draw BG in the direction of motion of B to represent it. Complete the parallelogram BFHG, then BH represents the absolute velocity of the water, in magnitude and direction,

leaving the wheel. Altering the vane angle at discharge will not alter the velocity BF; it will alter the absolute velocity of discharge. Since the flow takes place in atmosphere, the kinetic energy at entrance is $\frac{1}{2}\sigma Q \frac{1}{2g} AD^2$, and at exit $\frac{1}{2g}\sigma Q \cdot BH^2$, where Q is the water in cubic feet per second; thus, the efficiency is

$$\frac{AD^2 - BH^2}{AD^2}.$$

The efficiency will be a maximum and equal to unity when BH is equal to zero, in which case the water is discharged tangentially. The water, under these conditions, would not drain away from the machine. In practice BH must have such a value that its component perpendicular to BG is sufficient to drain the wheel. The effective component of the velocity BH, which drains the wheel, is perpendicular to the direction of motion BG, as any component perpendicular to that direction will, whilst increasing the urging force, decrease the efficiency. The magnitude of the drainage velocity can only be determined by experience. Thus, to make the wheel work under the most favourable conditions, the water must be guided to the wheel; it must be received by the wheel without shock, and it must leave the wheel with as little velocity as possible in a direction perpendicular to the direction of motion.

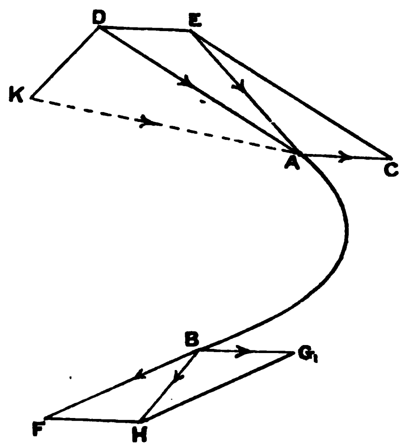


FIG. 187.

To find the total change of pressure on the vane, draw (Fig. 186) a line DK parallel and equal to BH, and join KA, then KA is the total change of velocity on the vane, and $\sigma Q \cdot KA$ is the total resultant force on the vane. It acts in the direction KA, and causes a turning moment on the wheel.

If the velocity of the wheel at A be equal to and parallel to that of B, Fig. 187 represents the diagram of velocities. In that case EK = GH = FB = EA; and for such a wheel the construction in Fig. 188 applies. If the directions of motion of water at the inlet and outlet edges are known, namely, BA and BK may be plotted. If a line be drawn bisecting AK at right angles to meet the direction in E, BE will represent that of the velocity of the wheel u. The relative velocity is represented by EA and EK, at the inlet and outlet edges respectively.

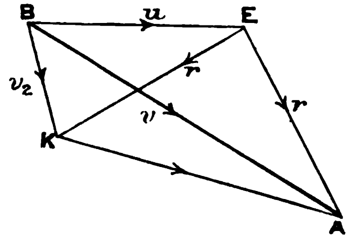


FIG. 188.

§ **128. Barker's Wheel.**—The case just discussed represents an impulse wheel. The flow takes place in the atmosphere, so that it is under constant pressure, and—barring friction—the relative velocity is constant.

Barker's wheel is a simple type of reaction wheel. It is shown diagrammatically in Fig. 189, which represents a plan. A is a vertical pipe, B is a hollow cylindrical casing, and C, C are orifices which direct the water tangentially from the orifices. The wheel rotates about the axis of the pipe A, being guided at the top and working in a footstep bearing at the bottom; the water is admitted at the top, and floods the casing B. The discharge of water from the orifices causes the wheel to rotate in a clockwise direction.

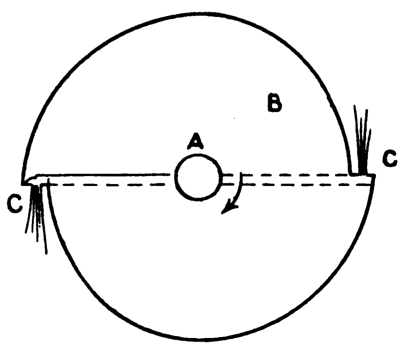


FIG. 189.

Imagine the orifices plugged up, and the static pressure head due to the column of water is h. If the wheel rotate with an angular ω, the motion is that of a forced vortex. If dr be the

thickness of an elementary circular strip at radius r, dp the increase of radial pressure, then

$$dp = \frac{\sigma\omega^2 r dr}{g}$$

whence

$$(p - p_0) = \frac{\sigma\omega^2}{2g}(r^2 - r_0^2)$$

where p is the pressure at radius r, and p_0 is the pressure at the axis. The increase of pressure at the orifice is

$$\frac{\sigma}{2g}\omega^2 r^2$$

and the pressure head at the orifice is

$$h + \frac{\omega^2 r^2}{2g}$$

If the plugs be removed, the velocity of discharge through the orifices relative to the arm will be

$$v = \sqrt{2gh + \omega^2 r^2}$$

neglecting the flow through the orifices. The absolute velocity of discharge through the orifices will be

$$\sqrt{2gh + \omega^2 r^2} - \omega r.$$

There may be two or more orifices, so that if a is the combined effective area of the discharge, the discharge per second is

$$Q = a\sqrt{2gh + \omega^2 r^2}$$

and the reaction at radius r is

$$\frac{\sigma Q}{g}\{\sqrt{2gh + \omega^2 r^2} - \omega r\}.$$

The turning moment is

$$\frac{\sigma Q r}{g}\{\sqrt{2gh + \omega^2 r^2} - \omega r\}$$

and the useful work per second is

$$\frac{\sigma Q \omega r}{g}\{\sqrt{2gh + \omega^2 r^2} - \omega r\}.$$

The total work available per second is σQh

$$\text{efficiency} = \frac{\omega r}{gh}\{\sqrt{2gh + \omega^2 r^2} - \omega r\}$$
$$= \frac{\omega^2 r^2}{gh}\left\{\sqrt{1 + \frac{2gh}{\omega^2 r^2}} - 1\right\}$$
$$= \frac{\omega^2 r^2}{gh}\left\{1 + \frac{gh}{\omega^2 r^2} - \frac{1}{8}\left(\frac{2gh}{\omega^2 r^2}\right)^2 \dots - 1\right\}$$
$$= 1 - \frac{1}{2} \cdot \frac{2gh}{\omega^2 r^2}$$

which increases as ωr increases, and becomes unity when ωr is infinite. In practice, the maximum efficiency is approximately when the velocity of orifices is the same as that of discharge, in which case the efficiency is

$$\frac{\omega r}{gh}\{\sqrt{2\omega^2 r^2} - \omega r\}\omega r = \frac{0{\cdot}414\omega^2 r^2}{gh} = 0{\cdot}828.$$

Hydraulic losses may be taken into account in the following way:—

Let the coefficient of hydraulic resistance of the wheel, referred to the velocity of discharge relative to the wheel v, be denoted by F, then the previous equation becomes

$$(1 + \mathrm{F})\frac{v^2}{2g} = h + \frac{\omega^2 r^2}{2g}$$

or
$$v = \sqrt{\frac{2gh + \omega^2 r^2}{1 + \mathrm{F}}}.$$

The expression for the efficiency is then—

$$\eta = \frac{\omega^2 r^2}{gh}\left\{\sqrt{\frac{1 + \frac{2gh}{\omega^2 r^2}}{1 + \mathrm{F}}} - 1\right\}.$$

If F be assumed constant, the efficiency will be a maximum when

$$\frac{2gh}{\omega^2 r^2} = 2\{\mathrm{F} + \sqrt{\mathrm{F}(1 + \mathrm{F})}\}$$

and the velocity of the orifice for maximum efficiency is—

$$\sqrt{\frac{gh}{\mathrm{F} + \sqrt{\mathrm{F}(1 + \mathrm{F})}}}$$

Experiment bears out the conclusion that there is a certain speed which gives maximum efficiency. Frequently it happens when $a = 1$, in which case $F = 0{\cdot}125$, and the maximum efficiency is 0·66 The friction of the bearings reduce this to about 0·6.

In this machine, it will be noticed that the only loss, barring hydraulic resistances, is due to the fact that the kinetic energy in the water discharged is entirely wasted. It is an inevitable loss, because the water enters the casing radially, and a turning moment can only be obtained by a tangential backward discharge at the orifices. But suppose that, by suitable guides, the water is directed to the machine by guides which give a circumferential velocity in the direction of motion. This would involve no expenditure of energy, and, by curving the arms, the water could be directed at discharge with little velocity of discharge in a tangential direction. The change of angular momentum would be obtained with greater efficiency.

TURBINES.

§ **129. Types of Turbines.**—In a turbine the water is guided to the moving parts of the machine by suitable guides, so as to give the water a circumferential velocity at entrance; it is received by the wheel without shock; it is guided through the wheel between vanes; and is discharged with little, if any, velocity in a tangential direction. Turbines have the advantage of being small for the power developed. The axis of the wheel may be horizontal or vertical.

Turbines are divided into three classes, according to the direction in which the water flows:

(1) *Parallel-flow, or axial, turbines,* in which the water is supplied and discharged in a current parallel to the axis.

(2) *Outward-flow turbines,* in which the water is supplied and discharged in currents radiating from the axis.

(3) *Inward-flow turbines,* in which the water is supplied and discharged in currents converging radially to the axis.

The different types are represented in Figs. 190, 191, 192, whilst Fig. 193 is a development of guides and vanes. In each

case A is the supply chamber, C the guides which are fixed, and which are curved so as to allow the water to enter the wheel with a considerable tangential velocity. The wheel B consists of a drum

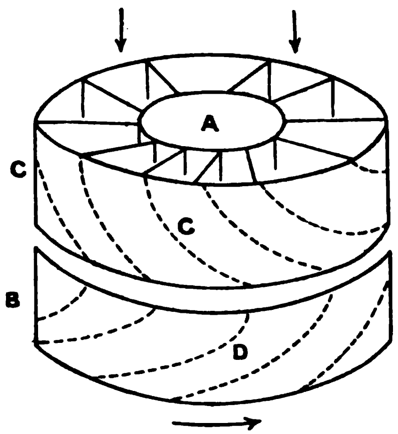


Fig. 190.

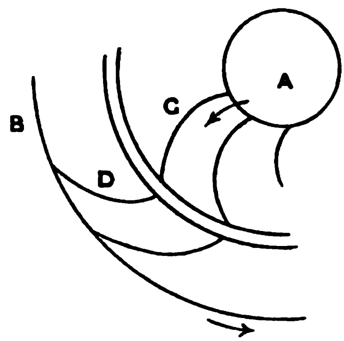


Fig. 191.

or annular passage, containing a set of suitably curved vanes D, which are curved backwards in such a manner that the water, after glancing off them, is left behind with as little energy as

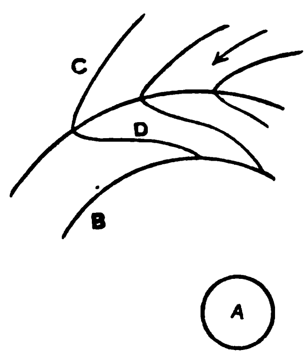


Fig. 192.

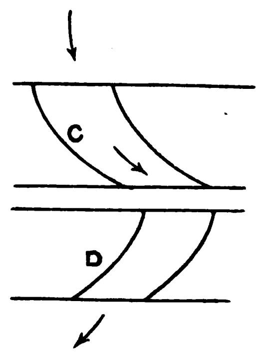


Fig. 193.

possible. In some cases the water is guided away from the wheel.

In some cases the wheel passages run full. The velocity at

any section depends on the sectional area of the passages through the wheel, and the turbine is called a *reaction* turbine. The energy

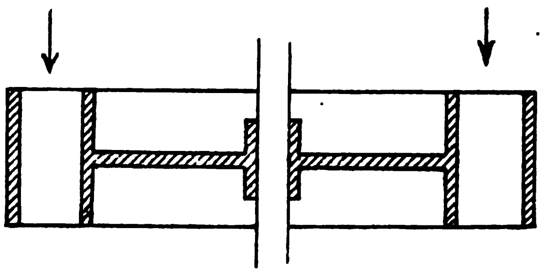


FIG. 194.

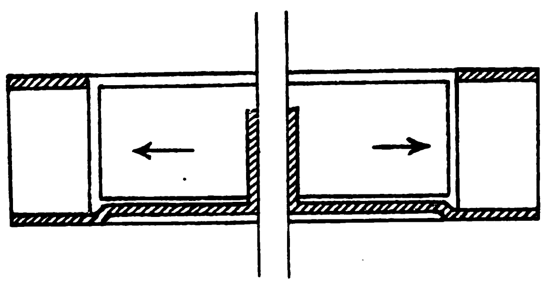


FIG. 195.

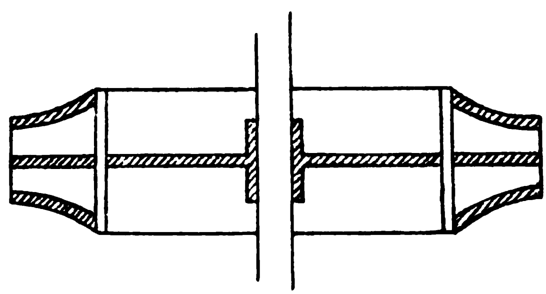


FIG. 196.

in the supply chamber may be partly kinetic and partly pressure energy, the pressure energy being converted into kinetic energy in the wheel itself. The wheel may be worked "drowned."

In others, already noticed, the wheel passages never run full, and the pressure is constant. The energy in the supply chamber is all kinetic, and the turbine must be placed above tail water. Such turbines are called *impulse* turbines.

In parallel-flow turbines (Fig. 194)—neglecting the thickness of the vanes—the receiving and discharging areas are equal, and the velocity of flow—that is to say, the component velocity parallel to the axis—is constant. In radial-flow turbines the crowns may be parallel plates perpendicular to the axis (Fig. 195) or curved plates (Fig. 196). In Fig. 195, neglecting thickness of vanes, the velocity of flow varies inversely as the distance from the axis; in Fig. 196 the velocity of flow depends on the curve of the crowns. If the crowns be hyperbolic curves having axes along the axis of the shaft and the central radial line, the velocity of flow will be the same at every section.

The velocity of a particle of water, whether considered in the guides or vanes, may be resolved into two components: (1) a component perpendicular to the direction of motion (whether parallel to the axis or radial); (2) a component tangential to the direction of motion. The former is that component by which the water is carried towards, through, or away from the wheel, and is called the *velocity of flow*. The latter is called the *velocity of whirl*. By reducing the velocity of whirl as the water flows through the machine, a turning moment is exerted on the wheel.

§ **130. Reaction Turbines.**—In reaction turbines, the guide passages and buckets run full.

For simplicity, let the notation be as follows:

Let	v	refer to the	absolute velocity of a particle of water
"	f	"	flow " " " "
"	w	"	whirl " " " "
"	r	"	relative velocity of a particle to the moving vane
"	u	"	velocity of the rim
"	A	"	"flow area;" that is, the section perpendicular to the direction of flow
"	Q	"	discharge in cubic feet per second
"	a_2, a_3	"	inlet and outlet radii

§ 131. **Geometrical Relations between Guide and Vane Angles and Velocities.**—Let suffixes (1), (2), (3) refer to the entrance of guide, the entrance to the vanes, and the exit from the vanes respectively, α, β, γ, the guide, vane angle at inlet, and vane angle at exit. Then

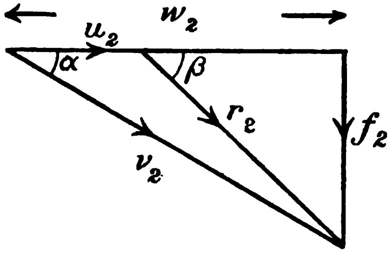


FIG. 197.

$$\frac{p_1}{\sigma} + \frac{v_1^2}{2g} = h_1$$

where h_1 is the effective head on the turbine. If u_2 is given, Fig. 197 shows the velocity at entrance to vanes, a line to represent u_2 be plotted and lines inclined at angles α and β to the direction of motion. The triangle of velocity gives v_2 and r_2, and a perpendicular line through the point of intersection to the direction of motion gives f_2, the horizontal component of v_2 being w_2. Thus, the triangle of velocities is obtained. At the discharging edge, u_3 and f_3, are known from the proportions of the turbine. Fig. 198 is the velocity diagram for the discharging edge, giving w_3 and v_3. For maximum efficiency $w_3 = o$ (§ 127)

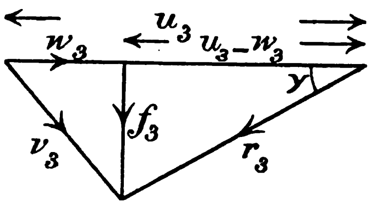


FIG. 198.

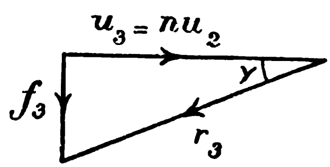


FIG. 199.

and Fig. 199 represents the diagram of velocities, in which n is the ratio of the outlet and inlet radius.

§ 132. **Analytical Relations.**—The diagrams in the last article give the relation between the angles; but a further relation is required before a solution can be obtained.

The pressure head h_2 exists part as pressure head and part as kinetic head, the relationship is

$$h_2 = \frac{p_2}{\sigma} + \frac{v_2^2}{2g} = \frac{p_2}{\sigma} + \frac{f_2^2 + w_2^2}{2g}.$$

The only energy lost is the energy at discharge, and this is equal to

$$\frac{v_3^2}{2g}$$

since the pressure at discharge is atmospheric. The head utilized is therefore represented by

$$\left(h_2 - \frac{v_3^2}{2g}\right)$$

and this represents the useful energy per pound.

The useful energy per pound may be estimated in another way. The momentum per second at the inlet edge is

$$\frac{\sigma Q w_2}{g}$$

and at the outlet

$$\frac{\sigma Q w_3}{g}.$$

The moment of momentum on the wheel is therefore

$$\frac{\sigma Q w_2 a_2 - \sigma Q w_3 a_3}{g} = \frac{\sigma Q}{g}(w_2 a_2 - w_3 a_3).$$

The work per second is therefore

$$\frac{\sigma Q}{g}\omega(w_2 a_2 - w_3 a_3)$$

where ω is the angular velocity of the wheel; and this is

$$\frac{\sigma Q}{g}(u_2 w_2 - u_3 w_3).$$

Thus, per pound, the work per second is

$$\frac{u_2 w_2 - u_3 w_3}{g}.$$

The velocity diagrams in the last article have also to be satisfied, and in addition

$$\frac{u_2 w_2 - u_3 w_3}{g} = h_2 - \frac{v_0^2}{2g} = \frac{p_2}{\sigma} + \frac{v_2^2 - v_3^2}{2g}.$$

If $p_3 = 0$, and $w_3 = 0$

$$\frac{u_2 w_2}{g} = h_2 - \frac{f_3^2}{2g} = \frac{p_2}{\sigma} + \frac{w_2^2 + f_2^2 - f_3^2}{2g}.$$

If $f_2 = f_3$, or if the velocities of flow are small compared to the other terms, then

$$\frac{u_2 w_2}{g} = \frac{p_2}{\sigma} + \frac{w_2^2}{2g}.$$

If p_2 be zero the turbine is an impulse turbine, then

$$u_2 = \frac{w_2}{2}.$$

If the pressure and kinetic heads are equal, then $u_2 = w_2$. Altering the ratio of pressure and kinetic heads in the supply chamber alters the speed of the wheel and vane angles.

§ 133. **Illustration.**—Consider a particular case. Suppose the vane angle at inlet is 90°, and that the vanes are radial at inlet; and suppose the crowns of the wheel be so designed that the velocity of flow is constant. Assume $w_3 = 0$, as is generally the case, and take the outer radius equal to n times the inner. The velocity diagrams are shown in Figs. 200, 201, etc. Also

$$h_2 = \frac{u_2 w_2}{g} + \frac{f_3^2}{2g} = \frac{u_2^2}{g} + \frac{u^2 u_2^2 \tan^2 \gamma}{2g}$$

$$\therefore u_2^2 = \frac{2gh_2}{2 + u^2 \tan^2 \gamma} = \frac{V_2^2}{2 + u^2 \tan^2 \gamma} = \frac{V_2^2}{2 + \tan^2 a}$$

where V_2 = velocity due to whole head $= \sqrt{2gh_2}$

$$\therefore \frac{u_2}{V_2} = \frac{1}{\sqrt{2 + \tan^2 a}}$$

since $\tan a = \frac{f_2}{u_2} = \frac{u f_3}{u_3} = u \tan \gamma$.

Also
$$\eta = \frac{u_2 w_2}{g h_2} = \frac{2u_2^3}{V_2^2} = \frac{2}{2 + \tan^2 a}$$

and
$$\frac{p_2}{\sigma} = h_2 - \frac{w_2^2 + f_2^2}{2g} = h_2 - \frac{u_2^2(1 + \tan^2 a)}{2g}$$
$$= h_2\left(1 - \frac{1 + \tan^2 a}{2 + \tan^2 a}\right)$$
$$= h_2 \frac{1}{2 + \tan^2 a}$$

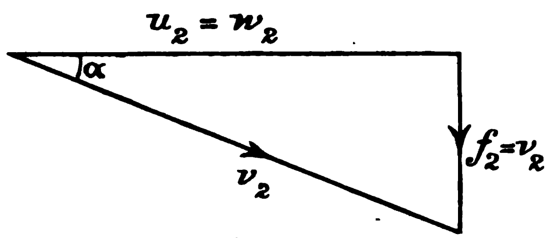


FIG. 200.

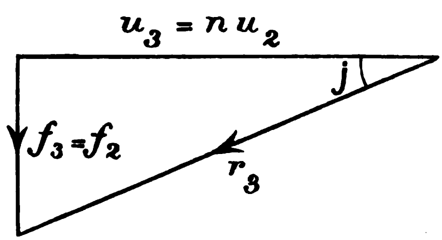


FIG. 201.

These quantities are all expressed in terms of a, the angle of the guides, and the following table gives the results in the case of a parallel flow turbine in which $n = 1$, an outward flow turbine in which $n = \sqrt{2}$, and an inward flow turbine in which $n = \frac{1}{2}$.

a	$n \tan \gamma = \tan a$.	Value of γ parallel $n = 1$.	outward $n = \sqrt{2}$.	inward $n = \frac{1}{2}$	$\frac{u_2}{V_2}$	η	$\frac{p_2}{g h_2}$	$\frac{v_2^2}{V^2}$	$\frac{f_3^2}{2g h_2}$
20°	0·364	20°	14½°	36°	0·685	0·938	0·469	0·531	0·062
25°	0·466	25°	18¼°	43°	0·672	0·902	0·451	0·549	0·098
30°	0·577	30°	22¼°	49°	0·655	0·857	0·428	0·572	0·143
35°	0·700	35°	26½°	54½°	0·634	0·804	0·403	0·597	0·196

The table shows that with radial vanes at inlet, increasing the guide angle increases the angle at discharge, decreases the speed of the wheel, the pressure head and the efficiency, and increases the velocity of flow.

§ **134. Change of Pressure in the Wheel.** The equation of angular momentum may be expressed in the form

$$\frac{u_2 w_2 - u_3 w_3}{g} = \left(\frac{p_2}{\sigma} + \frac{v_2^2}{2g}\right) - \left(\frac{p_3}{\sigma} + \frac{v_3^2}{2g}\right)$$

$$\text{or } \frac{p_2 - p_3}{\sigma} + \frac{f_2^2 - f_3^2}{2g} + \frac{w_2^2 - w_3^2}{2g} = \frac{u_2 w_2 - u_3 w_3}{g}$$

which may be written

$$\frac{p_2 - p_3}{\sigma} = \frac{1}{2g}\left\{(w_3^2 - 2u_3 w_3 + f_3^2) - (w_2^2 - 2u_2 w_2 + f_2^2)\right\}$$

$$= \frac{1}{2g}[\overline{(u_3 - w_3)^2 + f_3^2} - \overline{(w_2 - u_2)^2 + f_2^2} + (u_2^2 - u_3)^2]$$

$$= \frac{1}{2g}\{r_3^2 - r_2^2 + u_2^2 - u_3^2\}$$

$$= \frac{u_2^2 - u_3^2}{2g} + \frac{r_3^2 - r_2^2}{2g}.$$

Thus the change of pressure depends on the velocities of the inlet and outlet edges, and the change of relative velocity in the wheel passages at exit and inlet. The first is the change of pressure on the assumption that the water in the wheel rotates as a solid body; the second, that the wheel is fixed, and the water flows in stream-line motion through the passage.

§ **135. Stable and Unstable Motion.**—In an outward flow reaction turbine $u_3 > u_2$, and the first term is negative. Assuming that $p_3 = 0$, the pressure at the inlet is less than it otherwise would be if the wheel were at rest, and the difference is more marked the greater the speed of the wheel. If for any cause the wheel races—that is, acquires a sudden increase in speed—and the first term in the expression for the pressure increases negatively and the pressure in the supply chamber is reduced, the supply per

minute is increased, and the work is also increased. The efficiency will fall off because, although the whirling speed increases, it will probably not increase to such an extent as to prevent shock. Thus, in an outward-flow turbine, there is an element of unstability—a tendency to race increasing. For this reason $\frac{a_3}{a_2}$ should not be made too large, usually $1\frac{1}{4}$ to $1\frac{1}{3}$.

In an inward-flow turbine the reverse takes place. In this case $u_2 > u_3$, the first term is positive, and if the turbine race, the pressure in the supply chamber increases, the supply is reduced, and there is a tendency to reduce the speed; that is to say, the motion is stable.

Barker's wheel is an illustration. Here $w_2 = 0$, $f_3 = 0$, and the value of w_3 is negative, and the first equation becomes

$$\frac{u_3 w_3}{g} = \left(\frac{p_2}{\sigma} + \frac{v_2^2}{2g}\right) - \frac{w_3^2}{2g}$$

$$\text{taking } \frac{p_3}{\sigma} = 0$$

$$\begin{aligned} \text{or} \quad h_2 &= \frac{u_3 w_3}{g} + \frac{w_3^2}{2g} \\ &= \frac{(w_3 + u_3)^2 - u_3^2}{2g} \\ w_3 &= -u_3 + \sqrt{2gh_2 + u_3^2} \\ &= -\omega r + \sqrt{2gh_2 + \omega^2 r^2} \end{aligned}$$

as before (§ 128).

§ **136. Impulse Turbines.**—As already pointed out in impulse turbines, the wheel passages never run full. The energy equation becomes

$$\frac{u_2 w_2}{g} = \frac{v_2^2}{2g} - \frac{f_3^2}{2g}$$

which is identical in form to

$$\frac{u_2^2 - u_3^2}{2g} = \frac{r_2^2 - r_3^2}{2g}$$

In a parallel-flow turbine $u_2 = u_3$ and $r_2 = r_3$.

Taking the general case of an impulse turbine, the energy of equation is

$$\frac{u_2 w_2}{g} = h_2 - \frac{f_3^2}{2g}$$

in which $v_2 = V_2$, and therefore

$$V_2^2 - 2u_2w_2 - f_3^2 = 0$$

$$u_2 = \frac{V_2^2 - f_3^2}{2w_2}$$

If a be the guide angles (Fig. 197), then

$$w_2 = V_2 \cos a$$

$$\therefore u_2 = \frac{V_2^2 - f_3^2}{2V_2 \cos a}$$

If V_2, f_3, a be given, this enables u_2, the speed of the wheel, to be calculated. The value of f_2 is obtained from

$$f_2 = V_2 \sin a$$

and hence β is obtained from (Fig. 197).

$$\tan \beta = \frac{f_2}{w_2 - u_2}$$

The value of $r_2 = \sqrt{f_2^2 + (w_2 - u_2)^2}$.

At exit u_3 and f_3 are known, and therefore (Fig. 199)

$$\tan \gamma = \frac{f_3}{u_3} \qquad \text{and } r_3^2 = f_3^2 + u_3^2.$$

§ **137. Resistances in a Turbine.**—In considering the resistance of turbines, the resistances in the head and tail race must be excluded. The turbine must be debited with all losses between the section at which the delivery pipe enters the turbine casing, and at which the suction pipe leaves the casing. This would give the true hydraulic efficiency of the turbine looked upon as a hydraulic machine. But the actual efficiency of the whole plant must include all losses in the delivery and suction pipes from whatever cause arising.

The most important loss in a turbine is that due to skin friction. The speeds of flow in a turbine are high, and, unless the curvature of the guides and vanes are gradual, might increase, perceptibly, the frictional loss.

The second loss which, in an indifferently designed turbine, may be considerable, is the loss due to eddies at the entrance and exit from the guides and vanes. Fig. 202 shows a badly designed

FIG. 202.

vane, blunt at exit edges, and in which eddies are formed, and a good design, in which the edges are fined down to leave a convergent stream (compare § 60).

A third loss is that due to the energy in the discharge water. This is inevitable, because the machine has to be drained of the water, and gravity—in turbines—is not sufficient for the purpose.

§ 138. **Law of Comparison in Turbines.**[1]—Although it is possible to make approximate calculations of the above losses, given in the next article, it is evident that if two similar and similarly situated turbines work under similar conditions, there must be some definite relation between the resistances and speeds in the two turbines; since the laws of skin friction, eddy resistance, and flow loss can be expressed in general terms without reference to quantitative data.

Consider, first, the loss due to *skin friction*, and take a small element of surface, either of the guides or vanes. Let the velocity of rubbing over this element be v. Then, the dissipation of energy is proportional to the product of the square of the speed,

[1] This law of comparison was given by Professor Osborne Reynolds, in his lectures on Hydro-mechanics.

and the area of rubbing surface per second. That is to say, if v is the velocity of rubbing in feet per second, and D denotes a linear dimension, say the diameter, then the dissipation of energy is proportional to $D^2v \times v^2 = D^2v^3$, the first term representing the rubbing area per second, and the second term being proportional to the skin friction. If the discharge be Q cubic feet per second, the loss of head per cubic foot of water is proportional to

$$\frac{D^2v^3}{Q}.$$

The second loss is due to the formation of eddies, whether those eddies are produced by discontinuities or diverging streams (§§ 34, 60). A general expression for the loss of head is

$$\left(\frac{a_2}{a_1} - 1\right)^2 \cdot \frac{v^2}{2g}$$

and thus the eddy loss is proportional to v^2, since the ratio of $\frac{a_2}{a_1}$ is the same in similar turbines. Since the velocity through the guides and vane passages is proportional to the discharge, and inversely proportional to the sectional area; the loss of head per pound is proportional to

$$\left(\frac{Q}{D^2}\right)^2$$

which may be taken to be proportional to the loss per cubic foot.

The third loss is due to the flow loss at exit. This may be taken to vary as the square of the velocity, and follows the same law as eddy resistance. Thus the total loss of head is

$$H^1 = A\frac{D^2v^3}{Q} + \frac{BQ^2}{D^4}.$$

Moreover, since the turbines are geometrically similar, and work at corresponding speeds under corresponding conditions, v must be proportional to H, the head of water. Thus

$$\frac{H^1}{H} = A\frac{D^2H^{\frac{1}{2}}}{Q} + B\frac{Q^2}{D^4H}$$

in which A and B are coefficients to be determined for each design of turbine.

In a turbine, the problem is to design a turbine to work with a given quantity of water and a given head. Different proportions of the guides and wheels will give different efficiencies; and the question arises whether any particular design gives the best results. The last equation points to this conclusion. For, if H′ be a minimum, the last equation gives, differentiating with respect to D

$$\frac{2ADH^{\frac{1}{2}}}{Q} - \frac{4BQ^2}{D^5H} = 0$$

$$\text{or} \quad \frac{D^6H^{\frac{3}{2}}}{Q^3} = \frac{2B}{A}.$$

Hence if H and Q are known, D, the diameter for maximum efficiency, is known.

Again, if Δ denote the gross area of the orifices, and w the velocity of whirl at the inlet

$$\Delta = \frac{Q}{w} = \frac{Q}{\sqrt{gh}}$$

taking radial vanes. For a particular turbine, $\frac{\Delta}{D^2}$ must have a certain value.

The inclination of the vanes is given

$$\tan a = \frac{f}{w} = \frac{Q}{\Delta\sqrt{gh}} :: \frac{Q}{D^2\sqrt{h}}$$

$$\text{or} \quad \cot a \text{ is } :: \frac{D^2\sqrt{H}}{Q}.$$

This method of consideration for the design of a turbine may be applied to all other types of hydraulic machines. The efficiencies of the two machines will be the same, and if the only loss in the machine be taken to be the frictional loss, the law of comparison becomes

$$\frac{D^2H^{\frac{1}{2}}}{Q} = \text{constant}.$$

If the discharge of the machines be the same, then $D^2\sqrt{H}$ will be constant. If the difference of head, H, be small, then D must be large—thus, if a pressure engine be used, there must be a large cylinder; or, if a turbine, it must be large, and, therefore, it must

be of the type of an undershot water-wheel, or, better, an overshot wheel. If H is large, then D is small, and the type of turbine discussed is used.

The efficiencies of the different types of machine are—

Undershot water-wheel	0·6
Overshot	0·75
Pressure engines (slow moving) . .	0·66 to 0·8
Turbines	0·8

§ **139. Design of Turbine.**—The determination of the different losses in a turbine, and their separation, is not an easy matter. In a careful analysis made of a number of well-designed Jouval parallel-flow reaction turbines, the type usually used on the Continent, the average results were—

Collision on tops of guides and vanes	3·5 to 5 per cent.
Leakage between guides and wheel	4·5 „ „
Friction in guides and vanes . .	7 to 9 „ „
Flow loss	2 to 8 „ „
Mechanical friction . . .	2 to 4 „ „
	19 to 30½

Thus, the efficiency varies from 81 to 69½. The first would be an exceptional result, and the average efficiency may be taken as about 75 per cent.

Generally, the shaft friction absorbs about 2 or 3 per cent.; the flow loss, 5 to 7 per cent.; frictional and other hydraulic losses, 12 to 14 per cent.; giving an efficiency of 74 to 81 per cent.

In designing a turbine, the head and speed would probably be known. The type of turbine must be selected—whether radial or parallel flow—and the shape of the crowns must be fixed. Moreover, some relation must be assumed between the speed of the wheel and the velocity due to the head; or, what is the same thing, between the speed of the wheel and the whirling velocity at inlet.

An illustration may be given showing the difference (1) when only flow loss is taken into account; (2) when the hydraulic resistances are included.

Take an outward flow-turbine, the outer radius being 1¼ times

the inner, with parallel crowns, and the velocity of the wheel 0·9 velocity of whirl, the flow loss being 7 per cent. of the head, and all other hydraulic losses 15 per cent.

If only the flow loss be included,

$$\frac{u_2 w_2}{g} = 0{\cdot}93 h_2$$
$$u_2 = 0{\cdot}9 w_2$$
$$w_2 = 0{\cdot}719\sqrt{2gh_2}$$
$$u_2 = 0{\cdot}646\sqrt{2gh_2}$$
$$u_3 = 0{\cdot}806\sqrt{2gh_2}$$
$$f_3 = 0{\cdot}265\sqrt{2gh_2}$$
$$f_2 = 0{\cdot}33\sqrt{2gh_2}$$
$$\frac{p_2}{\sigma} = 0{\cdot}375 h_2$$
$$\frac{v_2^2}{2gh_2} = 0{\cdot}625 h_2$$
$$\alpha = 25^\circ, \quad \beta = 77\tfrac{1}{2}^\circ, \quad \gamma = 18^\circ.$$

This may be considered to be a first approximation.

Then, with friction—

$$\frac{u_2 w_2}{g} = 0{\cdot}78 h_2$$
$$\text{and since} \quad u_2 = 0{\cdot}9 w_2$$
$$w_2 = 0{\cdot}688\sqrt{2gh_2}$$
$$u_2 = 0{\cdot}592\sqrt{2gh_2}$$
$$\text{also} \quad \frac{f_3^2}{2g} = 0{\cdot}07 h_2$$
$$\therefore f_3 = 0{\cdot}265\sqrt{2gh_2}$$
$$f_2 = 1{\cdot}25 f_3 = 0{\cdot}33\sqrt{2gh_2}$$
$$u_3 = 1{\cdot}25 u_2 = 0{\cdot}74\sqrt{2gh_2}$$
$$\frac{v_2^2}{2gh_2} = 0{\cdot}33^2 + 0{\cdot}658^2 = 0{\cdot}542$$
$$\frac{p_2}{\sigma} = 0{\cdot}458$$
$$\alpha = 27^\circ, \quad \beta = 79^\circ, \quad \gamma = 19\tfrac{1}{2}^\circ$$

This may be considered to be a second approximation.

The speed by the first calculation is 9 per cent. too high, and the pressure head at exit of the guides is 18 per cent. too small.

The above method gives the speed of the wheel and the guide and vane angles. There are, however, other quantities also required. The supply of water is usually known. If a_3 represents the exit radius of the wheel, and b_3 the width between the crown plates at exit, n the number of revolutions per second, then—

$$2\pi a_3 b_3 f_3 = Q$$
$$\text{and } u_3 = 2\pi n a_3.$$

Here f_3, Q, u_3 are known, and b_3, a_3, and n are, say, unknown. Usually the revolutions would be given, in which case a_3 and b_3 are determined. The eye of the wheel is generally of the same area as the area of flow through the wheel.

In the same way, it is possible to include friction in impulse turbines. But there is an important distinction. In reaction wheels, the velocity depends on the sectional area of the buckets. In an impulse wheel, this is not so. The assumption that the relative velocity is constant over the vane is not true—friction modifies it. Moreover, in impulse the effect of centrifugal force must tend to "pile" the water to the outer part of the wheel. An impulse wheel does not, therefore, admit of an exact treatment.

§ 140. Comparison between Reaction and Impulse Turbines.—*Position of Turbines—Reaction Turbines.*—In reaction turbines, the buckets always run full, and, therefore, the wheel may either work immersed or be placed at a height above tail water not exceeding the barometric pressure. In the former case, the pressure at discharge is greater than atmospheric pressure by at least the pressure due to the drowned height; in the latter, a suction pipe leads from the turbine to the tail water, and the pressure is less than that of the atmosphere by an amount not greater than the length of suction column. In the absence of losses, the available head is unaltered, being, in all cases $(p_2 - p_3)\sigma$, but the pressure $p_2 . p_3$ depend on the position of turbine. The advantage of placing the wheel immersed under ordinary conditions is that no part of the fall is lost in time of drought. The advantage of a suction tube is that the wheel is more accessible.

The friction in the suction tube is not a loss, because the delivery pipe is necessarily less by the same amount. If the arrangement of the suction tube is such that the water enters it without shock, and if the suction pipe gradually increase in area until it enters the turbine, the pressure will increase. A suction pipe is most conveniently applied to a parallel flow and an outward-flow turbine Sometimes—particularly in inward-flow turbines—a "diffuser" is used. The diffuser is a casing surrounding the turbine, with curved plates, being wider at the outer radius and contracting gradually to the entrance to the guides.

Impulse Turbines.—The buckets are designed so as to just run full in times of flood, so that under normal conditions they are not full. The wheel, therefore, must be placed above tail water. If the net fall is only small, and the turbine is clear of the tail water in flood, the loss in times of drought becomes excessive. It might, therefore, be allowed to run drowned in times of flood, when the supply is plentiful; or else the hydropneumatic system could be adopted. In Girard's parallel flow, the turbine is placed below tail water in a casing supplied with air by a small air-pump. The pressure is thus greater than atmospheric pressure by the height of the tail water above the discharge orifice, and consequently varies with the discharge. The free water surface inside the casing is maintained at an invariable level just below the discharge orifices of the wheel. They are not much used on account of extra trouble, cost of auxiliary plant, etc.

Speed of Wheel.—In reaction turbines the pressure and kinetic heads in the supply chamber are usually equal (§ 132), so that the velocity of the wheel is $\sqrt{gh}$. In impulse turbines, the velocity is usually half that due to the whole head, namely, $\frac{1}{2}\sqrt{2gh}$, that is, $0{\cdot}7\sqrt{gh}$. It is, therefore, less in impulse turbines than in reaction turbines. Impulse turbines are better suited for large heads than reaction turbines, because the speed in the latter might become excessive. This difficulty is removed by using compound turbines (§ 142); an impossible arrangement in impulse turbines.

Regulation.—Every turbine is designed to work most efficiently at a known head, a known speed, and a known supply. The head, within limits, remains constant, and the variable quantity

is usually the supply. Some kind of regulation must therefore be adopted which makes the wheel efficient at all loads. In reaction turbines of the parallel-flow type, the water is regulated by sluices, which slide vertically, between guides, in the guide passages (Fig. 203). If the sluices are all partially opened, there will be considerable losses due to a sudden enlargement, and the

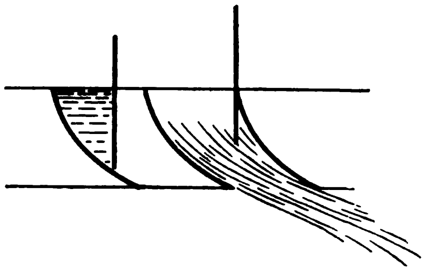


FIG. 203.

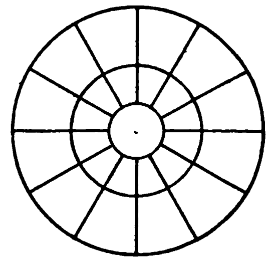


FIG. 204.

custom is to entirely shut one or more as required. The bucket is always full, and the intermediate supply of water probably causes a slight unsteadiness, and some eddy loss.

In the parallel-flow reaction turbines (*Fontaine*), double turbines have been used, consisting of a pair of concentric wheels, and made in one piece (Fig. 204), supplied with water by a similar pair

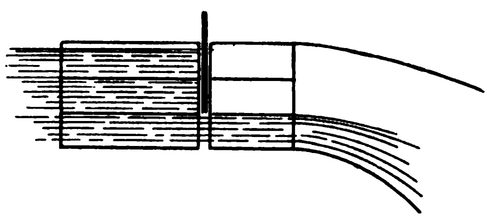


FIG. 205.

of concentric annular supply passages. Each passage has its own set of sluices so that either division of the double wheel can have its supply of water cut off.

In the Fourneyson outward-flow reaction turbine, the guide chamber and wheel are divided into a number of compartments by partitions perpendicular to the axis (Fig. 205). A cylindrical

sheeting can be raised or lowered between the guide chamber and wheel so as to cut off one or more compartments. There will be increased friction at ordinary loads, but for large turbines it is a very efficient mode of regulating.

In the *Thomson* inward-flow turbine, the guide blades—four in number—are pivoted near the tips by countersunk pins (Fig. 206). In the figure the supply pipe is E, the casing C, the wheel A, with its shaft B, and the guides G. The countersunk pins are at P. These guides are connected near their outer ends to four vertical spindles by bell-cranked levers, and these spindles, by gearing, work with a central spindle outside the casing. When the central

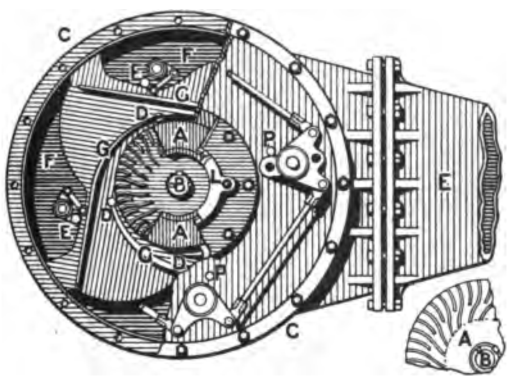


Fig. 206.

spindle is turned, each guide is equally affected, and the angle of the guides is altered. It is a very efficient means for small variations, but under large variations, if the turbine has to run at a constant speed, some sacrifice of efficiency takes place.

Impulse Turbines.—Impulse turbines lend themselves easier to efficient regulation than reaction turbines. In both, the guide passages are always full, but in an impulse turbine the wheel passages are never full.

In *Girard's* parallel-flow turbine, each guide passage is supplied with a shutter, which can be raised or lowered at will, as in Fontaine's reaction turbine—one or more shutters being closed according to the supply. The efficiency is unaltered—however

many guide passages are closed—because the water, escaping from a full-open guide passage, impinges on an empty bucket, and no loss in shock occurs. Thus, parallel-flow impulse turbines have an advantage over parallel-flow reaction turbines. The buckets of an impulse turbine must be ventilated, and the usual arrangement is as shown in Fig. 207.

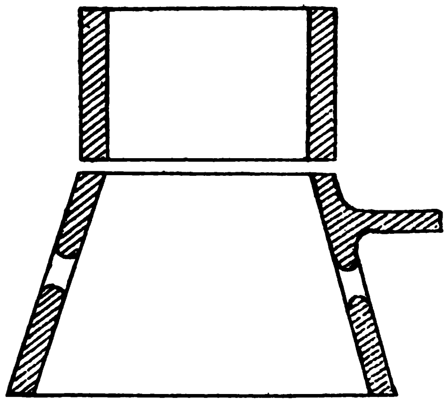


FIG. 207.

§ 141. **Devices to relieve Pressure of Shaft.**—An interesting point in connection with turbines on a large scale—as, for example, the Niagara Installations, where the power was transferred from the lower level to the higher by means of a vertical shaft—the weight of shafting and turbines might be as much as 60 to 70 tons. In parallel-flow turbines the upward pressure might be obtained by so arranging the velocities of flow that the change of momentum in a direction parallel to the axis would balance the thrust. The water, in that case, would enter at the bottom with a fairly high-flow velocity, and it would leave at the top with as low velocity of flow as is consistent with efficiency. This is the arrangement adopted in the Niagara Paper Mills turbines.

In a more recent installation at Niagara the turbines are of the outward-flow type, with diaphragms and cylindrical shutters. To balance the great weight of shaft and turbines, the water from the penstock passes up through the guide portion of the upper wheel, which thus forms a balancing piston; whereas, in the lower wheel the water acts as in the ordinary way.

§ **142. Compound Turbines.**—It has been pointed out (§ 140) that a disadvantage of reaction turbines is that, with high heads, their speed becomes excessive. Professor Osborne Reynolds, F.R.S.,

invented the compound turbine in order to overcome this objection. The turbine is of the inward-flow type. The principle of this turbine is that the fall of pressure is obtained in stages. The water flows from the delivery pipe, through gradually deviated channels, and is discharged on to the first wheel without shock. It is then led from the eye of the first wheel to the outside of the second wheel, and the same precautions are taken. There may be two or more wheels. In the one first designed by Professor Reynolds (in 1886)—and now working in the Whitworth Laboratory at Manchester, with unimpaired efficiency—was a four-stage turbine. In Chapter VI. is described the first compound centrifugal pump, also with four chambers. The four-stage turbine is an exact reproduction of the pump, but it is half the size.

§ **143. Test.**—The following table gives a number of tests of this four-stage turbine at different speeds—

Revolutions per minute	1200	1755	1770	1963	2072
Delivery head at turbine	85·2	87·7	85·8	88·7	92·0
" " tail race	3·4	2·6	2·3	2·2	2·7
Effective head in feet	81·8	85·1	83·5	86·5	89·3
Water per minute in pounds	398	350	355	327	328
Turning moment in foot-pounds	3·00	2·00	2·2	1·5	1·3
Efficiency	0·69	0·74	0·83	0·65	0·59

It appears that the maximum efficiency—which is exceedingly high—is 83 per cent at 1770 revolutions per minute, the efficiency ranging from 69 per cent. at 1200 revolutions per minute to 59 per cent. at 2070 revolutions per minute. The performance of the turbine is remarkable.

To measure the brake horse-power, a hydraulic brake was fitted on the same shaft. This brake is very fully illustrated in Chapter VI.

§ **144. Turbine Installations.**[1]—Professor Unwin, F.R.S., in an address given before the graduates of the Institution of Mechanical Engineers, summarizes the installations at Niagara. He points out that the ordinary discharge of the Niagara River is about 250,000 cubic feet per second, or about 6000 tons per second. If

[1] *Proceedings of the Institute of Mechanical Engineers*, February, 1906.

the total fall between the lakes could be utilized, 7,000,000 horse-power would be available, and at the Falls themselves about 4,000,000 horse-power is available. The first important effort to obtain power was made in 1861, when the so-called hydraulic canal was constructed, 38 feet in width, 8 feet in depth, and 4400 feet in length, from a point above the upper cataracts to a basin at the top of the bluff below the falls. On the bluff were constructed mills, having turbines supplied with water from the basin and discharging it through short tunnels on the face of the bluff. In these cases only part of the available fall was utilized, water being plentiful, and the cost of excavating pits for the turbines considerable. In 1885 about 10,000 horse-power was utilized in this way on the whole available supply of the hydraulic canal.

When engaged in forming the reservation, Mr. Thomas Evershed considered the possibility of utilizing part of the power of the Falls in a more effective manner and on a larger scale than at Niagara. In 1886 a company obtained a charter giving the right to utilize 200,000 horse-power on the American side, and a little later they obtained the further right to utilize 250,000 horse-power on the Canadian side. A surface canal, 250 feet wide, 12 feet deep, and 1700 feet long was commenced; and a tail-race tunnel, discharging into the lower river, 21 feet high, 19 feet wide, and 7000 feet long, was commenced. These works were adequate for developing 100,000 horse-power. Then an investigation was commenced to find out the best method of utilizing the power which could be thus made available. A large paper-mill was erected, using 8000 horse-power, which put in its own turbines.

The first decision was to distribute the power electrically. The hydraulic part of the project did not present great difficulty. Although no turbine had been constructed of more than 1000 horse-power, designs were obtained for turbines of 5000 horse-power. It was resolved to adopt turbines of Swiss design—in Switzerland turbines are extensively used—of 5000 horse-power, each placed at the bottom of a slot excavated in the rock, and driving dynamos on the ground by vertical shafts. There were

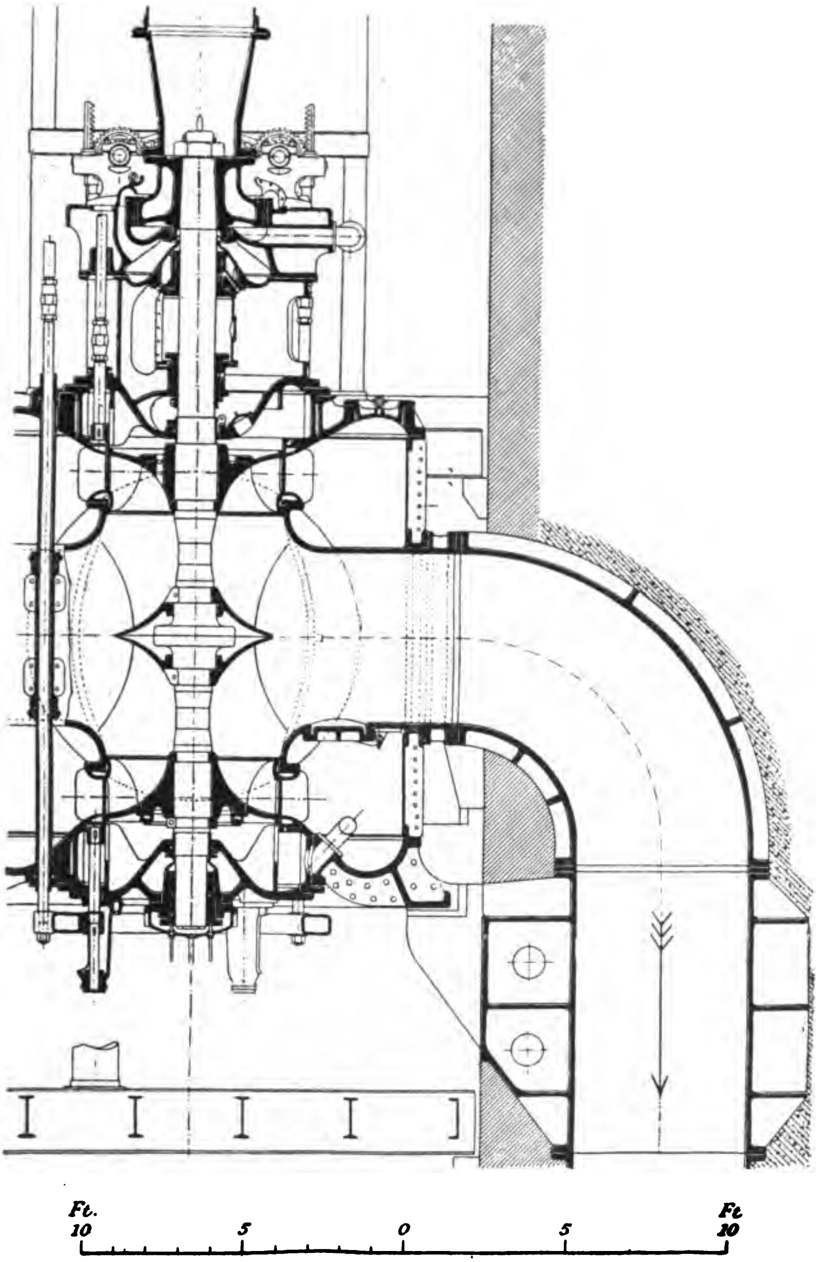


FIG. 208.

difficulties in the plans for electrical distribution, but these were overcome.[1]

§ 145. The Niagara Falls Power Company on the American Side.— This company began work in 1890, and first delivered power in 1895. It has now two power-houses. Power-house No. 1 has ten units of 5000 horse-power each. The wheel slot is 178 feet long, and 178 feet deep, and the effective fall is 136 feet. The tur-

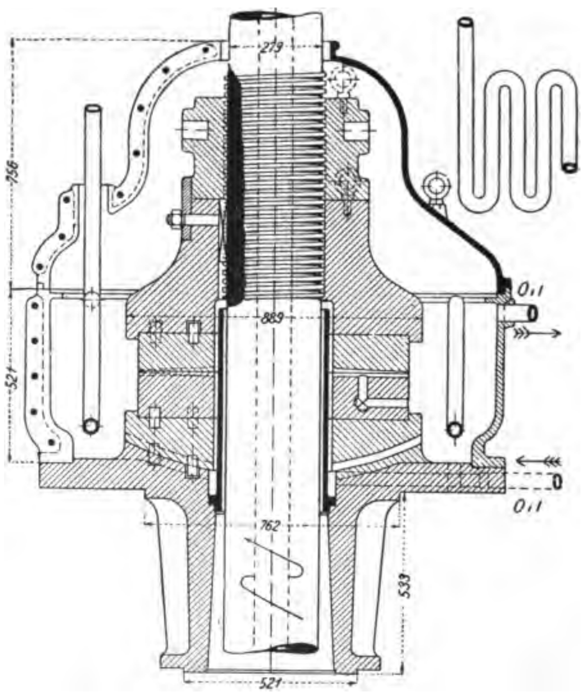


FIG. 209.

bines (Figs. 208 and 209) are twin outward-flow turbines, designed by Messrs Faesch and Picard, of Geneva, and built in America, with penstocks $7\frac{1}{2}$ feet diameter. The turbines run at 250 revolutions per minute. The vertical shafts are hollow, of $\frac{3}{4}$-inch rolled tube, 38 inches diameter, with solid bearings 11 inches diameter.

[1] For reference, see Professor Unwin's paper. It is beyond the scope of this work to discuss the electrical side.

The turbines are regulated by cylindrical sluices, which rise and fall on the outside of the wheels. In this power-house the weight of each turbine wheel and shaft is about 35 tons, and that of the revolving field ring of the dynamo about 35 tons, altogether 70 tons, which could not be carried by any pivot or collar-bearing at the speed of the turbines (compare § 141). The water pressure acts on the cover of the upper turbine, and is relieved from acting on the lower turbine, giving an upward force of 65 to 70 tons to balance the

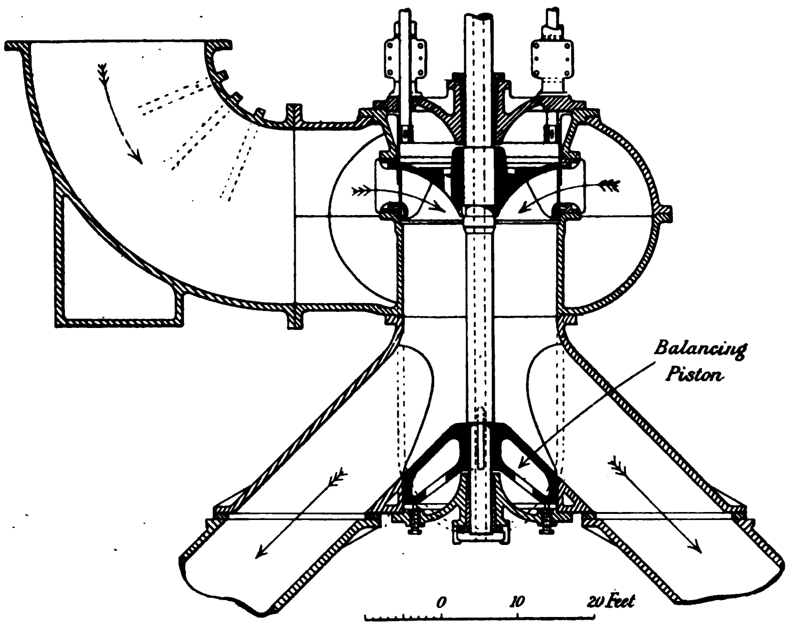


FIG. 210.

weight. It will be noticed that in Fig. 208, which represents a section of the turbine and penstock, in the upper crown, there are two openings which permit the high-pressure water to flow through to the top; whilst the bottom crown plate is solid. The upward pressure acts on the upper turbine wheel, which is keyed to the shaft. Fig. 209 shows an enlarged view of the lower turbine with the collar-bearing.

In the second power-house, the installation was by Messrs.

Eschen & Wyss, of Zurich. These are single inward-flow turbines, each of 5500 horse-power, with bronze wheels and cylindrical sluices and suction-pipes. In this installation, with radial-flow turbines another arrangement is necessary. A special piston (Fig. 210) is provided, 4 feet 6 inches diameter, the water pressure on which, due to the head, balances the weight. The excess unbalanced load in this case is taken by the bearing shown in Fig. 210, oil being forced between the bearing surfaces.

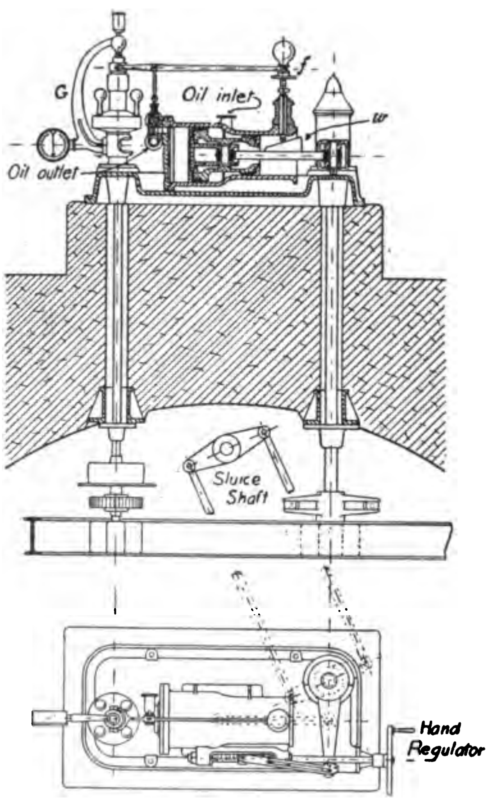


FIG. 211.

The most difficult problem in designing was the speed regulation, which required to be very efficient for driving alternators in parallel. Steam engines using a light fluid can be easily regulated by governors acting direct on the throttle or the expansion valve. It is different with water turbines using a fluid of great inertia. In one of the Niagara penstocks there is about 400 tons of water, flowing at 10 feet per second, opposing great resistance to any quick variation of flow. The governor in power-house No. 1 consists of a sensitive governor acting on a ratchet relay. The governor puts one or other of two ratchets in gear with the ratchet-wheel connected with the sluices. The ratchets reciprocate, being driven by the turbine.

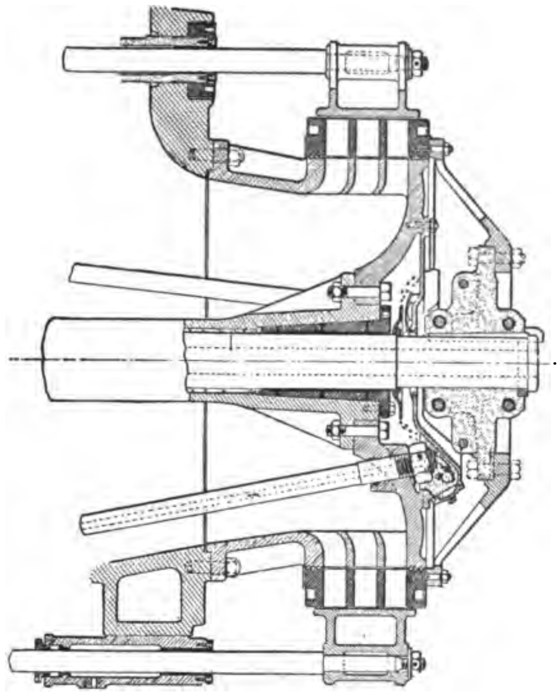


FIG. 213.

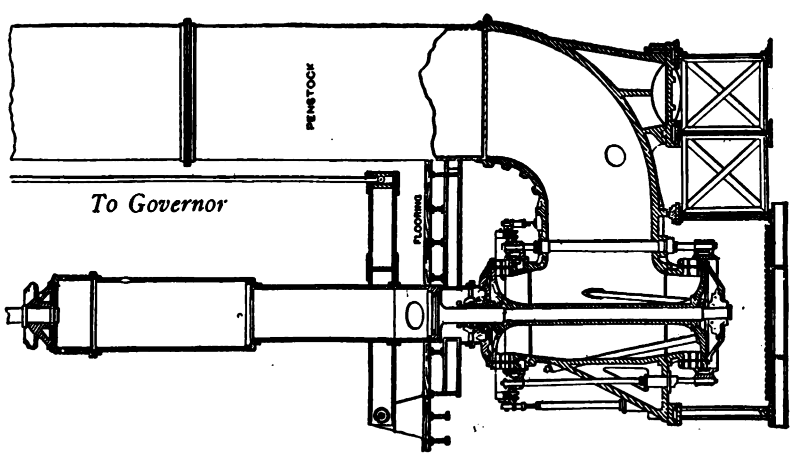


FIG. 212.

According as one or other ratchet is in gear the sluices are raised or lowered.

In power-house No. 2, the relay is an hydraulic relay. This is shown in principle in Fig. 211, which, however, is not exactly the arrangement adopted at Niagara. In this case the sensitive governor opens a valve and puts in action a ram driven by oil from an oil reservoir at a pressure of 1200 pounds per square inch. The millimetre of movement of the governor sleeve fully opens the relay valve, and the rams move the turbine sluice with a force of 50 tons. An automatic sluice relieves any excess pressure in the penstock due to water-hammer action. The tendency to hunt in relay governors is prevented by a subsidiary arrangement. The ram of the relay as it moves forward gradually closes, by lowering the fulcrum end, *f*, of the lever, which rests on the wedge, *w*, the relay valve, which admits pressure oil, unless the sensitive governor reopens it. The turbine sluice can be completely opened or shut in 12 seconds. The ordinary variation of speed is not more than one per cent. The momentary variation, if all the load is thrown off, is not more than five per cent.

§ **146. The Canadian Niagara Power Company.**—The installation of this company will be worked in combination with that of the Niagara Falls Company. When complete the power-house will have eleven units of 10,250 horse-power each, on 133 feet effective fall. Five turbines are now installed. The turbines are double inward-flow turbines with suction pipes (Fig. 212). The tail-race tunnel is 2200 feet long, 21 feet high, by 19 feet wide, and the water will flow at 27 feet per second. The wheel slot is 570 feet long, 165 feet deep, and 18 feet wide. The weight of the turbine-wheel shaft and field ring is 120 tons, which is carried by a balancing piston, as already described, on which water acts at the pressure due to the fall. An enlarged section of the lower wheel is shown in Fig. 213.

§ **147. The Ontario Power Company on the Canadian Side.**—The plans of this company differ essentially from the others. The intention is to develop 200,000 horse-power. The intake is specially designed with reference to ice difficulties. The openings in the intake dam have a curtain dipping nine feet into the water. The

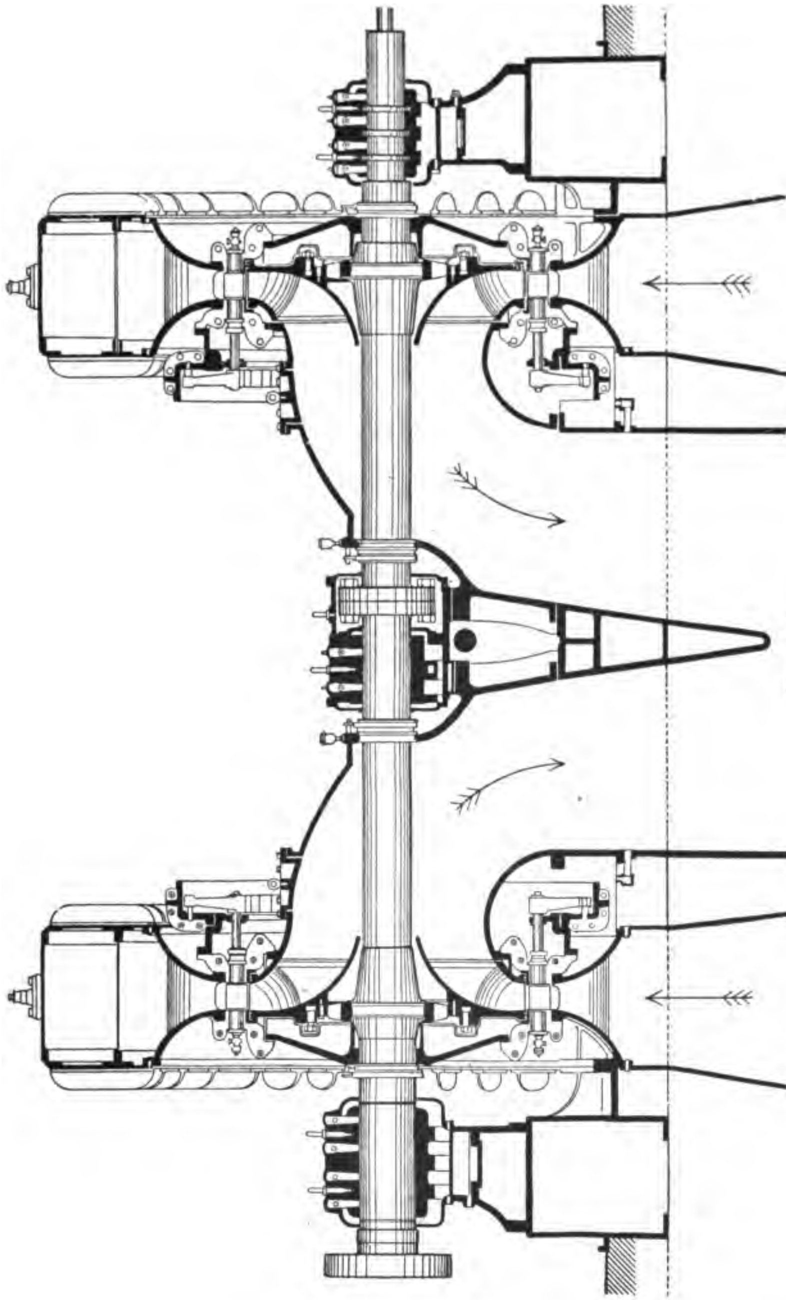


FIG. 214.

flow to the turbines is under the curtain, the floating ice being carried past. A second curtain on the same principle is constructed between the firebay and inner basin, and the ice in the outer basis is carried forward over the lower part of the outer dam. The ice in winter is a serious difficulty, cake ice floats down from the upper lakes, and "mush" ice is formed in the turbulent rapids primarily by the freezing of spray and foam. For ice in this latter form there are screen frames.

From the intake three great steel conduits, 18 feet and 20 feet in diameter, convey the water round the other power-houses to the top of the bluff below the Fall. These conduits, of which one is already constructed, are of ½-inch steel plates, stiffened with bulb irons and encased in concrete. The velocity in the conduits will be 15 feet per second. There is a spillway at the end, formed by a weir to prevent water-hammer in the pipes. The flow over the weir passes down through a helical culvert or spillway in the rock to the lower river. From the conduits the water will be taken down to the turbines through twenty-two steel pipes, 9 feet in diameter, passing down the face of the bluff.

The power-house in course of erection is on a platform at the foot of the bluff and just above the level of the lower river. The turbines (Fig. 214) are to be inward-flow twin turbines, each of 12,000 horse-power under 175 feet head. The axis of the turbines is horizontal, and the shaft is 24 inches diameter. The turbine wheels are 78 inches diameter, and have movable guide-blades, which are more efficient than the cylindrical sluices used in the other power-houses, though of course ecomony of water is not of great importance at Niagara.

CHAPTER VI

CENTRIFUGAL PUMPS

§ 148. **Comparison between Turbines and Pumps.**—A reciprocating pump is a reversed pressure engine; so a centrifugal pump is a reversed turbine. In reciprocating pumps the chief losses are in the resistance of the valves and slip; and this is practically the same for a given discharge for all pressures. Thus the efficiency of a reciprocating pump working against a high head is large; but it is small for a small head. On the other hand, in a centrifugal pump there are no valves, and consequently they are better adapted for low lifts than reciprocating pumps.

A centrifugal pump is merely a reversed turbine. In a turbine, at the receiving edge, the water is received with an absolute velocity v_2 and pressure p_2, the total energy being

$$\frac{p_2}{\sigma} + \frac{v_2}{2g} \text{ ....(§ 131).}$$

It is discharged at the discharging edge with a pressure p_3 (generally atmospheric) and velocity v_3—or, more usually, f_3—the velocity of which being usually zero. Its energy is then

$$\frac{p_3}{\sigma} + \frac{f_3^2}{2g}.$$

The difference, in the absence of frictional and other losses, represents the work done on the turbine, and is equal to $\frac{u_2 w_2}{g}$. The vanes at the receiving edge are curved backwards, are radial,

or are curved forwards, according as $u_2 \overset{<}{\underset{>}{=}} w_2$. Moreover (Fig. 215), the energy equation

$$\frac{u_2 w_2}{g} = \frac{p_2 - p_3}{\sigma} + \frac{v_2^2 - f_3^2}{2g}$$

$$= \frac{p_2}{\sigma} + \frac{w_2^2}{2g} + \frac{f_2^2 - f_3^2}{2g}, \quad p_3 = 0,$$

and assuming $f_2 = f_3$, then $\dfrac{u_2 w_2}{g} = \dfrac{p_2}{\sigma} + \dfrac{w_2^2}{2g}$.

If $u_2 = \dfrac{w_2}{2}$, $p_2 = 0$, and the vanes are curved backwards,

$u_2 = w_2$, $\dfrac{p_2}{\sigma} = \dfrac{w_2^2}{2g}$, the vanes are radial,

$u_2 = \dfrac{3}{2} w_2$, $\dfrac{p_2}{\sigma} = 2 \cdot \dfrac{w_2^2}{2g}$, the vanes are curved forwards.

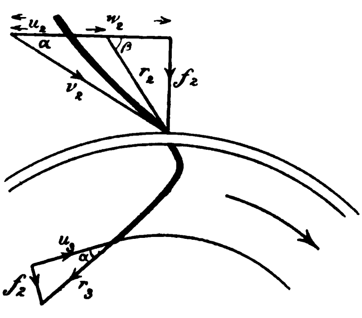


FIG. 215.

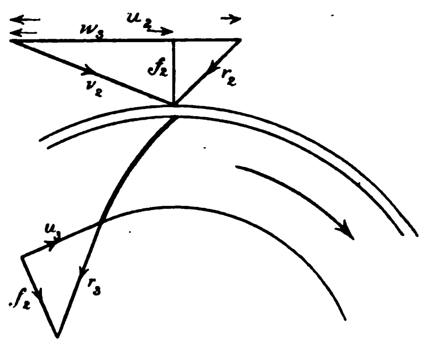


FIG. 216.

Thus the pressure head may be varied to any extent by altering the vane angle. If the vane be curved forwards (Fig. 216) the pressure head preponderates.

§ **149. General Consideration affecting Centrifugal Pumps.**—In a pump the operations are reversed, and, using the same notation as in turbines, the same amount of work is expended on the pump as the water previously expended on the turbine (Fig. 217). The water enters the wheel at the centre with, as assumed above, no tangential velocity, but at a pressure p_3. It leaves the wheel

with a velocity in the direction of motion v_2 and a pressure p_2. Other things being unaltered, the relation between pressure and velocity at the discharging edge depends on the angle β. The less this angle, the greater the pressure head at discharge.

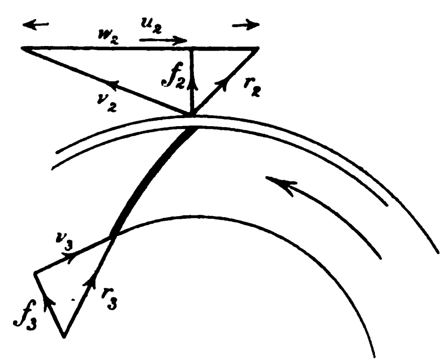


FIG. 217.

But there is this difference between a turbine and a pump—that whereas, in the turbine, both the pressure and kinetic energy are available for doing useful work, in a pump, although work has to be spent in producing both pressure and kinetic energy—except special precautions be taken—only the pressure head is available. The reason for this is, that to reduce the velocity of a mass of water is invariably accompanied by loss. For example, as an extreme case, in a pump with radial vanes at the discharging edge, if all the pressure energy is lost, the efficiency would be half; in a turbine the efficiency would be unity.

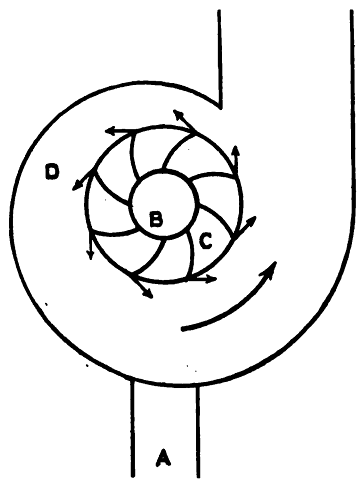


FIG. 218.

§ **150. Theoretical Consideration in Centrifugal Pump.**—As in turbines, fractional and other losses will, in the first place, be neglected. The working efficiency will be increased by making the pressure head at the commencement of the delivery pipe as great as possible. This may be done by either decreasing the kinetic head at discharge by suitably curving the vanes, or arranging the discharge chamber in such

a way that the velocity of the water is gradually reduced before the delivery pipe is reached.

In Fig. 218 the suction pipe is represented by A, the eye of the wheel by B, the pump wheel by C, and the discharge chamber by D.

Consider, first, the effect of curving back the vanes. Let suffix (3) refer to the eye-wheel, and (2) to the discharging edge of the wheel. Let b denote the absolute barometric head, h_3 the suction head, h_2 the absolute delivery head just outside the pump, p_3 the absolute pressure at the eye of the wheel, f_3 the velocity of flow into the pump, and v_2 the absolute velocity of discharge into the chamber (Fig. 218). Neglecting losses in suction pipe

$$b = h_3 + \frac{p_3}{\sigma} + \frac{f_3^2}{2g}.$$

Also

$$\frac{u_2 w_2}{g} = \left(\frac{p_2}{\sigma} + \frac{v_2^2}{2g}\right) - \left(\frac{p_3}{\sigma} + \frac{f_3^2}{2g}\right)$$

$$= \left(\frac{p_2}{\sigma} + h_3 - b\right) + \frac{v_2^2}{2g}$$

$$= \text{net pressure head} + \frac{v_2^2}{2g}.$$

In this expression the pressure head is immediately available for lifting purposes, but a large proportion of the kinetic head will be lost. The precise proportion depends on circumstances; but the formula may be expressed in the form

$$\text{net lift} = h = \frac{u_2 w_2}{g} - x \cdot \frac{v_2^2}{2g}$$

$$\text{and the efficiency} = \frac{gh}{u_2 w_2}.$$

In addition, the velocity diagrams must be satisfied. Referring to Fig. 217,

$$w_2 = (u_2 - f_2 \cot \beta)$$

whence

$$h = \frac{u_2(u_2 - f_2 \cot \beta)}{g} - x\frac{f_2^2 + (u_2 - f_2 \cot \beta)^2}{2g}$$

$$\text{efficiency} = \frac{gh}{u_2(u_2 - f_2 \cot \beta)}.$$

Usually, f_2 is made a certain fraction of the velocity due to the lift, say

$$f_2 = a\sqrt{2gh}$$

where $a \propto \frac{1}{8}$ to $\frac{1}{4}$. The two equations give a relation between u_2, β, and x, so that, assuming two of them, the values of u_2 and the efficiency may be obtained. Taking the higher value for a, the following table gives the velocity of the rim, and the efficiency when all the kinetic energy is lost, when half is lost, and when one quarter is lost:—

β	$x = 1$.		$x = \frac{1}{2}$.		$x = \frac{1}{4}$.	
	Velocity of rim.	Efficiency.	Velocity of rim.	Efficiency.	Velocity of rim.	Efficiency.
90°	1·03	0·47	0·83	0·73	0·76	0·86
45°	1·06	0·58	0·93	0·79	0·88	0·89
30°	1·12	0·65	1·02	0·83	0·99	0·91
20°	1·24	0·75	1·17	0·87	1·15	0·93
10°	1·75	0·86	1·73	0·93	1·72	0·97
5°	3·08	0·93	3·03	0·97	3·03	0·97

It will be noticed that when x is unity, the increased efficiency obtained by decreasing β is considerable, but that the advantage is not so marked in the third case. To get a high efficiency in the first case, it is necessary to have a small angle; but a much larger angle may be used in the second and third cases. Thus, if $x = \frac{1}{2}$, an angle of 20° gives a slightly higher efficiency than one at 10° when $x = 1$, and it runs at a much less speed. Below 20° the speed increases rapidly, and, in practice, to prevent a prohibitive speed, β is seldom made less than 20°. In the first and second cases this would give an efficiency of 0·75 and 0·87 respectively. In an actual case the efficiency would not be greater than 0·6, on account of losses in friction and slip.

§ **151. Arrangement of Pump Chamber.**—The value of x is difficult to determine, as it depends on the arrangement adopted. The three arrangements are—

(1) When the wheel is central with the casing (Fig. 219). In this the water area of the chamber round the wheel is constant,

and therefore the velocity will continually increase as the delivery pipe is reached. There will be streams from the wheel impinging with streams of different velocity in the chamber, and there will be losses in shock.

(2) *Eccentric Chamber.*—The wheel is eccentric with the casing, and at A (Fig. 220) the wheel touches the casing. Since the wheel discharges uniformly all round, the sectional area between the casing and wheel is made proportional to the flow area from A to the section considered. Thus, if the breadth perpendicular to the plane is constant all round, the width BC will be proportional to

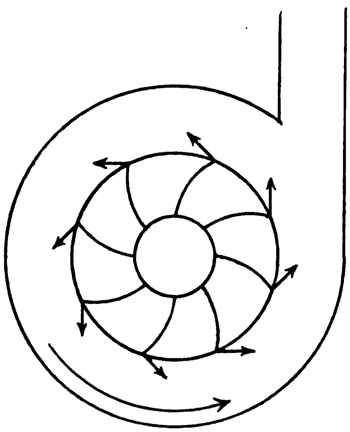


FIG. 219.

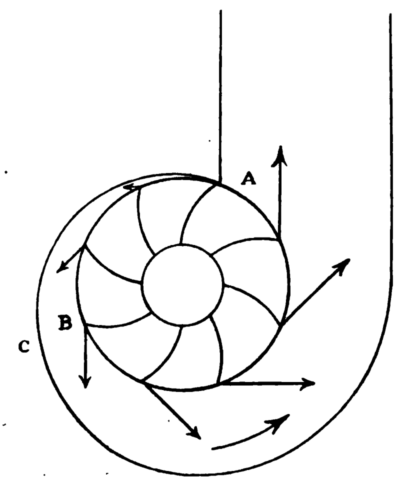


FIG. 220.

the area AB. The casing shown satisfies this condition. For compactness, the increased area would probably be obtained by increasing the width of the casing. Thus the velocity round the casing is uniform, and probably there is much less head lost in shock than in the first case. Guides may be fitted to give the proper direction of discharge; in that case, the pump must be run at a constant speed to produce best effect. They also would cause extra frictional resistances (compare § 137).

(3) *Volute Chamber.*—In a volute casing of this kind (Fig. 220), the direction of the water discharged from the pump is not

tangential; so that there is a stream of velocity v_2 impinging on a stream of uniform velocity f_1, say. Neglecting this, an estimate may be made of the loss in the chamber. If a stream, moving with velocity v_2, is reduced in speed to a velocity f_1, then (§ 34) the loss of head

$$\frac{(v_2 - f_1)^2}{2g}.$$

The kinetic energy of the water in the chamber is $\frac{f_1^2}{2g}$, and therefore the pressure energy in the water is

$$\frac{v_2^2}{2g} - \frac{(v_2 - f_1)^2}{2g} - \frac{f_1^2}{2g}$$
$$= \frac{f_1(v_2 - f_1)}{g}.$$

This is a maximum where $f_1 = \frac{v_2}{2}$ and is then $\frac{1}{2} \cdot \frac{v_2^2}{2g}$; so that on the assumptions made the value of x in the table would be $\frac{1}{2}$.

(4) *Whirlpool Chamber.*—In the third arrangement a whirlpool chamber is used (Fig. 221). The wheel is placed in a large casing, and the water, on leaving the wheel, is guided for a short distance by means of guides, inclined to the tangent line of the pump wheel. The water is then discharged freely into the whirlpool chamber, and forms a free vortex. Neglecting hydraulic resistances, the pressure head along a spiral stream line in the chamber must be constant and equal to

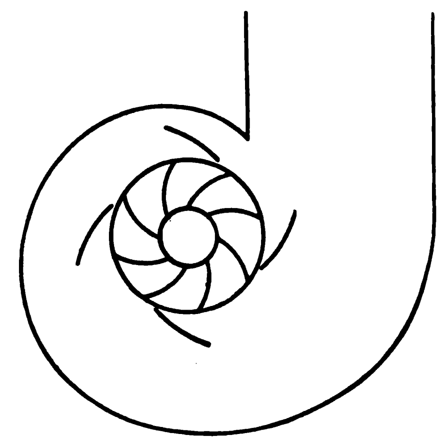


FIG. 221.

$$\frac{p_2}{\sigma} + \frac{v_2^2}{2g}.$$

As the stream flows outwards, the velocity decreases, since the

sectional area increases, since every stream line is similarly situated, va is the same for all streams, a being the radial distance when the velocity is v. The kinetic head at the commencement of the delivery pipe is, therefore

$$\frac{v_1^2}{2g} = \left(\frac{a_2}{a_1}\right)^2 \cdot \frac{v_2^2}{2g}$$

$$h = \frac{u_2 w_2}{g} - \left(\frac{a_2}{a_1}\right)^2 \cdot \frac{v_2^2}{2g}.$$

Usually, $\frac{a_2}{a_1} = \frac{1}{2}$, which corresponds to the third case in the table of § 150.

The following table gives the conditions that have to be satisfied in the three cases for an efficiency of 86° per cent.:—

	Concentric chamber.	"Volute" chamber.	Whirlpool chamber.
β	10°	20°	90°
$\frac{u_2}{\sqrt{2gh}}$	1·75	1·17	0·76

The speed with a whirlpool chamber is 66 per cent. of the speed with a volute chamber, and 42 per cent. of the speed with a concentric chamber. More, the smaller the angle at the discharging edge, the larger must be the vane in order to obtain the gradual deviation.

§ **152. Variation of Pressure in the Wheel and Chamber of a Pump.**—The variation of pressure in the wheel depends on the speed of wheel, and the change of relative velocity in the wheel. It is identical with that in a turbine (§ 134), and is given by the equation

$$\frac{p_2 - p_3}{\sigma} = \frac{u_2^2 - u_3^2}{2g} + \frac{r_3^2 - r_2^2}{2g}.$$

When the wheel is delivering a small supply of water, r_2 and r_3 are very small, and a forced vortex is formed, and

$$\frac{p_2 - p_2}{\sigma} = \frac{u_2^2 - u_3^2}{2g} = \frac{\omega^2}{2g}(a_2^2 - a_3^2)$$

in which ω is the angular velocity of the wheel.

After leaving the wheel with velocity v_2, the motion is that of a free vortex, provided a whirlpool chamber is used. The head remains constant along a stream line, with the result that if p be the pressure at radius a

$$\frac{p - p_2}{\sigma} = \frac{v_2^2 - v^2}{2g} = \frac{\omega^2}{2g}\left(a_2^2 - \frac{a_2}{a^2}\right)$$

since $av = a_2v_2$, and $v_2 = \omega a_2$.

The water in the wheel may be assumed to rotate as a solid body. The speed at which pumping occurs is given by

$$\frac{\omega^2(a_2^2 - a_3^2)}{2g} = h.$$

§ 153. Hydraulics Losses in Centrifugal Pumps.—The only loss so far considered in pumps is the kinetic energy in the delivery pipe. In addition to this source of loss, there are frictional losses in the suction pipe, in the wheel, and in the chamber, and in the delivery pipe. In addition there is loss due to "slip" in the wheel, and probably in the chamber. "Slip" in centrifugal pumps is due to entirely different causes to those in reciprocating pumps (§ 93). It is caused partly by leakage along the shaft; but the circulation of water which takes place in the wheel passages and pump chamber is responsible for much of the loss. The magnitude of these losses can only be found by experiment.

Usually, in a good pump, the efficiency varies from 0·55 to 0·66, and the energy equation is

$$\frac{u_2w_2}{g} = (1\tfrac{1}{2} \text{ to } 2)h.$$

Illustration.—Take, in a particular case, the following data:

$h = 16{\cdot}4$ feet, $p = 15°$, $a = 25°$, f_1 (velocity in delivery pipe) to be 5 per cent. of net lift, other losses in suction pipe, pump, and chamber to 45 per cent. of net lift, and let the required discharge be 3·54 cubic feet per second; so that the efficiency is 0·66. Assume that the outer radius is twice the inner. To find the speed horse-power the energy equation gives

$$\begin{aligned}\frac{u_2 w_2}{g} &= h + 0{\cdot}45h + \frac{f_1^2}{2g} \\ &= h + 0{\cdot}45h + 0{\cdot}05h \\ &= 1{\cdot}5h = 24{\cdot}6.\end{aligned}$$

In Fig. 217

$$\begin{aligned}w_2 \tan 25° &= (u_2 - w_2) \tan 15° \\ \therefore w_2 &= 0{\cdot}38u_2 \\ u_2 &= 45{\cdot}6 \text{ feet per second} \\ w_2 &= 17{\cdot}34 \text{ feet per second} \\ f_2 &= w_2 \tan 25° = 7{\cdot}33 \text{ feet per second} \\ \text{and } u_3 &= \tfrac{1}{2}u_2 = 0{\cdot}228 \\ \therefore f_3 &= 2f_2 = 14{\cdot}66 \\ \therefore \tan \gamma &= 0{\cdot}644 \\ \gamma &= 33° \\ \text{horse-power} &= \frac{3}{2} \times \frac{3{\cdot}54 \times 62{\cdot}5 \times 16{\cdot}4}{550} = 985.\end{aligned}$$

§ **154. Law of Comparison in Centrifugal Pumps.**—The law of comparison deduced for turbines may be applied to centrifugal pumps, the one being the reversed action of the other (§ 138). Consider two pumps, similar in all respects, and similarly situated in similar casings. Let the pumps be run at such speeds that the velocity conditions are the same, and that in all respects their behaviour is similar. The ratio of the pressure head and kinetic head, and all the velocities will be proportional to $\sqrt{H}$. The velocity of flow $\propto \sqrt{H}$ and as $\frac{Q}{D^2}$, in which Q represents the discharge in cubic feet per second, and D is a dimension of length. The friction loss per pound in flow loss, frictional loss, and eddy

loss is proportional to $\frac{L^2h^{\frac{3}{2}}}{Q}$; and therefore varies as H. In a pump, as in a turbine

$$\text{the speed} \propto \sqrt{H}$$
$$Q \propto D^2\sqrt{H}.$$

In ordinary cases pumps work against a definite head of water. As the speed increases, the frictional and other losses increase, or the lifting head is reduced proportionately, and the pump will work with less efficiency. But in one case—the circulating pump for a condenser—the pumping head is small, and the pump is simply used to circulate the water against the resistance of the inlet and outlet pipes, the tubes, and the water heads. In that case, the head against which the pump works varies as the velocity of the water, and therefore as the discharge. Thus the pumps will work with the same efficiency, and the speed will be proportional to the quantity of water circulated.

§ **155. Test of a Centrifugal Pump.**—In a test of a centrifugal pump, the quantities to be measured are the suction and delivery heads; the pressure in the suction pipe before the wheel, and the pressure in the delivery pipe and after leaving the wheel; water in pounds per minute and the revolutions; the turning moment transmitted to the pump shaft. A spring dynamometer connected to the shaft is, probably, the most convenient method.

Let h_1, h_2 be the actual suction and delivery heads in feet, p_1 p_2 the absolute actual pressures at the top of the suction main and entrance to delivery pipe; let Q be the discharge in cubic feet per minute, N the revolutions per minute, and T the twisting moment in foot-pounds, transmitted to the pump spindle.

Then

$$\text{hydraulic efficiency of pump} = \frac{(p_1 + p_2)Q}{T \,.\, 2\pi N}$$

$$\text{efficiency of plant} = \frac{(h_1 + h_2)\sigma Q}{T + 2\pi N}.$$

Take an experiment on the Reynolds-Mather pump, illustrated in Fig. 222. This is a four-stage pump, pumping against a delivery

head of 125 feet, the pump being 8·5 feet above the suction-tank level. The data in one trial were

Pressure in pounds per square inch above atmosphere in rising main	= 57·5
Pressure in pounds per square inch below atmosphere in rising main	= 6·2
Gallons of water per minute	= 135·6 gallons
Turning moment on pump spindle . . .	= 34·7 foot-pounds
Revolutions per minute	= 1480

Results.—The head at entrance to the rising main is 132·3 feet, on the suction side it is $-14{\cdot}3$, making 5·8 feet lost in friction and flow in suction pipe, and 7·3 feet lost in rising main. The work done by the pump per minute is

$$146{\cdot}6 \times 135{\cdot}6 = 198600 \text{ foot-pounds.}$$

The work transmitted by the pump is

$$34{\cdot}7 \times 1480 \times 2\pi = 323000.$$

Thus the efficiency of the *pump* is 0·615; whilst the efficiency of the whole plant is 0·525.

§ **156. Compound Centrifugal Pumps.—Description.**—When a centrifugal pump has to pump to high heads, the speed of a single pump would become excessive. Professor Osborne Reynolds, F.R.S., invented the compound pump. The principle of this pump is to obtain the rise of pressure in stages. The water flows from the suction pipe into the eye of the first wheel. It is discharged, and is led through gradually deviated passages to the eye of the second wheel, and so on successively. In the final chamber it is discharged into the delivery pipe. The original pump had four chambers; but Messrs. Mather & Platt, who were associated with Professor Reynolds in the construction of the pump, now, for mine work, use as many as twelve chambers, pumping to a height of 600 feet.

Professor Reynolds's first pump was specially designed for the Whitworth Laboratories, Manchester University, by Messrs.

Mather & Platt. It is running with the same efficiency now as it did when erected twenty years ago. It pumps against a head of

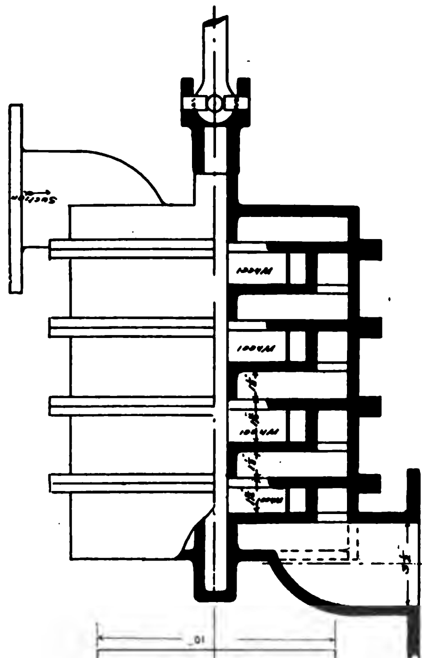


FIG. 222.

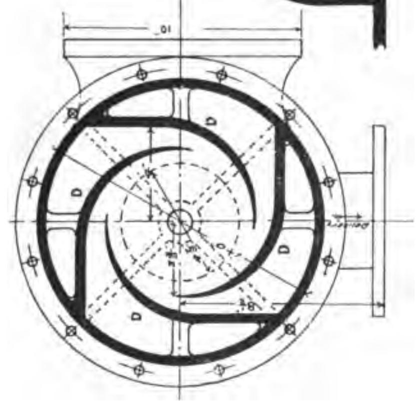


FIG. 223.

130 feet, and runs at about 1500 revolutions per minute. The pump is illustrated in Fig. 222, being a half longitudinal section and half elevation, and Fig. 223 the cross section. The water

passes up the suction pipe, fills the passage at the right of the pump, enters the pump at the eye, flows through the first pump wheel, which has radial vanes, and is then received by the guides D, D, D, D. It then flows in the next chamber, through the remaining three chambers, which are exactly similar and keyed to the same shaft, and thence up the delivery pipe. The pressure head produced by each wheel is about 35 feet. The guides, it will be seen, are very carefully designed, the inner edge running to a point. The eddy losses are therefore reduced to a minimum, and in each stage a considerable portion of the kinetic energy is converted into pressure head.

The advantages of centrifugal pumps over reciprocating pumps are (1) no valves, (2) uniform flow in delivery pipes, (3) may be driven direct by electric motor, (4) capable of pumping great heights, as in mines, (5) suitable for installation in mines, dispensing with engines and boilers, (6) light foundations, (7) compactness, (8) high efficiency.

§ **157. Test of the Four-stage Centrifugal Pump.**—The following table gives the results of tests of Professor Reynolds's pump at different speeds.

Revolutions per minute	1439	1462	1508	1586
Turning moment in foot-pounds	32·16	27·3	36·1	38·27
Work done *on* pump in foot-pounds per minute	290,700	250,600	238,000	381,400
Water raised per minute in pounds	1183	1191	1320	1690
Nett virtual lift in feet	145·5	144·1	151·5	15·5
Work done *on* pump in foot-pounds per minute	172,400	171,600	343,000	261,600
Efficiency	0·593	0·685	0·695	0·625

These results are remarkable, especially considering the head against which the pump works. The maximum efficiency takes place at about 1500 revolutions when the efficiency is 70 per cent.

The pump was driven by belting from the first motion shaft. The dynamometer for measuring the turning moment transmitted to the pump spindle consisted of a coiled square helical spring placed between the driving pulley and the pump shaft. A light wooden lever, pivoted near shaft, was actuated by the spaces

between the helical coils. The ratio of levers was large, and the reading was obtained by using a fixed graduated brass quadrant. Very accurate and sensitive readings were obtained.

§ 158. **Actual Tests on High-lift Pumps.**—It is important to have information of comparative tests of pumps when working under varying conditions. I am indebted to Messrs. Mather and Platt, Salford, Manchester, for kindly supplying me with much useful information. The test is on one of Messrs. Mather and Platt's high-lift turbine pumps, the maximum head being 140 feet.

The curves are of interest. In Fig. 224 the speed has been

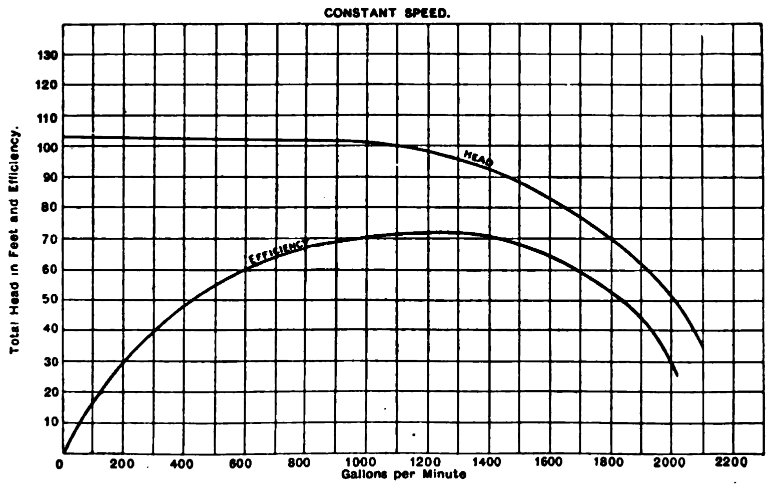


FIG. 224.

kept constant at 700 revolutions per minute. The abscissæ represent the discharge in gallons per minute, and the ordinates the lift of the pump. The lift remains practically constant at 102 feet up to 1000 gallons per minute; it then decreases slowly until the maximum efficiency of 0·72 is reached, when both rapidly fall. At 2000 gallons per minute the lift is only 50 feet, and the efficiency has been reduced to 0·3. It shows how overloading a pump reduces its efficiency.

In Fig. 225 the abscissæ are revolutions per minute, the ordinates are lift and efficiency, the gallons per minute being

constant. The efficiency increases fairly uniformly up to 450 revolutions per minute, and reaches a maximum at 750 revolu-

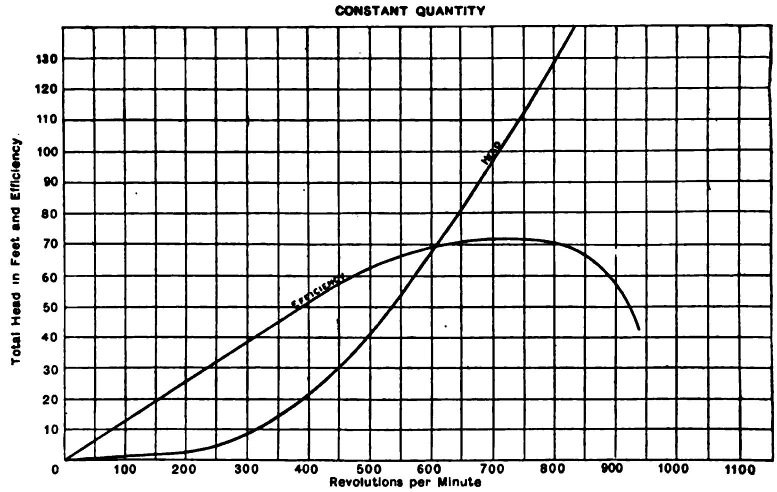


FIG 225.

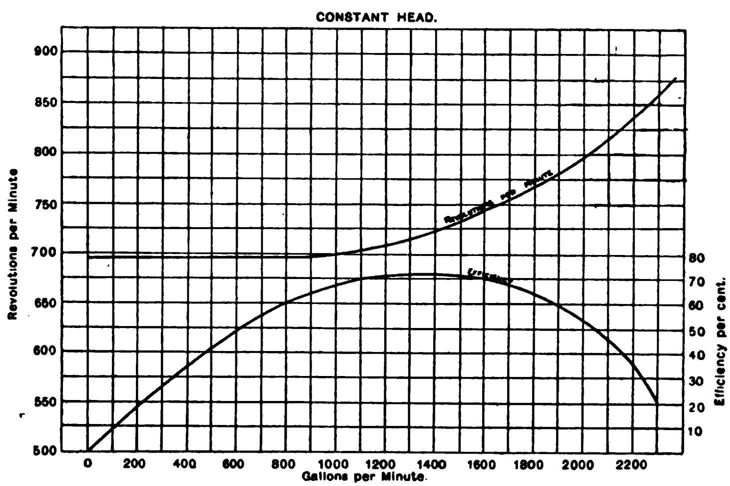


FIG. 226.

tions, after which it rapidly declines, being 0·42 at 940 revolutions per minute.

In Fig. 226 the abscissæ are gallons per minute, the ordinates are revolutions per minute and efficiency, whilst the pump is pumping against a constant lift. The revolutions per minute are practically constant up to 900 gallons, and when the maximum efficiency of 0·72 is reached, the revolutions are 730. When the efficiency falls to 0·2, the revolutions are 875 per minute.

§ 159. **Experiments on Centrifugal Pumps.**[1]—The Hon. R. C. Parsons made experiments on wheels of 14 inches diameter with a vortex chamber, lifting water at heads varying from 6 feet to 8 feet at speeds varying from 165 to 430 revolutions per minute. It was found that the efficiency of the wheels with curved vanes was always higher than that of the wheels with radial vanes, and further, that the efficiency depended on the form of the vortex chamber. To quote an example, with a discharge of 3000 gallons per minute the efficiency of the wheel with curved vanes was 44·3 per cent., that of the wheel with radial vanes when discharging the same quantity of water was 37·9 per cent., and this although the speed in the latter case is 163 revolutions against 206 revolutions in the first case. With a specially designed casing, Mr. Parsons obtained efficiencies as high as 62·4 per cent. when lifting 1753 gallons per minute to a height of 17·6 feet.

Dr. Stanton has made experiments on a 7-inch pump working under a head of 48 feet to determine the effect of (1) curved and radial vanes at high speeds, (2) efficiency of vortex chamber, (3) efficiency of guide passages, (4) possibility of high lifts by centrifugal pumps with single wheel.

The pump casing exposed, together with two of the wheels, is shown in Fig. 227. The casing consists of two parallel plates 1 inch apart, with a circular collecting chamber, the sides of the casing being 18 inches diameter, so that wheels of from 4 to 18 inches diameter could be used with it. For the 7-inch wheels used in these experiments the free vortex was formed in the space between the tips of the vanes and the collecting chamber. In cases where guide passages were used, the guides were built up on a disc of thin brass and bolted to the cover.

[1] Dr. Stanton, *Proceedings of Institution of Mechanical Engineers*, November, 1903.

The dynamometer for measuring the turning movement given to the pump was simple. A hollow sleeve surrounded the shaft, carrying a thin disc at the end, and fastened to the shaft by a set pin at the other end; the shaft also carried a thin disc at the end. The relative motion of these discs represented the amount of bursting; and by waxing one disc and having a fine pointer attached to the other by a spring, the relative position could be obtained. Fig. 228 shows the result of the trials, the sections of the wheels being shown in Figs. 229 and 230. The results show that with a 7-inch wheel, discharging into a single guide passage, with

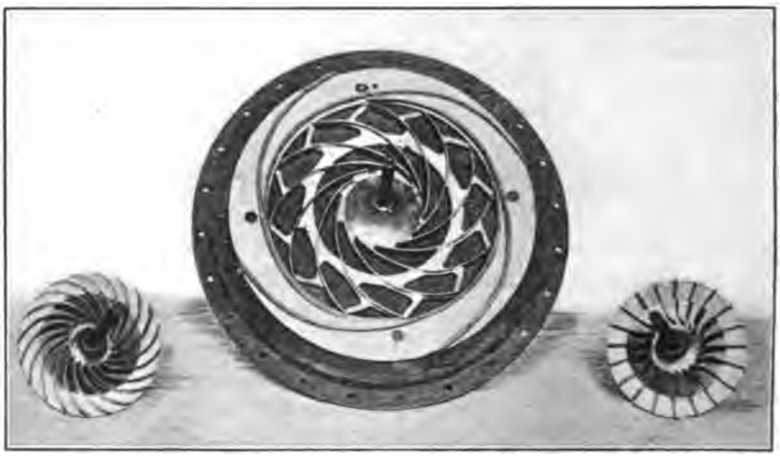


FIG. 227.

curved vanes, the efficiency of the pump was about 50 per cent., and with radial vanes was, at varying speeds, 48 to 53 per cent. With a 12-inch wheel, curved vanes, discharging into four guide passages, the efficiency of the pump varied at different speeds, from 28 to 35 per cent. With the same wheel discharging into a vortex the efficiency was 22 per cent.

§ **160. Circulating Pump for Marine Surface Condensers using Gutermuth Valves.**—Figs. 231 and 232 show an application of the Gutermuth valves in the circulating pump for a surface condenser. Fig. 231 is a vertical section showing the valve cones; and Fig. 232

is a horizontal section showing the Gutermuth valves in place.

Curves of Efficiency.—(1) *Used with 4 guide-passages;* (2) *With free vortex.*

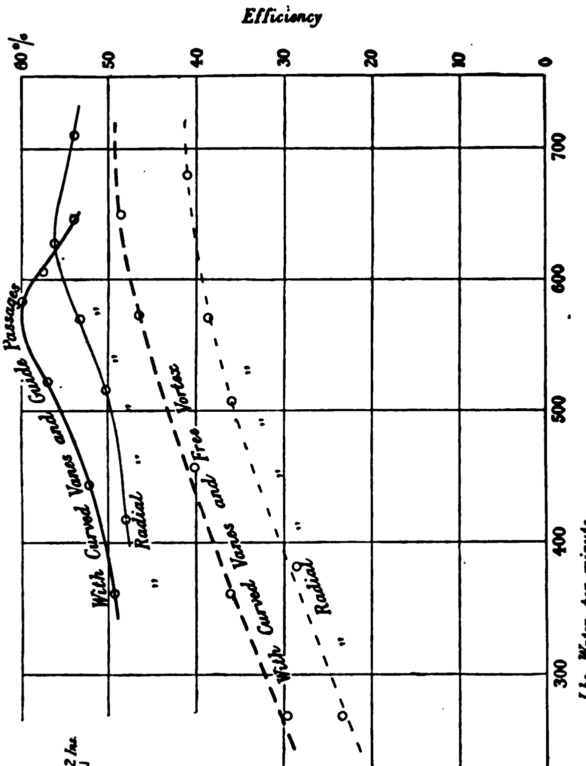
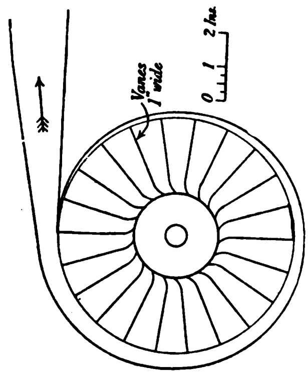


FIG. 228.

FIG. 229.
7" Radial Vaned Wheel used.



7" Curved Vaned Wheel.

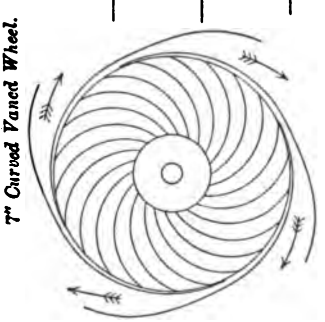


FIG. 230.

The two principal considerations are (1) the smallness of space

occupied, and (2) the amount of pressure to be overcome by these pumps; and they can therefore be of comparatively light con-

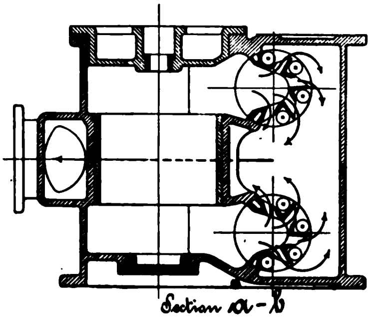


FIG. 231.

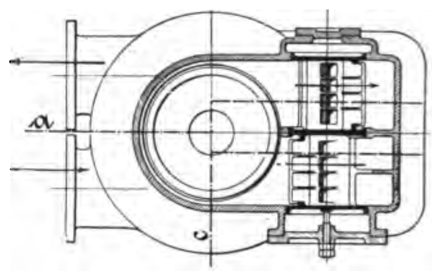


FIG. 232.

struction, so that the working speed can, in emergencies, be increased to 150 revolutions per minute, at which the pump illustrated will circulate 48,600 gallons per hour.

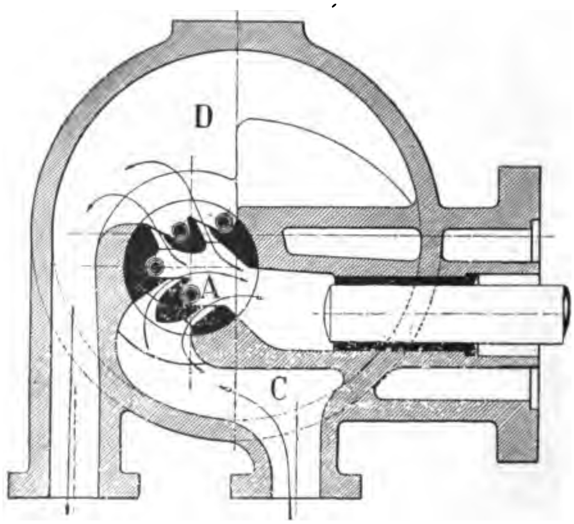


FIG. 233.

§ **161. Spherical Feed Pump.**—In a boiler feed pump of the spherical type, Fig. 234 shows a longitudinal section, and Fig. 233

a cross-section. A is the inserted valve cone, C the suction air-vessel, and D the pressure air-vessel.

§ **162. Reynolds's Hydraulic Brake.**[1]—Professor Reynolds, F.R.S., discussing the question of friction brakes, pointed out that although it is possible to construct such brakes as to work with almost any degree of accuracy, certain inconveniences and drawbacks attend their use, which in all cases leave much to be desired, particularly when—as in an experimental engine—work

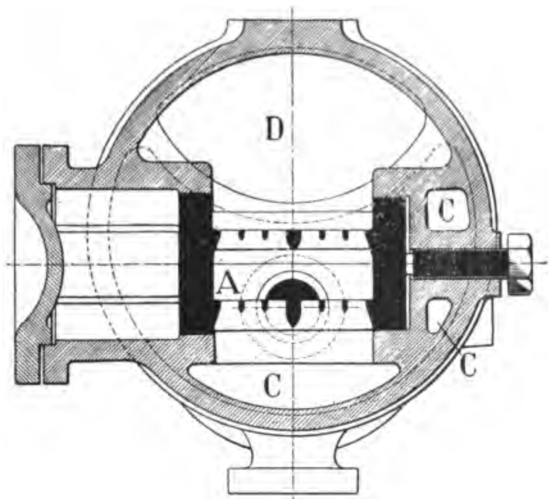


FIG. 234.

on the brake is the sole object of the engine. He summarizes the objections:

(1) Such brakes require constant observation and watching.

(2) A single engine cannot be started without relieving the load.

(3) Such brakes are cumbersome, and are not easily adapted to measure greatly different powers.

(4) Any particular brake cannot without considerable pulling, such as altogether removing the brake and brake-wheel, be altogether nugatory.

[1] *Institution Civil Engineers*, December 10, 1889.

§ 163. Conditions for a Perfect Brake.—Professor Reynolds gives the following conditions that brakes ought to satisfy:

(1) That the brakes should be certain in their action without any attention while the engines are running.

(2) That they should leave the engines free to start, and then take up their load without any attention.

(3) That they should be put on and off by a simple operation.

(4) That, when turned off, they should offer no sensible resistance to the engines.

(5) That they should be capable of being so adjusted as to impose any particular resistance, from zero to the greatest, at any speed at which it was desired to run the engine.

(6) That the resistance of the brakes, when once adjusted, should be independent of the speed of the engine.

(7) That the necessary size and structure of the brakes should not be such as to incommode or hamper the engines.

(8) That the resistance of the brake should admit of absolute determination from a single observation of these attributes. (1) and (2) belong to all fluid resistances, such as that of screw propellers or centrifugal pumps, in which cases the resistance, varying as the square of the speed, is zero when the engines start.

If the casing of a centrifugal pump, or the tank in which a paddle or screw works, be suspended on the crank shaft, making a complete balance when the shaft is at rest, then, when the shaft is in motion, the moment of resistance on the shaft will be exactly equal to the moment to turn the casing round the shaft. The first published account of this form of brake having been made use of for dynamometric measurement is by Hirn, in his investigation for the verification of Joule's mechanical equivalent of heat, and was subsequently adopted by Joule in his second determination. The brakes described below were used by Professor Reynolds, in conjunction with Mr. Moorby, to determine the average specific heat between limits of temperature of 32° and 212°.[1]

Professor Reynolds, when measuring the resistance on the

[1] *Philosophical Transactions of the Royal Society of London*, 1897.

shaft of a multiple steam turbine at speeds of 12,000 revolutions per minute, in 1876, made use of a centrifugal pump suspended on the shaft and working into the pump. The resistance, or head against which the pump was working, was regulated by a valve between the exit and inlet passages, that is, in the external circuit made by the water.[1]

At the same meeting, Mr. William Froude gave an account of his hydraulic brake, for measuring the power of large engines, in which the resistance was regulated on the same principle, by adjusting diaphragms or sluices in the passages between the revolving wheel and the casing. Mr. Froude's brake differed essentially from any of those previously used, being designed to obtain a maximum resistance with given-sized wheel. For this purpose, Mr. Froude invented an internal arrangement which affords a resistance out of all comparison with any other firm.

§ 164. **Considerations affecting the Working of the Brake.**—Professor Reynolds first constructed a model of Froude's brake with a 4-inch wheel—the object of which was to ascertain how far the sluices would act in maintaining a constant resistance at any particular speed, and what was the minimum resistance when the sluices were closed.

With this brake it was found that the minimum resistance was about 0·08 of the maximum; a hardly satisfactory range, considering it was desired to run the engines at a constant load at from 100 to 400 revolutions per minute, the maximum resistance of the brake ranging from 1 to 16, so that the minimum at 400 would be 26 per cent. greater than the maximum at 100 revolutions, apart from the fact that closing the sluices would not render the brake nugatory.

This, however, was of small importance compared with another fact revealed by experiments. When the speed of the brake exceeded a certain small limit, determined by the head of water under which it was working, the maximum resistance fell off in a surprising and somewhat irregular manner. This falling off was found to be owing to the brake partially emptying itself of water, due to the air from the water gradually accumulating in

[1] British Association, 1877.

the centre of the vortex—a fact which, if not dealt with, threatened to render such brakes useless for the purpose of these engines.

The argument was simple: in a vortex, the pressure at the centre is less than the pressure at the outside. The pressure at the outside in these brakes is determined by the atmosphere, and the small head under which they are working; and the outside forms a closed surface. The pressure at the centre will, therefore, at different speeds, fall below the pressure of the atmosphere. Air will be drawn from the water and accumulated in the centre, occupying the space of the water and diminishing the resistance: and, owing to various causes, the action will be irregular. This would be prevented if passages could be carried through the outside to the axis of the vortex, carrying a supply of water at or above the pressure of the atmosphere, so as to prevent the pressure at this point falling below the atmosphere. This was accomplished by perforating the vanes of the wheel, and supplying water through the perforation. It also appeared that, by having similar perforations in the casing open to the atmosphere, the pressure at the centre of the vortex could be rendered constant, whatever the supply of water or the speed of the wheel; so that it would then be possible to run the brake partially full, and regulate the resistance, from nothing to the maximum, without the sluices of Mr. Froude. These conclusions having been verified as a model, it was decided to arrange the engines with shafts in line, with three brakes on the shafts; and the brakes, with 18-inch wheels, were designed according to the resistance given by the model. The brakes promised all the attributes desirable, except that of running with a constant load under varying speeds. This matter was considered during their construction, and an automatic arrangement was devised, acting as cocks, regulating the supply and exit of the water to and from the brake necessary to keep it cool—the lifting of a lever opening the exit and closing the supply, so as to diminish the quantity in the brake, and *vice versâ.*

The danger of such an arrangement hunting was carefully considered, and precautions taken. The brakes have been running for twenty years—requiring no care nor attention. They are easily tested for balance. They have neither fixed nor spring

attachment, except the bearing on the shaft. They are loaded on a 4-foot lever, with 2-inch stay between the stops. When a speed of the engine reaches about 20 revolutions per minute, the levers rise—whatever the load—and, though always in slight motion, do not vary ½ inch until the engines stop. During the run the load on the brakes may be altered at will, without any other adjustment. The first brake, designed on this principle, was for

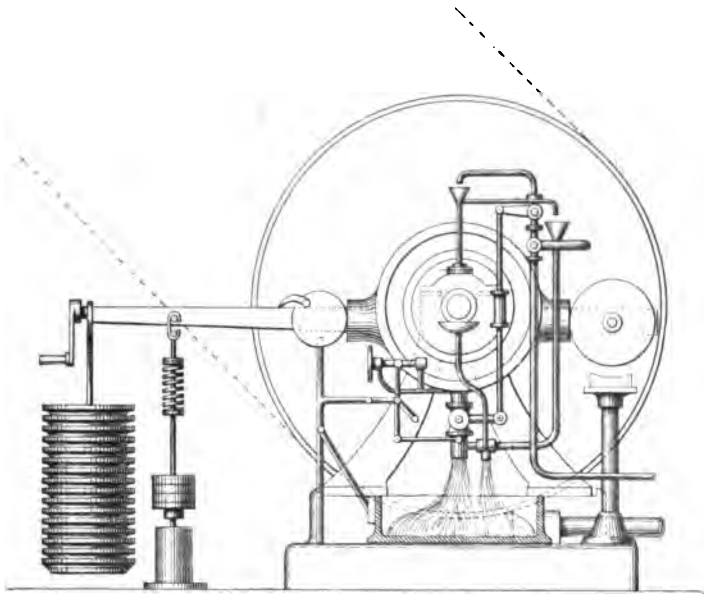


FIG. 235.

the Whitworth Laboratory, Owens College, and was made by Messrs Mather and Platt.[1]

§ **165. Description of Brake.**—The brake consists primarily of (1) a brake wheel, 18 inches in diameter, fixed on the 4-inch brake shaft by set pins, so that it revolves with the shaft (Figs. 235 and 236), and (2) a brake—a brake case—which encloses the wheel, the shaft passing through *bushed* openings in the case, which it fits

[1] See the Bakerian Lecture by Professor Reynolds and Mr. Moorby, *Transactions of the Royal Society*, 1897.

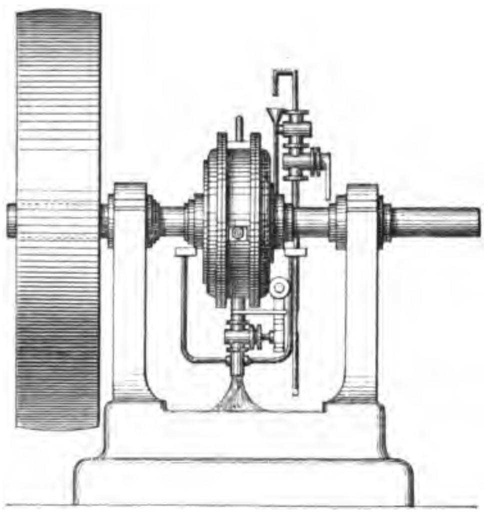


FIG. 236.

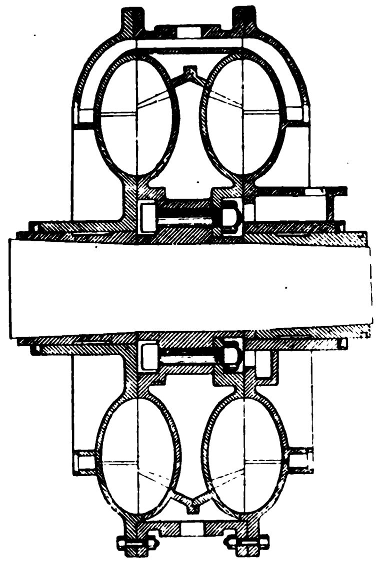


FIG. 237.

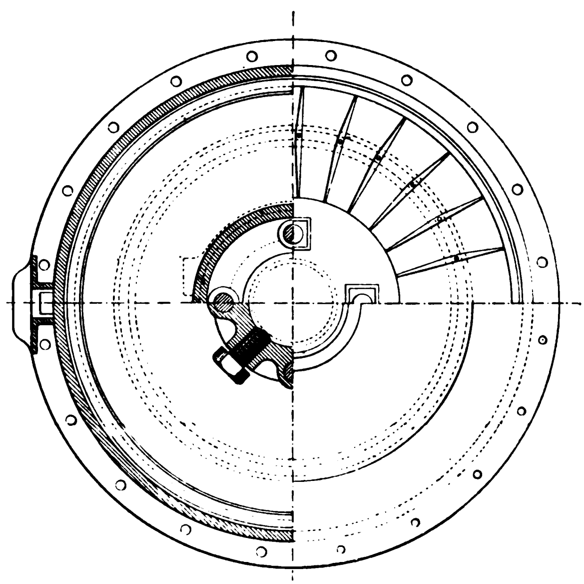


FIG. 238.

closely, so as to prevent undue leakage of water, while leaving the shaft and brake wheel free to turn the case, except for the slight friction of the shaft.

The outline of the axial-section of the brake wheel (Fig. 237) is that of a right cylinder 4 inches thick. The cylinder is hollow —in fact, made of two discs which fit together, forming an internal boss for attachment to the shaft, and also bolted together at the periphery, forming a closed annular box, except for apertures described further on. In each of the outer disc faces of the wheel are 24 pockets (carefully formed), $4\frac{1}{2}$ inches radial and $1\frac{1}{2}$ inches deep measured axially, but so inclined that the narrow partitions or vanes ($\frac{1}{4}$ inch) are nearly semicircular discs inclined at 45° to the axis; the vane or arc face being perpendicular to the vane on the opposite face. A section perpendicular to the axis is shown in Fig. 238, one quarter exposed to show the vanes and ventilating holes. Fig. 239 is a photograph of the brake, showing the brake case, lever, and arrangement of linkages for water valves. Fig. 240 is a photograph showing the brake exposed, the pockets in the casing and wheel and the core-holes being shown.

The internal disc faces of the brake case, as far as the pockets are concerned, are the exact counterparts of the disc faces of the wheel (except that there are 25 pockets), so that the partitions in the case are in the same planes as the partitions meeting them in the wheel, there being $\frac{1}{64}$ clearance between the faces.

The pairs of opposite pockets when they come together form nearly closed chambers, with their sections, parallel to the vanes, circular. In such spaces, vortices in a plane inclined at 45° to the axis of the shaft may exist, in which case the centrifugal pressure on the outside of each vortex will urge the case and the wheel in opposite directions inclined at 45° to the direction of motion of the wheel, which will give a tangential force over the disc faces of the wheel of $\frac{1}{\sqrt{2}}$ of these vortex pressures; whilst the forces parallel to the axis neutralize, so that there is no thrust, and the brake case floats on the shaft. To ensure the constant pressure, and at the same time to allow of the pockets

FIG. 239.

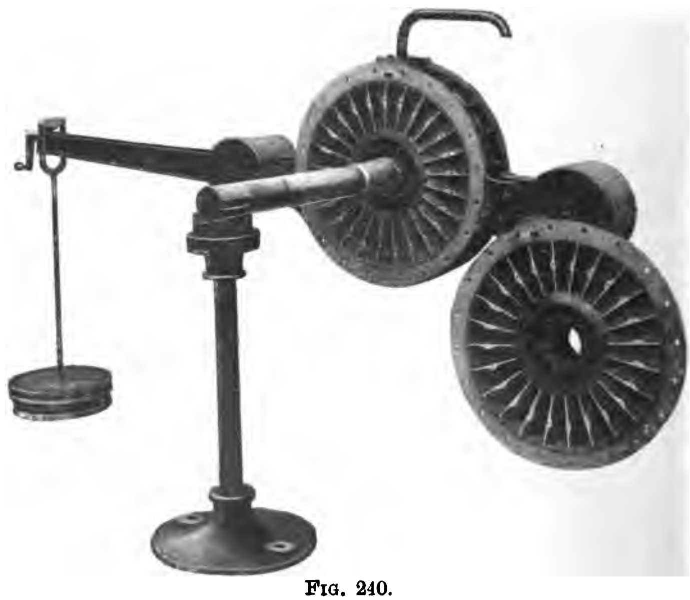


FIG. 240.

being only partially full—that is, to allow of hollow vortices with air-cores at atmospheric pressure, it is necessary that there should be free access of air to the centre of the vortices, and as this access cannot be obtained through the water, which completely surrounds these centres, it is obtained by passages ($\frac{1}{8}$ inch diameter) with the metal of the guides, which lead to a common passage opening to the air on the top of the case (Figs. 237 and 239).

To supply the brake with water, there are similar passages in the vanes of the wheel leading from the box cavity, which again receives water through parts which open opposite an annular recess in one of the disc faces of the case, into which the supply of water is led by means of a flexible indiarubber pipe from the supply regulating valve.

The water on which work has been done leaves the vortex pockets by the clearance between the disc surfaces of the wheel and case, and enters the annular chamber between the outer periphery of the wheel and the cylindrical portion of the case, which is always full of water when the wheel is running, whence its escape is controlled by a valve in the bottom of the case, from which it passes to waste.

By means of linkage (Fig. 235) connected with a fixed support and the brake case, an automatic adjustment of the inlet and outlet valves, according to the position of the lever, is secured without affecting the mass-moment on the brake case. And this also affords means of adjusting the position of the lever. To admit of adjustment for wear, the shaft is covered over that portion which passes through the bushes, the bushes being similarly coned and screwed into short sleeves on the casing, so that by unscrewing them the wear can be followed up and leakage prevented.

The brake levers for carrying the load and balance weight are such as to allow the load to be suspended from a groove parallel to the shaft, at 4 feet from the shaft, by a carrier with a knife-edge, the carrier and the weights each being adjusted to 25 lbs. In addition to this load, a weight is suspended from a knife-edge on the lever nearer the shaft, this weight being the

piston of a dash-pot in which it hangs freely, except for the viscous resistance of the oil. This weight, being adjusted to exert a moment of 100 foot-pounds, and again a travelling weight of 48 pounds, is carried on the lever linked by a screw of ¼-inch pitch, so that one turn changes the moment by 2 foot-pounds, while a scale on the lever shows the position. A shorter lever on the opposite side of the case carries a weight of 746 pounds, which is adjusted to balance the lever and sliding weight when the load is removed.

§ **166. Principle of Brake.**—The principle of these hydraulic dynamometers is as follows:—The brake wheel imparts moment of momentum to the water in the case, and the friction of the shaft imparts moment of momentum to the case. The water in the case, when its moment of momentum is steady, imparts moment of momentum to the case as fast as it receives it, and the time mean of the moment of the load is equal to the time mean of the moment of effort of the shaft. This is not affected by water entering and leaving the case at equal rates, provided it enters and leaves radially.

§ **167. Statical Determination of Balance of Brake.**—To test this, a bar is attached to the horizontal bars at the back of the brake (Fig. 235), and marks on the brake and bar exactly coincide. The moment of this bar about the centre of the shaft has to be accurately determined. Adjust the rider to some known reading and level the brake. Water is then added to a bucket suspended from the end of a lever, distant 4 feet from the centre of the shaft, until the friction of the glands, together with any lack of balance of the brake, is overcome, or the lever floats. The buckets and water are then weighed, and the experiment repeated with different positions of the rider, the results being tabulated. The procedure is then repeated for the other side of the brake, the bucket being now hung on the bar at a distance from the axis equal to d.

Let W be the weight of water and bucket when hanging from the end of the lever at 4 feet from axis; W′ the weight when hanging from the bar attached to the back end at distance d from axis; M_0 represent the brake error moment, if any; Mb the

moment of the bar; M_w the moment of the rider; F the moment of friction. Then the equations are:

(1) Bucket on bar side

$$M_w + F = M_b + M_0 + W'd.$$

(2) Bucket on lever side

$$\begin{aligned} 4W + M_w &= M_b + M_0 + F \\ \therefore F + M_0 &= 4W + M_0 - M_b \\ F - M_0 &= W'd - M_w + M_b \\ \therefore M_0 &= M_w - M_b + \tfrac{1}{2}(4W - W'd). \end{aligned}$$

Determine M_0 for, say, four positions of the rider, noting in each case whether M_0 is positive or negative. That is to say, whether the brake reading is too large or too small.

A test of the brake on the low-pressure engine gave

Reading on rider scale	2	4	6	8	10	12	14	16	18	20	22	24
Error in foot-pounds	−4·3	−2·7	−3·6	−1·5	−1·5	−1·7	−2·4	−1·8	−2·7	−2·2	−2·2	−2·2

Apparently, therefore, the brake is out of balance. The reason is that when Professor Osborne Reynolds and Mr. Moorby made their experiments on the determination of Joule's equivalent, the range of temperature was taken between 32° and 212° F., and the brake had to be well lagged. The brake has not been corrected, so that the error is entirely due to the removal of the lagging.

CHAPTER VII

PROFESSOR OSBORNE REYNOLDS'S RESEARCHES

§ **168. Viscous Flow in Capillary Tubes.**—Consider a circular tube (Fig. 241) of small dimensions, and suppose a viscous fluid flow along it. Take an element *abcd*, of small thickness dr, and at radius v; let p_1, p_2 be the intensities of pressures over the areas *ca* and *bd* parallel to the axis, u and $u + dr$ the velocities over the faces *ab* and *cd*; and l the length of the element. The force tending to drag the element along is the difference between the viscous force on *ab* and *cd*. The force over *ab* is

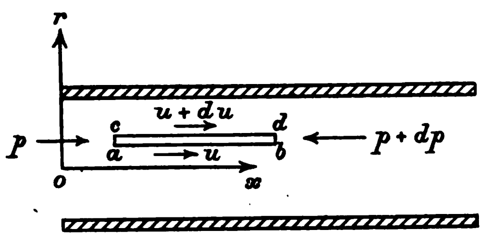


Fig. 241.

$$\mu\frac{du}{dr} \times 2\pi rl = f, \text{ say}$$

opposing motion, and over *cd*

$$f + \frac{df}{dr}dr$$

in the direction of motion. The difference

$$2\pi\mu l\frac{d}{dr}\left(r\frac{du}{dr}\right)dr$$

is the net force with which the element is dragged along, and which, therefore, represents the resistance which could be overcome—that is to say, equal to, since the pressure is constant along the radius

$$2\pi r(p_2 - p_1)dr.$$

Thus $$\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{p_2 - p_1}{\mu l} \cdot r$$

$$\therefore u = \frac{p_2 - p_1}{4\mu l} \cdot r^2 + \mathrm{A} \log_e r + \mathrm{B}.$$

When $v = 0$, u is finite, and therefore A is zero; when $r = a$ (the radius of the tube), $u = 0$, so that

$$u = \frac{p_1 - p_2}{4\mu l}(a^2 - r^2).$$

The total flux is

$$\int_0^a u \times 2\pi r dr = \frac{\pi a^4}{8\mu} \cdot \frac{p_1 - p_2}{l}.$$

This result was verified by Poiseuille in his experiments on the flow of water in capillary tubes; namely, that the time of efflux of a given quantity of water varies as the length of the tube, inversely as the difference of pressure at the two ends, and inversely as the fourth power of the diameter.

Let v = mean velocity of flow.
d = diameter of tube.
σ = weight per cubic foot of water.

Then loss of pressure head

$$= \frac{p_1 - p_2}{\sigma} = \frac{32\mu}{\sigma} \cdot \frac{vl}{d^2}.$$

Thus, in a capillary tube, the loss of head varies directly as the velocity, directly as the length, and inversely as the square of the diameter.

It will be noticed that in § 23 the loss of pressure head in a large pipe at ordinary velocities has been found to be given by

$$\frac{p_1 - p_2}{\sigma} = f \cdot \frac{4l}{d} \cdot \frac{v^2}{2g}$$

which shows that the loss of head varies directly as the length, directly as the square of the velocity, and inversely as the diameter. Both these formula have been verified by experiment under the conditions under which they hold. It is desirable, therefore, to discuss the matter at greater length.

§ 169. Viscous Flow between Parallel Plates very near together.

—Consider two plates very near together, and suppose a viscous fluid to flow between them (Fig. 242). Take an element, *abcd*.

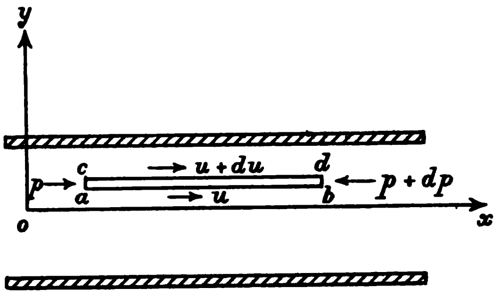


FIG. 242.

Let the axes of x, y, z be along the direction of motion, perpendicular to the axis of x, and parallel to the plates, and z perpendicular to the plates — the origin being midway between the plates. Let u be the velocity at distance z from the xy-plate. Then the viscous force a over the face ab is

$$\mu\frac{du}{dz} \cdot dxdy = f, \text{ say,}$$

opposing motion. And over cd, in the direction of motion,

$$f + \frac{df}{dz}dz;$$

so that the net force tending to drag the element along is

$$\frac{d}{dz}\left(\mu\frac{du}{dz}\right) \cdot dxdy = \mu\frac{d^2u}{dz^2}dxdydz$$

and which, therefore, represents the resistance which could be overcome; that is to say, since the pressure is constant along the axes of y and z, the difference of pressure is

$$(p_2 - p_1)dy\,dz$$

p_1 and p_2 being the pressures over the two end areas. Thus

$$\mu\frac{d^2u}{dz^2} \cdot du = p_2 - p_1.$$

Now, the difference of pressure is proportional to the length; hence

$$\mu\frac{d^2u}{dz^2} = \frac{p_2 - p_1}{l}$$

$$\frac{d^2u}{dz^2} = \frac{p_2 - p_1}{ul}$$

$$u = \frac{p_2 - p_1}{2ul}z^2 + \mathrm{A}z + \mathrm{B}.$$

If $2h$ be the distance between the plates, then $u = 0$ when $z = \pm h$, and, therefore

$$u = \frac{p_1 - p_2}{zul}(h^2 - z^2).$$

Thus the variation of velocity in the direction of the axis of z is represented by a parabolic curve, as in the first case, and the maximum velocity is at the centre plane when $z = 0$; that is, the maximum velocity is

$$u = \frac{p_1 - p_2}{zul} . h^2.$$

The average velocity is, therefore,

$$u_a = \frac{p_1 - p_2}{zul} . h^2.$$

Hence

$$\frac{p_2 - p_1}{l} = - \frac{3\mu}{h^2} \cdot \frac{u_a}{l}$$

or, to express it differently,

$$\frac{dp}{dx} = - \frac{3\mu}{h^2} du.$$

§ 170. An Experimental Investigation of the Circumstances which determine whether the Motion of Water shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channels.[1]

Leading Features of the Motion of Action Fluid.—The exact manner in which water moves is difficult to perceive, and still more difficult to define, as also are the forces attending such motion, yet there are certain general features both of forces and motions which require solution.

The relations between the resistance encountered by and the velocity of a solid body moving steadily through a fluid in which it is completely immersed, or of water moving through a tube, present themselves mostly in one or other of two simple forms. The resistance is generally proportional to the square of the

[1] Professor Osborne Reynolds, F.R.S., *Philosophical Transactions of the Royal Society*, pp. 935–982; or Collected Papers, Cambridge University Press, pp. 51–103, Part II.

velocity; and, when this is not the case, it takes a simpler form, and is proportional to the velocity.

Again, the internal motion of water assumes one or other of two broadly distinguishable forms — either the elements follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths the most indirect possible.

The transparency or the uniform opacity of most fluids renders it impossible to see the internal motions, so that, broadly distinct as are the two classes (direct and sinuous) of motion, their existence would not have been perceived were it not that the surface of water, where otherwise undisturbed, indicates the nature of the motion beneath.

The motion of the water may be shown by adding a few streaks of highly coloured water to the clear moving water. Then, although the coloured streaks may at first be irregular, they will, if there are no eddies, soon be drawn out into even colour bands; whereas, if there are eddies, they will be curled and whirled about.

Certain circumstances have been definitely associated with particular laws of force. Resistance, as the square of the velocity, is associated with motion in tubes of more than capillary, and with the motion of bodies through the water at more than insensibly small velocities, while resistance as the velocity is associated with capillary tubes and small velocities.

§ 171. Dimensional Properties in the Equations of Motion.— Professor Reynolds, before making experiments, obtained a relation between the dimensions without integration. In a fluid in which inertia forces dominate, the equation is

$$\frac{dp}{ds} = -\sigma u\frac{du}{ds} \quad \text{.....................(§ 1)}$$

In a fluid in which viscosity dominates

$$\frac{dp}{ds} = -\frac{3\mu}{h^2}u \quad \text{.....................(§ 169)}$$

Thus, when the motion depends on inertia and viscous forces

$$\frac{dp}{ds} = -\sigma u\frac{du}{ds} - \frac{3\mu}{h^2}u.$$

The relative value of the terms on the right-hand side of this equation—h being a linear function—is

$$\frac{\dfrac{\sigma V^2}{L}}{\dfrac{\mu}{L^2}V}$$

V and L having of the dimension of velocity and distance.

This reduces to

$$\frac{\sigma VL}{\mu}.$$

Thus, if the eddies were owing to one particular cause, the argument shows that the birth of eddies depends on some definite value of

$$\frac{\sigma VL}{\mu}.$$

§ **172. The Cause of Eddies.**—The general cause of the change from steady to eddying motion was, in 1843, pointed out by Professor Stokes as being that, under certain circumstances, the steady motion becomes unstable, so that an indefinitely small disturbance may lead to a change of sinuous motion. Both the causes above referred to are of this kind, and yet they are distinct, lying in the part taken in the instability by viscosity. If a fluid free from viscosity be compared with a viscous fluid, then

(1) The frictionless fluid might be unstable and the viscous fluid stable. Under these circumstances the cause of eddies is the instability as a perfect fluid, the effect of viscosity being in the direction of stability.

(2) The frictionless fluid might be stable and the viscous fluid unstable, under which circumstances the cause of instability would be the viscosity.

When, in a channel, the water is all moving in the same direction, the velocity being greatest in the middle and diminishing to zero at the sides, as indicated by the curve in Fig. 243, eddies showed themselves reluctantly and irregularly; whereas, when the

water on one side of the channel was moving in the opposite direction to that on the other, as shown in Fig. 244, eddies appeared in the middle regularly and readily.

Methods of Investigation.—There are two methods of experimenting:

(1) Measuring the resistances and velocities of different diameters, and with different temperatures of water.

(2) Visual observation as to the appearance of eddies during the flow of water along tubes or open channels.

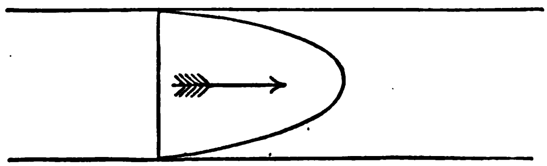


FIG. 243.

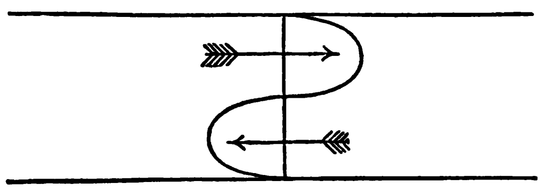


FIG. 244.

Experiments of Visual Observation.—The most important of these experiments related to water moving in one direction along glass tubes. Besides this, however, experiments on fluids flowing in opposite directions in the same tube were made; also a third class of experiments, which related to motion in a flat channel of indefinite breadth.

Experiments by Means of Colour-bands in Glass Tubes.—The diameters of the tubes experimented upon were 1 inch, $\frac{1}{2}$ inch, and $\frac{1}{4}$ inch (called No. 1, No. 2, and No. 3). They were all about 4 feet 6 inches long, and fitted with trumpet mouthpieces, so that the water might enter without disturbance. The water was drawn through the tubes out of a large glass tank, in which the tubes were immersed, arrangements being made so that a streak or

streaks of highly coloured water entered the tubes with the clear water.

The general results were as follows:

(1) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube (Fig. 245).

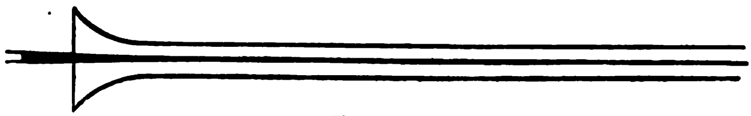


FIG. 245.

(2) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or uptake, the colour band would all at once mix up with

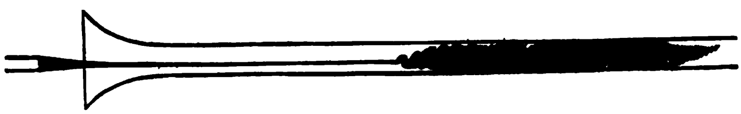


FIG. 246.

the surrounding water, and fill the rest of the tube with a mass of coloured, as in Fig. 246.

Any increase in the velocity caused the point of breakdown to approach the trumpet; but with no velocities that were tried did it reach this.

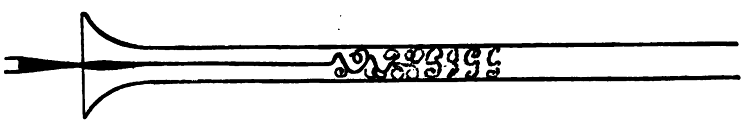


FIG. 247.

On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in Fig. 247.

The experiments clearly showed the existence of eddies and

a critical velocity. They also showed that, instead of the eddies coming gradually into existence, they came suddenly into existence. In order to get the law of the critical velocity, the diameters of the tube were carefully measured, also the temperature of the water, and the rate of discharge.

(4) It was then found that, with water at a constant temperature, and the tank as still as could by any means be brought about, the critical velocities at which the eddies showed themselves were almost exactly in the inverse ratio of the diameters of the tubes.

(5) That in all the tubes the critical velocity diminished as the temperature increased, the range being from 5° C. to 22° C.; and the law of this diminution, so far as could be determined, was in accordance with Poiseuille's experiments (§ 168). Taking T to express degrees Centigrade, then, by Poiseuille's experiments

$$\frac{\mu}{\sigma} \propto P = (1 + 0{\cdot}0336T + 0{\cdot}00221T^2)^{-1}.$$

Taking a metre as the unit, U_s the critical velocity, and D the diameter of the tube, the law of the critical point is completely expressed by the formula

$$U_s = \frac{1}{B_s} \cdot \frac{P}{D}$$

where

$$B_s = 43{\cdot}79$$

$$\log B_s = 1{\cdot}64139.$$

This shows that the eddies follow the law deduced by Professor Reynolds (§ 171) by theoretical consideration.

During the experiments the critical velocity was much higher than had been expected in pipes of such magnitude, resistance varying as the square of the velocity had been found at very much smaller velocities than those at which the eddies appeared when the water in the tank was steady; and, in the second place, it was observed that the critical velocity was very sensitive to disturbance in the water before entering the tubes; and it was only by the greatest care as to the uniformity of the temperature of the tank and the stillness of the water that consistent results were obtained.

This showed that the steady motion was unstable for large disturbances long before the critical velocity was reached—a fact which agreed with the full-blown manner in which the eddies appeared.

Professor Reynolds showed that for frictionless liquids, in the parallel flow, the flow was stable in one direction and unstable in the other. He then took account of viscosity, and taking into account the boundary, which had been omitted in the first investigation—that is to say, the friction at the solid surface—then the motion of the fluid, irrespective of viscosity, would be unstable. The integration depends on exactly the same relation

$$U \propto \frac{\mu}{cP}$$

as that previously found. This explained the sudden way in which eddies came into existence in the experiments with the colour band, and the effect of disturbances to lower the critical velocity. These were also explained, for as long as the motion was steady, the instability depended upon the boundary action alone; but once eddies were introduced, the stability would be broken down.

But as it appeared that the critical velocity, in the case of motion in one direction, did not depend on the curve of instability with a view to which it was investigated, it followed that there must be another critical velocity, which would be the velocity at which previously existing eddies would die out, and the motion became steady as the water proceeded along the tube. This conclusion was verified.

§ 173. **Results of Experiments on the Law of Resistance in Tubes.**—The existence of the critical velocity could only be tested by allowing water in a high state of disturbance to enter a tube, and after flowing a sufficient distance for the eddies to die out, if they were going to die out, to test the motion. It was impossible to apply the method of colour bands, and the method adopted was to test the law of resistance by experiment.

Two straight lead pipes, No. 4 and No. 5, each 16 feet long, and having diameters of $\frac{1}{4}$ and $\frac{1}{2}$ inch respectively, were used. The water was allowed to flow through 10 feet of pipe before

coming to the first gauge hole, the second gauge hole being 5 feet further along the pipe. The results were very definite, and are shown in Fig. 248. They were—

(1) At the lower velocities the pressure was proportional to the velocity, and the velocities at which a deviation from the law first named were in exact inverse ratio of the diameters of the pipes.

(2) Up to these critical velocities the discharge from the pipes agreed exactly with those given by Poiseuille's formula for capillary tubes.

(3) For some little distance after passing the critical velocity no very simple relation appeared to hold between the pressures and velocities. But by the time the velocity reached 1·2 × the

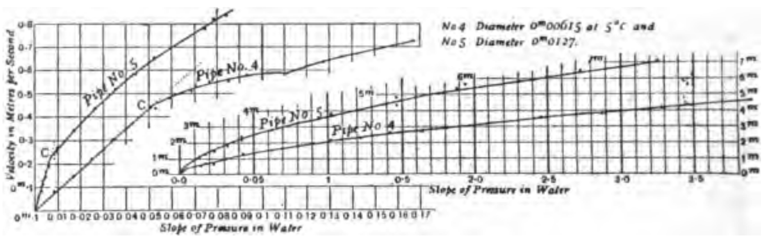


FIG. 248.

critical velocity, the relation became again simple. The pressure did not vary as the square of the velocity, but as 1·722 power of the velocity. This law held in both tubes and through velocities ranging from 1 to 20, where it showed no signs of breaking down. A diagram is shown in Fig. 248.

(4) The most striking result was that not only at the critical velocity, but throughout the entire motion, the laws of resistance exactly corresponded for velocities in the ratio of

$$\frac{\mu}{\sigma L}.$$

This last result was brought out in the most striking manner on reducing the results by the graphic method of logarithmic homologues (§ 21) first used by Professor Osborne Reynolds. Calling the resistance per unit length as measured in weight per cubic unit of water i, and the velocity v, log i is taken for abscissa and

log v for ordinate, and the curve plotted. In this way the experimental result for each tube was represented by a curve. These curves are shown on an enlarged scale in Fig. 249, and only differ in position. The experimental results are given in the diagram.

The further point noticed was, starting from the lowest speed,

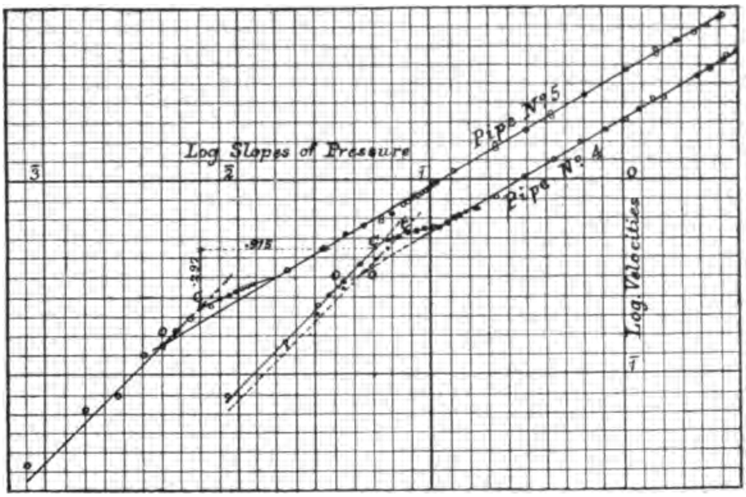


FIG. 249.

it was found that the fluid in the differential gauge was at first very steady, lowering steadily as the velocity was increased by stages, until, when a certain speed was reached, the fluid jumped about, and the smallest adjustment of the tap sent the fluid in the gauge outside of the field of the microscope.

Either of the curves may be brought into exact coincidence (Fig. 249) with the other by a rectangular shift, and the horizontal shifts are given by the difference of the logarithm of

$$\frac{D^3}{\mu^2}$$

D representing diameter for the two tubes, the vertical shift being the difference of the logarithms of

$$\frac{D}{\mu}.$$

The temperatures at which the experiments were made were nearly the same, but not quite, so that the effect of variations of μ showed themselves.

§ 174. Comparisons with D'Arcy's Experiment.—The definiteness of these results, their agreement with Poiseuille's law, and the new form which they indicated for the law of resistance above the critical velocities, led Professor Reynolds to the results with those of D'Arcy on pipes ranging from 0·014 to 0·5 metre in diameter.

Professor Reynolds had the logarithmic homologues drawn from D'Arcy's published work. If the law quoted above was general, then these logarithm curves must all shift into coincidence, if each were shifted horizontally through

$$\frac{D^3}{P^2}$$

and vertically through

$$\frac{D}{P}.$$

There were many doubtful points, but it was rather a question of seeing if there was any systematic disagreement. When the curves came to be shifted the agreement was remarkable; the only difference was that in Professor Reynolds's experiment, in the slopes of the higher portions of the curve, Professor Reynolds's slope was 1·722 : 1, and D'Arcy's varied, according to the nature of the material of the pipe, from the lead pipes, which were the same as those of Professor Reynolds, varying from 1·92 to 1 with cast-iron pipes. This shows that the nature of the surface of the pipe has an effect on the law of comparison above the critical velocity (Fig. 250).

The Critical Velocities.—All the experiments agreed in giving

$$v_c = \frac{1}{278} \cdot \frac{P}{D}$$

as the critical velocity, to which corresponds the critical slope of pressure

$$i_c = \frac{1}{477{,}000{,}000} \cdot \frac{P^2}{D^3}$$

the units being metres and degrees Centigrade. It will be observed that the value is much less than the critical velocity at which steady motion broke down, the ratio being 43·7 to 278.

The General Law of Resistance.—The logarithmic homologues all consist of two straight branches, the lower branch inclined at 45° and the upper one at n horizontal to 1 vertical. Except for the small distance beyond the critical velocity these branches constitute the curves. These two branches meet in a point on the curve at a definite distance below the critical pressure, so that, ignoring the small portion of the curve above the point

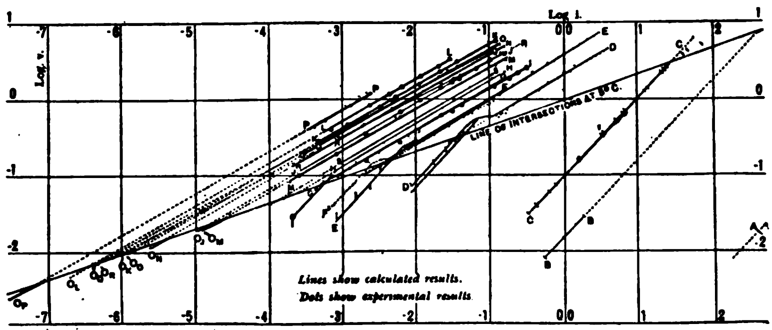


FIG. 250.

before it again coincides with the upper branch, the logarithmic homologue gives the law of resistance for all pipe velocities

$$A\frac{D^2}{\sigma^2}i = \left(B\frac{D}{\sigma}v\right)^n$$

where n has the value unity so long as either number is below unity, and then takes the value of the slope n to 1 for the particular surface of the pipe.

If the units are metres and degrees Centigrade,

$$A = 67,700,000$$
$$B = 396$$
$$P = (1 + 0{\cdot}0336T + 0{\cdot}000221T^2)^{-1}.$$

For feet

$$A = 1,935,000$$
$$B = 36{\cdot}9.$$

This equation, then, excluding the region immediately about the critical velocity, gives the law of resistance in Poiseuille's tubes (those of the present investigation and D'Arcy's), the range of diameters being

from 0·000613 (Poiseuille, 1845)
to 0·5 (D'Arcy, 1857).

This algebraical formula shows that the experiments entirely accord with the theoretical conclusions.

The empirical constants are A, B, P, and n; the first three relate solely to the dimensional properties of the fluid summed up in the viscosity, and it seems probable that the last relates to the properties of the surface of the pipe. The following table gives the results of different surfaces of tubes:—

Lead-jointed	= 1·79
Varnished	= 1·82
Glass	= 1·79
New cast-iron	= 1·88
Incrusted pipe	= 2·00
Cleaned pipe	= 1·91

§ **175. Experiments on Glass Tubes by means of Colour Bands.**— Professor Reynolds experimented on the motion of water by colour bands. A straight tube, nearly 5 feet long and about an inch in diameter, was selected from a large number as being the most nearly uniform, the variation of the diameter being less than $\frac{1}{32}$ inch. The ends of the tube were ground off plane, and on the end which appeared the slightly larger was fitted a trumpet mouth of varnished wood, great care being taken to make the surface of the wood continuous with that of the glass. The other end of the glass pipe was connected by an indiarubber washer with an iron pipe nearly 2 inches in diameter. The iron pipe passed horizontally through the end of a tank 6 feet long, 18 inches broad, and 18 inches deep, and then through a quadrant so that it became vertical, and reached 7 feet below the glass tube. It then terminated in a large cock, when open, a clear way of nearly a square inch. The cock was controlled by a long lever

reaching up to the level of the tank. The tank was raised upon trestles about 7 feet above the floor. The glass tube thus extended in a horizontal direction along the middle of the tank, and the trumpet mouth was about a foot from the end. Through this end, just opposite the trumpet, was a straight length of tube $\frac{3}{4}$ inch in diameter, and this tube was connected, by means of an indiarubber tube with a clip upon it, with a reservoir of colour. With a view to determining the velocity of flow, an instrument was fitted for showing the change of level of the water in the tank to $\frac{1}{100}$ inch, a floating weight rotating a horizontal spindle to which was attached a pointer. The remainder were hung at various levels in the tank.

In making the experiment, the colour was allowed to flow very slowly, and the cock slightly opened. The colour band established itself much as before (Fig. 245), and remained beautifully steady as the velocity was increased until, at once, on a slight further opening of the valve, at a point about 2 feet from the pipe, the colour band appeared to expand and mix with the water so as to fill the remainder of the pipe with a coloured cloud, of what appeared at first sight to be of a uniform tint (Fig. 246). Closer inspection, however, showed the nature of the cloud. By moving the eye so to follow the motion of the water, the expansion of the colour band resolved itself into a well-defined waving motion of the band, at first without other disturbance, but after two or three waves came a succession of well-defined and distinct eddies. They were sufficiently recognizable by following with the eye, but more distinctly seen by a flash from a spark, when they appeared as in Fig. 247. On slightly closing the valve the eddies disappeared, and the straight colour band established itself.

§ 176. Relation between the Critical Velocity, Size of Tube, and Viscosity.—Two more tubes (2 and 3) were prepared similar in length and mounting to the first, but having diameters of about $\frac{1}{2}$ inch and $\frac{1}{4}$ inch respectively.

In the mean time an attempt was made to ascertain the effect of viscosity by using water at different temperatures. The temperature of the water from the main was about 45° C., the temperature of the room about 54°; to obtain a still higher temperature,

the tank was heated to 70° by a jet of steam. Then, taking, as nearly as possible, similar disturbances, twenty experiments were made, and the results tabulated.[1]

To compare these for the viscosity, Poiseuille's experiments were available, but to prevent any accidental peculiarity of the water being overlooked, experiments after the same manner as Poiseuille's were made with the water in the tank. The results of these, however, agreed so exactly with those of Poiseuille that the comparative effect of viscosity was taken from Poiseuille's formula,

$$P^{-1} = 1 + 0{\cdot}03368\,T + 0{\cdot}000221\,T^2$$

where $P \propto \mu$ with the temperature, and T is temperature Centigrade. The relative values of P at 47° and 70° are as 1·3936 to 1;

FIG. 251.

whilst the relative critical velocities at these temperatures were as 1·45 to 1, which agreement is very close, considering the nature of the experiments.

The mean value of log. B_s is 1·64139 and for B_s 243·79.

Professor Reynolds came to the conclusion that the condition might be one of instability for disturbances of a certain magnitude, and stable for smaller disturbances. In order to test this, an open coil of wire was placed in the tube so as to create a definite disturbance (Fig. 251). Eddies now showed themselves at a velocity of less than half the previous critical velocity, and these eddies broke up the colour band, but it was difficult to say whether the motion was really unstable, or whether the eddies were the result of the initial disturbance, for the colour band having once broken up and become mixed with the water, it was impossible to say whether the motion did not tend to become steady again later in the tube.

[1] See Table I. *Trans. Royal Society*, p. 954, or *Collected Papers*, part ii. p. 74.

When there was any considerable disturbance in the water in the tank and the cock was opened very gradually, the state of disturbance would first show itself by the wavering about of the colour band in the tube; sometimes it would be driven against the glass and would spread out, and all without a symptom of eddies. Then, as the velocity increased, but was still comparatively small, eddies, and often very appreciable eddies, would show themselves along the latter part of the tube. On further opening the cock those eddies would disappear, and the colour band would become fixed, steady right through the tube, which condition it would maintain until the velocity reached its normal critical value, and then the eddies would appear suddenly as before.

§ 177. **Experiments to determine the Critical Velocity by means of Resistance in the Pipes.**—In the higher critical velocity, the water is in a state of eddy motion; so that the method of colour bands is inadmissible, and the only method left was to measure the resistance at the latter part of the tube in conjunction with the discharge.

The necessary condition was somewhat difficult to obtain. The change in the law of resistance could only be ascertained by a series of experiments which had to be carried out under similar conditions as regards temperature, kind of water, and condition of the pipe; and in order that the experiments might be satisfactory, it seemed necessary that the range of velocities should extend far on each side of the critical velocity. In order to best ensure these conditions, it was decided to draw water direct from the main, using the pressure in the main for forcing the water through the pipes.

The apparatus is shown in Fig. 252. As the critical value under consideration would be considerably below that found for the change for steady motion into eddies, a diameter of about ½ inch (12 mm.) was chosen for the larger pipe, and ¼ inch for the smaller, such pipes being the smallest used in the previous experiments. The pipes (4 and 5) were ordinary lead pipes of ¼-inch and ½-inch diameter. These are very uniform in diameter, and when new, present a bright metal surface inside. The pipes were 16 feet long, straightened by laying them in a

trough formed by two inch boards at right angles. This trough was then fixed so that one side of the trough was vertical and the other horizontal, serving as a ledge on which the pipes rested at a distance of 7 feet from the floor; on the outflow ends of the pipes cocks were fitted to control the discharge, and at the inlet end of the pipes were connected, by means of a T-branch, with an indiarubber hose from the main; this connection was purposely made in such

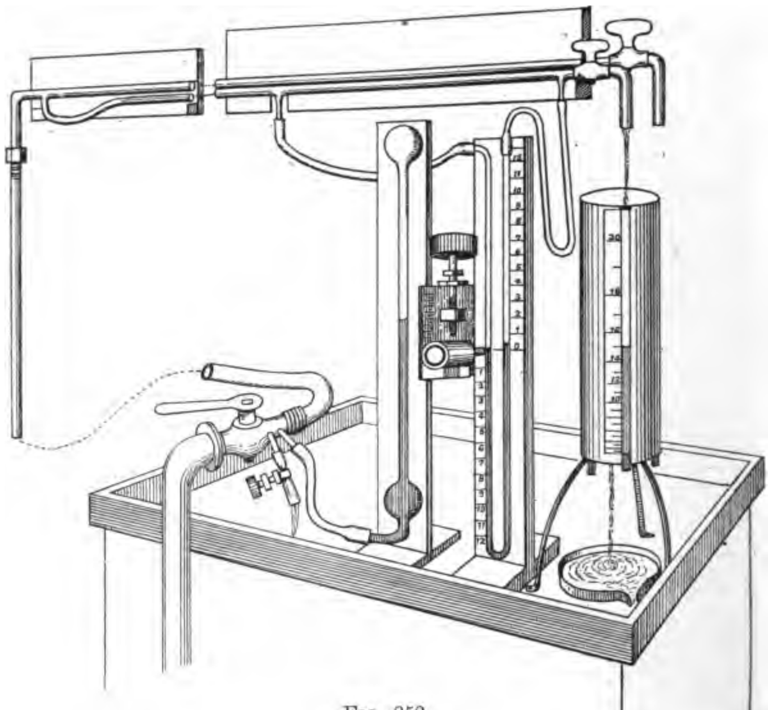


FIG. 252.

a manner as to necessitate considerable disturbance in the water entering the pipes from the hose. The hose was connected, by means of a $\frac{1}{4}$-inch cock, with a 4-inch branch from the main. With this arrangement the pressure on the inlet to the pipes was under the control of the cock from the main, and at the same time the discharge from the pipes was under control

from the cocks on their ends. This double control was necessary owing to the varying pressure from the main, and after a few preliminary experiments a third and more delicate control, together with a pressure gauge, were added, so as to enable the observer to keep the pressure in the hose, that is, on the inlets to the pipes, constant during the experiment. This arrangement was accomplished by two short branches between the hose and the control cock from the main, one of these being furnished with an india-rubber mouthpiece with a screw clip upon it, so that part of the water which passed the cock might be allowed to run to waste, the outer branch being connected with the lower end of a neutral glass tube, about 6 mm. in diameter and 30 inches long, having a bulb about 2 inches diameter near its lower extremity, and being closed by a similar bulb at its top.

The arrangement served as a delicate pressure gauge. The water entering at the lower end forced the air from the lower bulb into the upper, causing a pressure of about 30 inches of mercury. Any further rise increased the pressure by forcing the air in the tubes into the upper bulb, and by the weight of water in the tube. During an experiment the screw clip was continually adjusted, so as to keep the level of the water in the glass tube between the bulbs constant.

The Resistance Gauges.—Only the last 5 feet of the tube was used for measuring the resistance, the first 10 or 11 feet being allowed for the acquirement of a regular condition of flow. It was a matter of guessing that 10 feet would be sufficient for this, but since, compared with diameter, this length was double as great for the smaller tube, it was expected that any insufficiency would show itself in a greater irregularity of the results obtained with the larger tube, and as no such irregularity was noticed, it appears to have been sufficient.

At a distance of 5 feet near the ends of the pipe, two holes of about 1 millimetre were placed in each of the pipes for the purpose of gauging the pressures at these points of the pipes. As owing to the rapid motion of the water in the pipes past these holes, any burr or roughness caused in the pipe in piercing these holes would be apt to cause a disturbance in the pressure, it was very

important that this should be avoided. This was overcome by the simple expedient of drilling holes completely through the pipes, and then plugging the side on which the drill entered. Before drilling the holes short tubes had been soldered to the pipes, so that the holes communicated with these tubes; these tubes were then connected with the limbs of a syphon gauge by indiarubber pipes.

The gauges were about 30 inches long. Two were used, the one containing mercury, the other bisulphide of carbon. These gauges were constructed by bending a piece of glass tube into a V-form, so that the two limbs were parallel, and at about 1 inch apart. The tubes were fixed to stands with carefully graduated scales behind them, so that the height of the mercury or carbon in each limb could be read. It had been anticipated that readings taken in this way would be sufficient. But it turned out to be desirable to read variations of level of the smallness of $\frac{1}{1000}$th of an inch, or $\frac{1}{46}$th of a millimetre. A species of cathetometer was used. The water was carefully brought into connection with the fluid in the gauge, the india-rubber connections facilitating the removal of air.

Means adopted in measuring the Discharge—For this purpose a species of orifice or wire gauge was constructed, with a diaphragm consisting of many thicknesses of fine wire gauze about 2 inches from the bottom; a tube connected the bottom with a vertical glass tube, the height of water in which showed the pressure of water on the gauze; behind this tube was a scale divided so that the divisions were as the square roots of the height. Throughout the thin bottom were drilled six holes, one $\frac{1}{8}$ inch diameter, one $\frac{1}{4}$ inch, and four of $\frac{1}{2}$ inch. These holes were closed by corks, so that any one or any combination could be used.

The data obtained from these experiments have been given in § 174, but for fuller information reference must be made to the *Transaction of the Royal Society*, 1883, or Professor Reynolds's *Collected Papers.*

The Theory of Lubrication and its Applications.[1]

Mr. Beauchamp Tower, in a paper read before the Institution of Mechanical Engineers in November, 1883, gave the results of some interesting experiments on lubricated journals. In these reports Mr. Tower, making no attempt to formulate, states the results of experiments. These results, which were obtained under the ordinary conditions of lubrication, so far agree with the results of previous investigators as to show a want of any regularity. But one of the causes of this want of regularity, irregularity in the supply of the lubricant, appears to have occurred to Mr. Tower early in his investigations, and led him to include amongst his experiments the unusual circumstances of surfaces completely immersed in oil. The results so obtained show a great degree of regularity, but while making these experiments he was accidentally led to observe a phenomenon which, taken with the results of his experiments, amounts to a crucial proof that in these experiments with the oil bath, the surfaces were completely and continuously separated by a film of oil, this film being maintained by the motion of the journal, although the pressure in the oil at the crown was known, by actual measurement, to be as much as 625 pounds per square inch above the pressure of the oil bath.

These results obtained with the oil bath are very important. They show that with perfect lubrication a definite law of variation of the friction, with the pressure and velocity holds for a particular journal and brass. This strongly implies that the irregularity previously found was due to imperfect lubrication. Mr. Tower brought this out: Substituting for the bath an oily pad, pressed against the free part of the journal, and making it so slightly greasy that it was barely perceptible to the touch, he again found considerable regularity in the results; these, however, were very different from those of the bath. Then with intermediate lubrication, he obtained intermediate results, of which he says, "The

[1] "On the Theory of Lubrication and its Application to Mr. Beauchamp Tower's Experiment," by Professor Osborne Reynolds, F.R.S. *Collected Papers*, part ii. pp. 228–310, or *Transaction of the Royal Society*, 1886, pp. 156–239.

results, generally speaking, were so uncertain and irregular that they may be summed up in a few words. The friction depends on the quantity and uniform distribution of the oil, and may be anything between the oil-bath results and seizing, according to the perfection or imperfection of the lubrication."

§ **178. Viscosity of Lubricants.**—Oil is a viscous fluid, in that it can sustain a tangential resistance or sheer stress. Using the term *distortion* to express change of shape, apart from change of position, uniform expansion, or contraction, the viscosity of a fluid is defined as the shearing stress caused in the fluid while undergoing distortion, and the shearing stress divided by the rate of distortion is called the coefficient of distortion, or, commonly, the viscosity of the fluid. This is best considered by considering a mass of fluid bounded by two parallel planes at a distance a, and supposing the fluid between these planes to be in motion in a direction parallel to these surfaces, with a velocity which varies uniformly from zero at one of these surfaces to u at the other. Then the rate of distortion is

$$\frac{u}{a}$$

and the shearing stress on a plane parallel to the motion is expressed by

$$f = \mu\frac{u}{a}$$

μ being the coefficient of viscosity. The coefficient μ measures a physical property of the fluid which is independent of motion. This restriction is equivalent to restricting the applications of the equations of motion for a viscous fluid to the cases in which there are no eddies or sinuosities. It has been shown by Professor Reynolds that in parallel channels, so long as the product of the velocity, the width of channel, and the density of the fluid divided by μ, is less than a certain constant value, in a round tube this constant is 1400, or

$$\frac{\mathrm{D}v\sigma}{\mu} < 1400.$$

At a temperature of 50°, with a foot as the unit of length, for water,

$$\frac{\mu}{\sigma} = 0{\cdot}00001428$$

$$Dv < 0{\cdot}02$$

so that if D, the diameter of the channel, be 0·001 inch, v would have to be at least 240 feet per second for the resistance to vary other than the velocity.

As regards the slipping at the boundaries, Poiseuille's experiments, as well as those of Professor Reynolds, failed to show a trace of this, although f reached the value of 0·702 pound per square inch, so that within this limit it may be taken as proved that there is no slipping between any solid surface and

FIG. 253.

water. With other fluids, such as mercury in glass tubes, it is possible that the case may be different; but, as regards oils, the probability seems to be that the limit within which is no slipping will be much higher than with water.

§ **179. General View of the Action of Lubrication.**—Let AB and CD (Fig. 253) be perpendicular sections of the surfaces, CD being of limited but of great extent compared with the distance h between the surfaces, both surfaces being of unlimited length in a direction perpendicular to the paper.

CASE 1. *Parallel Surface in Relative Tangential Motion.*—In Fig. 254 the surface CD is supposed fixed, while AB moves to the left with a velocity U. Then, by the definition of viscosity (§ 178), there will be a tangential resistance,

$$F = \mu\frac{U}{h}$$

and the tangential motion of the fluid will vary uniformly from

U at AB to zero at CB. Thus if FG be taken to represent U, then PN will represent the velocity in the fluid at P. The slope of the line EG, therefore, may be taken to represent the force F, and the direction of the tangential force on either surfaces is the same as if EG were in tension. The sloping lines, therefore, represent the condition of motion stress throughout the film.

FIG. 254.

CASE 2. *Parallel Surfaces approaching with no Tangential Motion.*—The fluid has to be squeezed out between the surfaces, and since there is no motion at the surface, the horizontal velocity outward will be greatest halfway between the surfaces, nothing at O at the middle of CD, and greatest at the ends. If in a certain state of the motion (shown by the dotted line, Fig. 255) the

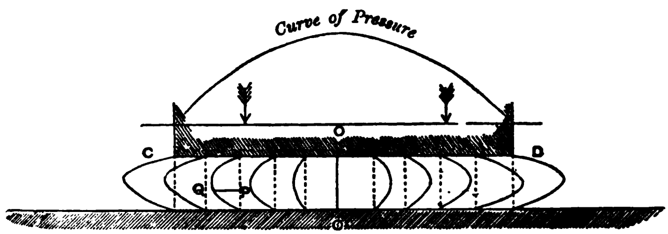


FIG. 255.

space between AB and CD be divided in ten equal parts by vertical dotted lines, and these lines be supposed to move with the fluid, then they will shortly after assume the positions of the curved lines, in which the areas included between each pair of curved lines is the same as in the dotted figure. In this case, as in Case 1, the distance QP will represent the motion at any point P, and the slope of the lines will represent the tangential force in the fluid, as if the lines were stretched with elastic strings. It

is at once seen from this that the fluid will be pulled towards the middle of CD by the viscosity, as though by the stretched elastic lines, and hence that the pressure will be greatest at O, and fall off towards the ends C and D, and would be approximately represented by the curve at the left of the figure.

CASE 3. *Parallel Surfaces approaching with Tangential Motion.*—The lines representing the motion in Case 1 and Case 2

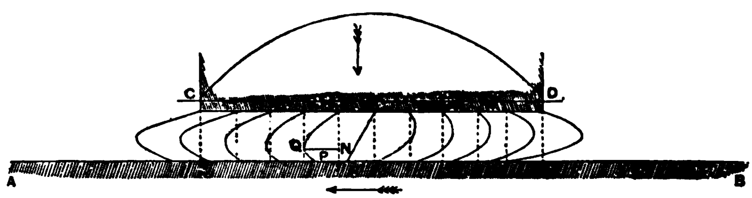


FIG. 256.

may be superimposed by adding the distance PQ in Fig. 255 to the distance PN in Fig. 254. The result will be as shown in Fig. 256, in which the lines represent, in the same way as before, the motions and stresses in the fluid where the surfaces are approaching with tangential motion. In this case the distribution of pressure over CD is nearly the same as in Case 2, and the mean

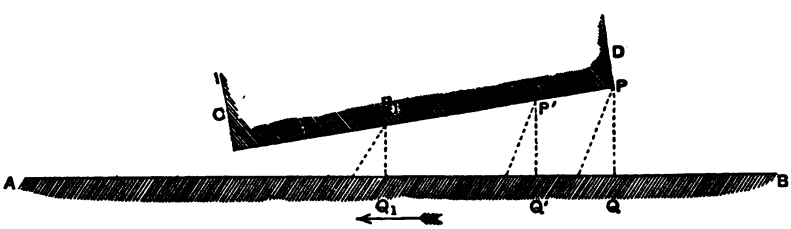


FIG. 257.

tangential force will be the same as in Case 1. The distribution of the friction over CD will, however, be different. This is shown by the inclination of the curves at the points where they meet the surface. Thus, on CD the slope is greater on the left and less on the right, which shows that the friction will be greater on the left and less on the right than in Case 1. On AB the slope is greater on the right and less on the left, as is also the friction.

CASE 4. *Surfaces inclined with Tangential Motion only.*—AB is in motion the same as in Case 1, and CD is inclined as in Fig. 257. The effect in this case will be nearly the same as in the compound movement (Case 3). For if corresponding to the uniform movement U of AB, the velocity of the fluid varied uniformly from the surface AB to CD, then the quantity carried across any section PQ would be

$$PQ \times \frac{U}{2}$$

and consequently would be proportional to PQ; but the quantities carried across all sections must be the same, as the surfaces

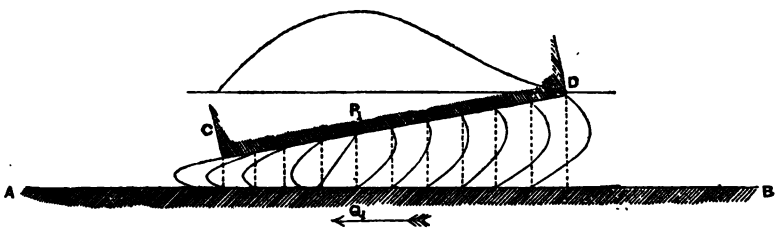


FIG. 258.

do not change their relative distances; therefore, there must be a general outflow from any vertical section PQ, P′Q′ (Fig. 257) given by

$$\frac{U}{2}(PQ - P'Q').$$

The outflow will take place to the right and left of the section of greatest pressure. Let this be P_1Q_1, then the flow past any other section PQ is

$$\frac{U}{2}(PQ - P_1Q_1)$$

as the right or left, according as PQ is to the right or left of P_1Q_1. Hence at this section the motion will be one of uniform variation, and to the right and left the lines showing the motion and friction will be nearly as in Fig. 256. This is shown in Fig. 258. This explanation is the explanation of continuous lubrication. The

pressure of the intervening film of fluid would cause a force tending to separate the surfaces. The mean line or resultant of this force could act through some point. This point does not necessarily coincide with P_1, the point of maximum pressure. In equilibrium of the surface AB, the point will be in the line of the resultant external force urging the surfaces together, otherwise the surface ACD would change its inclination. The resultant pressure must also be equal to the resultant external force perpendicular to AB (neglecting the obliquity of CD). If the surfaces were free to approach the pressure would adjust itself to the load, for the nearer the surfaces the greater would be the friction, and consequent pressure for the same velocity, so that the surfaces would approach, until the pressure balanced the load. As the distance between the surface diminished, the point of the resultant would change its position, and, therefore, to prevent an alteration of inclination, the surface CD must be constrained so that it could not turn round. It is to be noticed that continuous lubrication between plane surfaces can only take place with continuous motion in one direction, which is the direction of continuous inclination of the surfaces. With reciprocating motion, in order that there may be continuous lubrication, the surfaces must be other than plane.

Revolving Cylindrical Surface.—When the moving surface AB is cylindrical, and revolving about an axis, the general motion of the film will differ somewhat from what is the case with flat surfaces.

CASE 5. *Revolving Motion, CD, Flat and symmetrically placed.*—The surface velocity of AB may be expressed by U as before. The curves of motion found by the same method as in the previous cases are shown in Fig. 259. The curves to the right of GH, the shortest distance between the surfaces, will have the same character as those in Fig. 258, to the right of C, at which is also the shortest distance between the surfaces. On the left of GH the curves will be exactly similar to those on the right. This is because a uniformly varying motion would carry a quantity of fluid proportional to the thickness of the stratum from right to left, and thus while it would carry more fluid through the sections towards the right than it would carry across GH, necessitating an outward flow from the position P_1 in both directions, the same motion

would carry more fluid away from sections towards C than it would supply past GH, this necessitating an inward flow towards the position P_2. Since G is in the middle of CD, these two actions, though opposite, will be otherwise symmetrical, and

$$P_2G = GP_1.$$

From the convexity of the curves to the section at P_2, it appears that this section would be one of minimum pressure, just as P_1 is of maximum. Of course this is supposing the lubricant under sufficient pressure at C and D to allow of the pressure

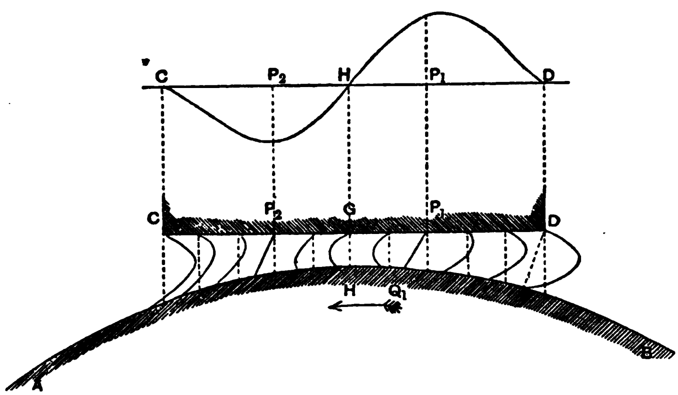


FIG. 259.

falling. The curve of pressure will be similar to that at the top of Fig. 259, in which C and D are points of equal pressure, P_1, H, P_2 the singular points in the curve.

Under such conditions the fluid pressure acts to separate the surfaces on the right, but as the pressure is negative on the left, the surface will be drawn together; so that the total effect is to produce a turning movement on the surface AB.

CASE 6.—The same as Case 5, except that G is not in the middle of CD. In this case the curves of motion will be symmetrical on each side of H at equal distances, as shown in Fig. 260. If C lies between H and P_2, the pressure will be altogether positive, as shown by the curve of Fig. 260—that is, will tend to separate the surface.

§ 180. The effect of a Limiting Supply of Lubricating Material.— In the cases already considered, C and D have been the actual limits of the upper surface. If the supply of lubricant is limited, C and D may be the extreme points to which the separating film reaches on the upper surface, which may be unlimited (Fig. 259).

CASE 7. *Supply of Lubricant limited.*—If the surface AB be supposed to have been covered with a film of oil, the oil adhering to the surface and moving with it, then the surface CD to have been brought up to a less distance than that occupied by the film

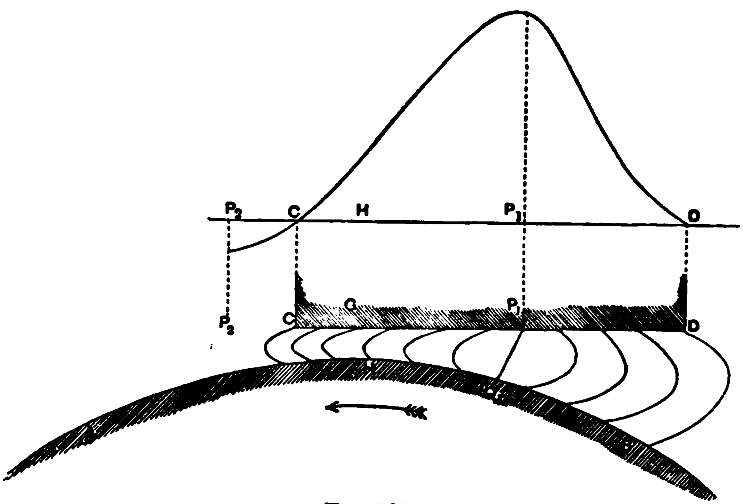


FIG. 260.

of oil, the oil will accumulate as it is brought up by the motion of AB, forming a pad between the surfaces, particularly on the side B. The thickness of the film as it leaves the side C is reduced until the whole surface AB is covered with a film of such thinness that as much leaves at C as is brought up to D, then the condition will be steady (Fig. 261). Putting b for the thickness of the film of oil outside the pad, the quantity of oil brought up to D by the motion of this film will be, per second, bU, and the quantity which passes the section P_1Q_1, across which the velocity varies uniformly, will be

$$\frac{P_1Q_1 \cdot U}{2}.$$

Therefore, since there is no further accumulation,

$$P_1Q_1 = 2b.$$

Also, since $GP_1 = GP_1$ (Fig. 259, Case 5),

$$P_2Q_2 = 2b.$$

And since the quantity which passes P_2Q_2 will not be sufficient to occupy the larger sections on the left, the fluid will not touch

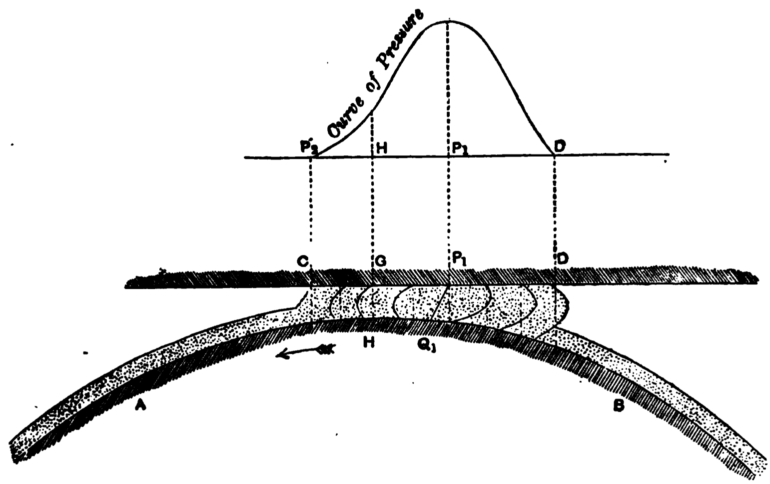


FIG. 261.

the upper surface to the left of P_2. The limit will therefore be at P_2, the fluid passing away with AB in a film of thickness b.

This is the ordinary case of partial lubrication :—AB, the surface of the journal, is covered with a film of oil; CD, the surface of the brass or bearing, is separated from AB by a pad of oil near H, the point of nearest approach. This pad is under pressure, which is a maximum at P_1, and slopes away to nothing at D and P_2, the extremities of the pad, as is shown by the curve above (Fig. 261).

§ **181. The Relation between Resistance, Load, and Speed for Limited Lubrication.**—In Case 7 a definite quantity of oil must be in the film round the journal or in the pad between the surfaces.

As the surfaces approach, the pad will increase and the film diminish, and *vice versâ*. The resistance increases with the length of the pad and with the diminution of the distance between the surfaces. The mean intensity of the pressure increases with the length of the pad, and inversely with the thickness of the film, but not in either case in the simple ratio. The total pressure, which is equal to the load, increases with the intensity of pressure and the length of the pad.

The definite expressions of these relations depend on certain integrations, which have not yet been effected. From the general relations it follows that an increase of load will diminish HG and P_1Q_1, and consequently the thickness of the film round the journal, and will increase the length of the pad. It will therefore increase the friction.

Thus, with a limited supply of oil, the friction will increase with the load in some ratio not precisely determined. Further, both the friction and the pressure increase in the direct ratio of the speed, provided the distance between the surfaces and the length of the pad remain constant; then, if the load remain constant, the thickness of the line must increase, and the length of the pad diminish with the speed; and both these effects will diminish friction in exactly the same ratio as the reduction of load diminishes friction. Thus if with a speed U, a load w, and friction F, a certain thickness of oil is maintained, the same will be maintained with a speed MU, a load MW, and the friction will be MF.

How far the increase of friction is to be attributed to the increased velocity, and how far to the increased load, is not yet shown in the theory for this case; but, as has been pointed out, if the load be altered from MW to W, the velocity remaining the same, the friction will be altered from MF in the direction of F. Therefore, with the load constant, it does appear from the theory that the friction will not increase on the first power of the velocity.

There is nothing in this theory contrary to the experience that, with very limited lubrication, the friction is proportional to the load and independent of velocity; while the theoretical conclusion

that the friction, with any particular load and speed, will depend on the supply of oil in the pad, is in strict accordance with Mr. Tower's conclusions and with the general disagreement of friction in different experiments.

§ 182. The Conditions of Equilibrium with Cylindrical Surfaces.— So far CB has been considered as a flat surface, in which case the equilibrium of CB requires that it should be so far constrained by external forces that it cannot either change its direction or move horizontally.

When AB is a cylindrical surface, having its axis parallel to that of AB, the only condition of constraint necessary for equilibrium is that CB shall not turn about its axis. This will appear on consideration of the following cases:—

CASE 8. *Surfaces cylindrical, and the Supply of Oil limited.*— Fig. 262 shows the surfaces AB and CD. The condition for the equilibrium of I, the centre of the brass, is that the resultant of the oil pressure on DC, together with friction, shall be in the direction OL, and the magnitude of this resultant shall be equal to the load.

As regards the magnitude of this resultant, it increases as HG diminishes to a certain limit, *i.e.* as the surfaces approach; so that in this respect equilibrium is obviously secured, and it is only the direction of the resultant pressure and friction that need be considered. Since the fluid film is in equilibrium under the forces exerted by the two opposite surfaces, those forces must be equal and opposite, so that it is only necessary to consider the force exerted by AB on the fluid.

From what has been already seen in Cases 6 and 7, it appears that the resultant line of pressure JM always lies on the right or *on* side of GH.[1] The resultant friction clearly acts to the left; so that if JM be taken to represent the resultant pressure, and MN the resultant friction, N is to the left of M, and JN, the resultant of pressure and friction, is to the left of JM. Taking LJ to represent the load, then LN will represent the resultant moving force

[1] "On" and "off" sides of the line of load are used by Mr. Tower to express respectively the sides of approach and succession, as D and C in Fig. 262, the arrow indicating the direction of rotation.

on CD, that is on I. Since H will move in the opposite direction to I, and since the direction of the resultant pressure moves in the same direction as H, the effect of a moving force, LN, on I will be to move N towards L until they coincide. Thus as long as JM is within the arc covered by the brass, a position of equilibrium is possible, and the equilibrium will be stable.

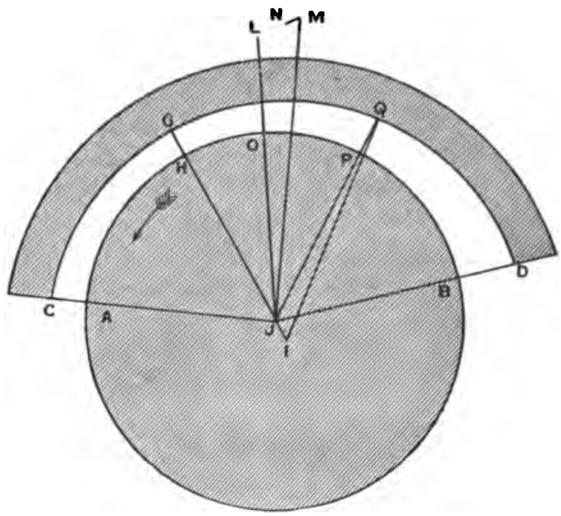


FIG. 262.

J is the axis of the journal AB.
I is the axis of the brass CD.
JL is the line in which the load acts.
O is the point in which JL meets AB.
$R = JP$.
$R + a = IQ$.
$h = PQ$.
$h_0 = HG$.

So far the condition of equilibrium shows that H will be on the left or off side of the line of load, and this holds whether the supply of oil is abundant or limited; but while with a very limited supply of oil, *i.e.* a very short oil pad, H must always be in the immediate neighbourhood of O, this is by no means the case as the length of the oil pad increases.

CASE 9. *Cylindrical Surfaces in Oil Bath.*—If the supply of oil

is sufficient, the oil film or pad between the surfaces will extend continuously from the extremities of the brass, unless first extension would cause negative pressure, which might lead to discontinuity. In this case the conditions of equilibrium determine the position of H.

The conditions of equilibrium are as before—

(1) That the horizontal component of the oil pressure on the brass shall balance the horizontal component of the friction.

(2) That the vertical components of the pressure and friction shall balance the load.

Taking the surface of the brass, as is usual, to embrace nearly half the circumference of the journal, and, to commence with, supposing the brass to be unloaded, the movement of H may be traced as the load increases. When there is no load, the conditions of equilibrium are satisfied if the position of H is such that the vertical components of pressure and friction are each zero, and the horizontal components are equal and opposite.

This will be when H is at O (Fig. 262); for then, as has been shown, Case 5, the pressure on the left of H will be negative, and will be exactly equal to the pressure at corresponding points on the right, so that the vertical components left and right balance each other. On the other hand, the horizontal components of the pressure on the left and right will both act on the brass to the right, and as these will increase as the surfaces approach, the distance JI must be exactly such that these components balance the resultant friction, which by symmetry will be horizontal and acting on the left.

It thus appears that when the brass is unloaded its point of nearest approach will be its middle point. This position, together with the curves of pressure, are shown in Fig. 263. As the load increases, the positive vertical components on the right of GH must overbalance the negative components on the left. This requires that H should be to the left of O. It is also necessary that the horizontal components of pressure and friction should balance. These two conditions determine the position of H and the value of JI.

As the load increases it appears from the exact equations that

OH reaches a maximum value, which passes H nearly, but not quite, at the left extremity of the brass, but leaves JI still small as compared with GH. For a further increase of load, H moves back again to O. In this condition the load has become so great that the friction, which remains nearly constant, is so small by comparison that it may be neglected, and the condition of

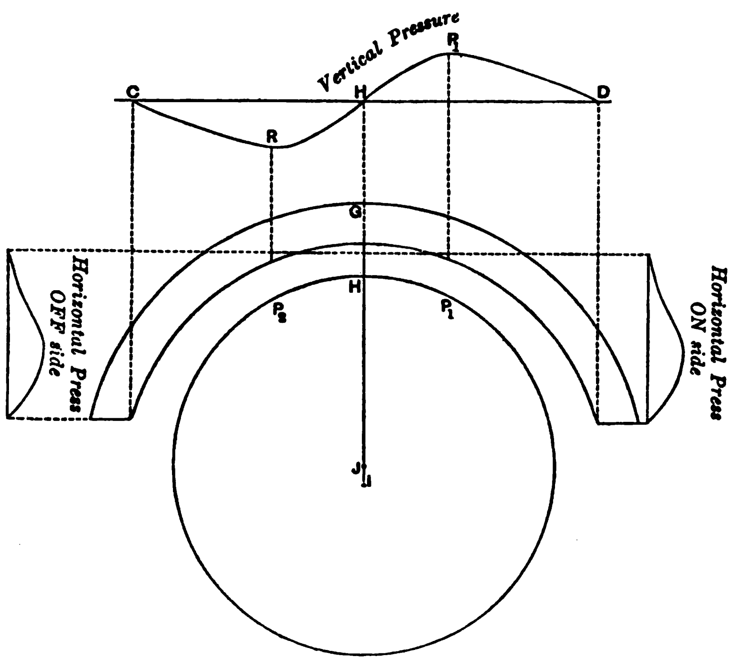


FIG. 263.

equilibrium is that the horizontal component of the pressure is zero, and the vertical component equal to the load. H continues to recede as the load increases. But when HC becomes greater than HP_2, the pressure between P_2 and C would become negative if the condition did not break down by discontinuity in the oil, which is sure to occur when the pressure falls below that of zero, and then the condition becomes the same as that of a limited supply of oil. This is important, as it shows that with extreme

loads the oil bath comes to be practically the same as that of a limited supply of oil, and hence that the extreme load which the brass would carry would be the same in both cases—as Mr. Tower has shown it to be. In all Mr. Tower's experiments with the oil bath it appears that the conditions were such that as the load

FIG. 264. FIG. 265.

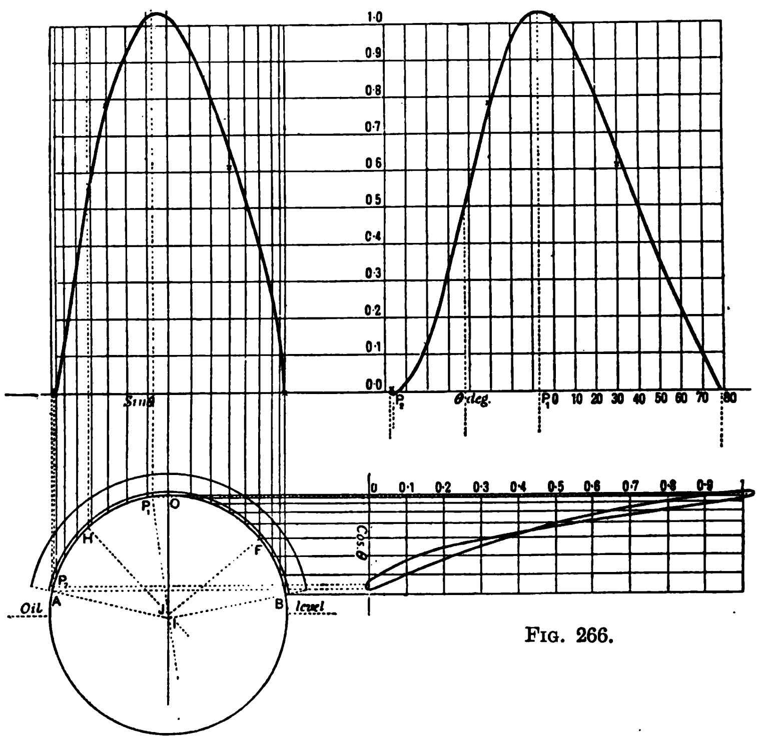


FIG. 266.

increased H was in retreat from C towards O, and that, except in the extreme cases, P_2 had not come up to C.

Figs. 264, 265, and 266 show the exact curves of pressure as calculated by the exact method which Professor Reynolds has adopted, for circumstances corresponding very closely to one of Mr. Tower's experiments, in which he actually measured the

pressures of oil at three points in the film. These measured pressures are shown by the crosses.

The results of the calculations for this experiment are to show, what could not be measured, that in Mr. Tower's experiment the difference in the radii of the brass and journal at 70°, and a load of 100 pounds per square inch, was

$$a = 0{\cdot}00077$$
$$GH = 0{\cdot}000375$$
$$\text{The angle } OJH = 48^\circ.$$

§ **183. The Wear and Heating of Bearings.**—Before the journal starts, the effect of the load will have brought the brass into contact with the journal at O (Fig. 262). At starting, the surfaces will be in contact, and the initial friction will be between solid surfaces, causing some abrasion. After motion commences, the surfaces gradually separate as the velocity increases, more particularly in the case of the oil bath, in which case at starting the friction will be much the same as with a limited supply of oil. As the speed increases according to the load, GH approaches, according to the supply of oil, to A, and varies but slightly with any further increase of speed; so that the resistance becomes more nearly proportional to the speed and less affected by load. When the condition of steady lubrication has been attained, if the surfaces are completely separated by oil, there should be no wear. But if there is wear, as it appears from one cause or another there generally is, it would take place most rapidly where the surfaces are nearest—that is, at GH on the off side of O. Thus, while the motion is in one direction, the tendency to wear the surfaces to a fit would be confined to the off side of O. This appears to offer a very simple and well-founded explanation of the important and common circumstance that new surfaces do not behave so well as old ones; and of the circumstances, observed by Mr. Tower, that in the case of the oil bath, running the journal in one direction does not prepare the brass for carrying a load where the journal is run in the opposite direction. This explanation, however, depends on the effect of misfit in the journal and brass which has yet to be considered.

§ 184. Approximately Cylindrical Surfaces of Limited Length in the Direction of the Axis of Rotation.—Nothing has so far been said of any possible motion of the fluid perpendicular to the direction of motion and parallel to the axis of the journal, it having been assumed that the surfaces were truly cylindrical and of unlimited length in the direction of their axes, and in such case there would be no flow.

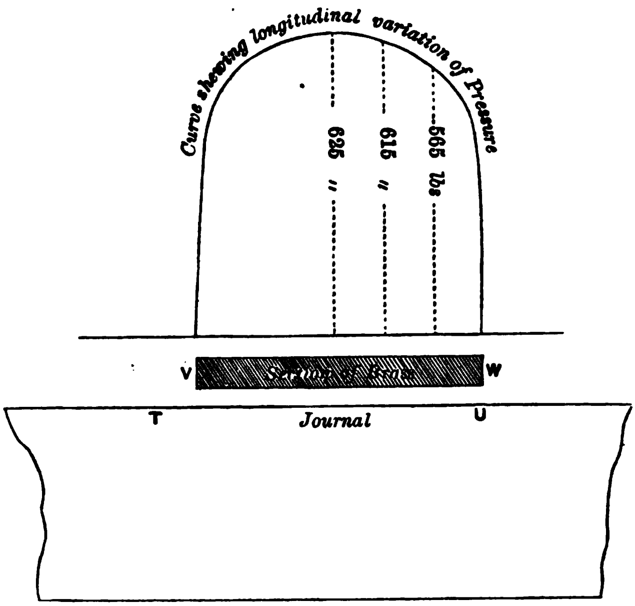


FIG. 267.

But in practice brasses are necessarily of limited length, so that the oil can escape from the ends of the brass. Such escape will obviously prevent the pressure of the film of oil from reaching its full height for some distance from the ends of the brass and cause it to fall to nothing at the extreme ends.

This was shown by Mr. Tower, who measured the pressure at several points along the brass in the line through O, and found it to follow a curve similar to that shown in Fig. 267, which corresponds to what might be expected from escape at the free ends.

If the surfaces are not strictly parallel in the directions TU and VW, the pressure will be greatest in the narrowest parts, causing axial flow from these into the broadest spaces. Hence, if the surfaces were considerably irregular, the lubricant would, by escaping into the broader spaces, allow the brass to approach and eventually to touch the journal at the narrowest spaces, and this would be particularly the case near the ends.

As a matter of fact, the general fit of new surfaces can also be approximate; and how near the approximation is, is a matter of the time and skill spent on preparing and, it is called, bedding them. This seems a small matter, but not when compared with the mean width of the interval between the brass and the journal, which is less than $\frac{1}{1000}$th of an inch.

The section of a new brass and journal taken at GH (Fig. 263) will therefore be, if sufficiently magnified, as shown in Fig. 268, the

FIG. 268.

thickness of the film, instead of being, say, of $\frac{3}{10000}$ths of an inch, varies from 0 to $\frac{5}{10000}$ths, and is less at the ends than at the middle.

In this condition the wear will be at the points of contact, which will be in the neighbourhood of GH on the off side of O (Fig. 262), so that if the journal runs in one direction only, the surfaces in the neighbourhood of GH (on the *off* side) will be gradually worn to a fit, during which the friction will be great and attended with heating, more or less, according to the rate of wear and the obstruction to the escape of heat. So long, however, as the journal runs in one direction only, GH will be on one side (the *off* side) of O, and the wear will be altogether or mainly on this side, according to the distance of H from O. In the mean time the brass on the *on* side is not similarly worn, so that if the motion of the journal is reversed, and the point H transferred to the late *on* side, the wear will have to be gone through again. That this is the true explanation is confirmed if, as seems from Mr. Tower's

report, the heating effect on first reversing the journal was much more evident in the case of the oil bath. For when the supply of oil is short, HG will be very small, and H will be close to O; so that the wearing area will probably extend to both sides of O, and thus the brass be partially, if not altogether, prepared for running in the opposite direction. When the supply of oil is complete, however, as has been shown, H is 50° or 60° from O, unless the load is in excess, so that the wear in the neighbourhood of H on the one side of O would not extend to a point 100° or 120° over to the other side.

Even in the case of a perfectly smooth brass, the running of the journal under a sufficient load in one direction should, supposing some wear, according to the theory, render the brass less well able to carry the load when running in the opposite direction; for, as has already appeared, the pressure between the journal and brass depends on the radius of curvation of the brass on the *on* side being greater than that of the journal. If, then, the effect of wear is to diminish the radius of the brass on the *off* side, so that when the motion is reversed the radius of the new *on* is equal to or less than that of the journal, while the radius of the new *off* side is greater, the one pressure would not rise. And this is the effect of wear; for, as Professor Reynolds proved by mathematical analysis, the effect of the oil pressure is to increase the radius of curvature of the brass, and as the centre of wear is well on the *off* side, the effect of sufficient wear will be to bring the radius on this side, while the pressure being removed, the brass on this side may resume a radius even less than that of the journal.

Professor Reynolds gives the mathematical analysis on pp. 258–310 in his Collected Papers.

EXAMPLES

THE Author wishes to point out that, during the eight years he was Professor of Applied Mechanics at the Royal Naval College, the Tutorial Work was a very important part of the course for Naval Engineers and Constructors. The following examples, and those in "Mechanism," were compiled for the use of those classes; and they, now, form part of the Tutorial Work in the Engineering Classes of the Manchester University.

CHAPTER I

1. Calculate, in tons per minute, the quantity of water which will enter a large tank through an orifice in the bottom, $1\frac{1}{2}$ feet square, the depth of the orifice below still water being originally 9 feet, and the coefficient of contraction being the same as a sharp-edged orifice. If the tank be 10 feet square, and the top originally was 5 feet above water level, find how long it will take to sink the tank.

Ans.—56·3 tons per minute.
14·9 seconds.

2. In making an experiment to find the different coefficients of orifices, a jet of water, issuing horizontal from an orifice 1 inch diameter, under a head of $5\frac{1}{2}$ feet, fell 2 feet 10 inches at distance 8 feet from the orifice, the discharge being $2\frac{1}{2}$ pounds per minute. Deduce the four coefficients.

Ans.—$C_v = 0·97$.
$C = 0·622$.
$C_c = 0·644$.
$C_r = 0·061$.

3. A vessel, containing water and having an orifice of 1 inch diameter in one side, which is at first plugged, is suspended so that the displacing force can be accurately measured. On the removal of the plug, the horizontal force required to keep the vessel in place—applied opposite the orifice—was 3·6 pounds. By the use of a measuring tank, the discharge was found to be

31 gallons per minute, the level of the water in the vessel being maintained at a constant height of 9 feet above the orifice. Determine the coefficient of velocity, contraction and discharge.

Ans.—$C = 0.92.$
$C_c = 0.677.$
$C = 0.62.$

4. In a rectangular notch, with two end contractions, the breadth was 8 feet. When the depth was 1·06 feet, the discharge in cubic feet per second was 28·4; when the depth was 1·65 feet, it was 54·5. Find the two constants in Francis formula.

Ans.—$\beta = 3.3.$
$x = 0.066.$

5. A lock on the Manchester Ship Canal is 600 feet long by 65 feet 6 inches wide, and the drop is 16 feet 6 inches. For filling, two culverts are provided 12 feet high and 6 feet wide. Find the time of filling.

Ans.—5 minutes nearly.

6. In a rectangular reservoir, the time taken to discharge a quantity of water equal to that in the reservoir, the head remaining constant, is t. Show that the time taken to empty the reservoir is $2t$. If the reservoir be wedge shaped (V) show that the time is $1\frac{1}{3}t$.

7. A turbine is supplied with water having an available fall of 50 feet. The flow, as gauged by a rectangular notch, and the height over the sill is 9 inches, and width 13 inches. The turbine developes $9\frac{1}{4}$ horse-power. Find its efficiency.

Ans.—$\eta = 0.608.$

8. At Salford Lock, the dimensions of the sluices, when full open, are (Fig. 26) $ab = 13$ feet, $ad = 20$ feet, $ac = 26$ feet. The width of the gate is 30 feet, and the coefficient of discharge is 0·597. Find the discharge in tons per hour.

Ans.—985,000.

CHAPTER II

9. It is found that the loss of head in a pipe 500 feet long, 4 inches diameter, when the velocity was 1·234 feet per second was 2·817 feet. Calculate the value of f, and compare it with D'Arcy's formula. If there is a difference, what is the probable cause?

Ans.—$f = 0.019.$

10. In the example in § 21, calculate the values of f, assuming

$$h' = f\frac{4l}{d} \cdot \frac{v^2}{2g}$$

and compare with D'Arcy's formula.

11. A 3-inch horizontal gradually contracts to a 1-inch mouthpiece, whence the water emerges into the air, the discharge being 600 pounds per minute. The pipe is 200 feet long, and receives water from an open tank, and $f = 0{\cdot}006$. Find the pressure at the nozzle end of the pipe, the velocity efflux, and the necessary height of tank.

Ans.—h = 150 feet.

12. It is desired to pump 10 cubic feet per minute through a nozzle at the end of a hose pipe of 3-inch diameter, and 100 yards long, and to deliver it at a height of 100 feet. If $f = 0{\cdot}036$, the combined efficiency of engine and pump is 0·6, the actual lift is $\frac{3}{4}$ the theoretical, find the proper diameter of nozzle, and the I.H.-P. of the engine.

Ans.— d = 0·575 feet.
Pumping horse-power = 3·12.
Indicated „ „ = 5·2.

13. A hydraulic motor is driven by means of an accumulator at a pressure of 750 pounds per square inch. The supply pipe is 900 feet long and 4 inches diameter, and $f = 0{\cdot}0075$. Find the maximum motor horse-power, and the velocity of flow under these conditions.

Ans.— 242·4 H.-P.
21·2 feet per second.

14. An accumulator works under a pressure of 150 pounds per square inch, d = 4 inches, l = 5280 feet, $f = 0{\cdot}00625$. If 75 H.-P. be received by the motor, find the horse-power sent out from the accumulator, and the efficiency of transmission.

Ans.— 84 and 235 H.-P.
Efficiency 0·892 and 0·32 H.-P.

15. In Q. 14, plot a curve of efficiency and horse-power delivered, on an H_1 base.

16. A river is 1000 feet wide at the surface of the water, the sides slope at 45 degrees, and the depth is 20 feet. Find the discharge in cubic feet per second with a fall of 2 feet to the mile, if $f = 0{\cdot}0075$.

Ans.—154,000.

17. A 9-inch drain pipe is laid at a slope of 1 in 150, and the value of c is 107 ($v = c\sqrt{mi}$). Find the angle subtended at the centre of the water line, and the velocity of flow when the discharge is $\frac{1}{2}$ cubic feet per second.

Ans.—$v = 3{\cdot}38$ feet per second.

18. A pipe 2 inches diameter is suddenly enlarged to 3 inches diameter. If it discharge 100 gallons per minute the water flowing from the small pipe into the large one, find the total loss of head and the gain of pressure at the sudden enlargement.

Ans.— Loss of head = $8\frac{1}{2}$ inches.
Gain of pressure = 14 inches.

19. If, in Q18, the motion be reversed, find the loss of head, and change of pressure head consequent on the sudden contraction, assuming a co-efficient of contraction of 0·66.

Ans.— Loss of head = $7\frac{1}{2}$ inches.
Diminution of pressure = $29\frac{3}{4}$ inches.

20. The injection orifices of a jet condenser of a marine engine are five feet below the surface of the sea, and the vacuum is 27″ Hg; with what velocity will the water enter the condenser, supposing three-fourths of the head lost by frictional resistances? Also, find the co-efficient of velocity and resistance, and the effective area of the orifices to deliver 100,000 gallons an hour.

Ans.—$v = 23{\cdot}6$ feet per second.
$c_r = \frac{1}{2}$.
$e_r = 3$.
area = 27 square inches.

21. In an experiment on the head necessary to cause flow through a condenser, the arrangement was: the water entered the water head at the bottom, passed through the lower nest of tubes in the second water head, through the upper nest of tubes, and through the outlet pipe.

The pressure was observed at the inlet pipe A, and at the upper water head B. The sectional area at A = 0·196 square foot, at B = 1·76 × 0·54 = 0·95 square foot. The number of tubes in the lower nest was 353, and in the upper, 326. The internal diameter of the tubes in the upper and lower nest was 0·65 inches. The length of all the tubes was 6 feet 2 inches.

When the circulating water was 1·21 cubic feet per second, the observed difference of pressure (by gauges) at A and B was 6·5 feet; when the circulating water was 1·72 cubic feet per second, the difference of level was 4·66 feet. Find the total head necessary to overcome frictional resistance in each case, and the coefficient of resistance referred to A. Find also the head due to skin friction alone taking f from D'Arcy's formula for clean pipes. If there is a considerable difference explain the cause.

Ans.—2·4 feet; 4·15 feet.

22. In a jet propeller (§ 41), the falling head is 26·25 feet, the suction head is 9·84 feet, and the barometric head is 33·9 feet. Moreover, the velocity in the suction main, and after mixing, were each 16·4 feet per second. Find the pressure in the mixing chamber, the velocity in the falling main, the ratio of the quantity of water pumping up to that coming from the tank, and the efficiency.

Ans.—Pressure = 8·65 pounds per square inch.
v_1 = 50·9 feet per second.
$\frac{Q_2}{Q_1}$ = 0·254.
y = 0·095.

23. A locomotive, running at forty miles an hour, scoops up water from a trough. The tank is 8 feet above the mouth of the scoop; and the delivery pipe has an area of 50 square inches. If half the available head is wasted at entrance, find the velocity at which the water is delivered into the tank, and the number of pounds lifted in a trench 500 yards long. What, under these circumstances, is the increased resistance; and what is the minimum speed of train at which the tank can be filled.

Ans.—velocity of delivery = 58·6 feet per second.
number of gallons = 752 pounds per second.
increased resistance = 1370 pounds.
minimum speed = 21·8 miles per hour.

CHAPTER III

24. A differential accumulator, loaded to 1400 pounds per square inch, has a 5-inch spindle and ½-inch bush, and supplies a riveter ram of 9-inch diameter, and 6-inch stroke, through 30 feet of 1-inch piping. Find the equivalent mass at the ram (1) accumulator cage remaining stationary, (2) accumulator cage descending, the pumps not working into accumulator during the operation.

Ans.—30 tons and 322 tons.

25. A single-acting hydraulic engine is connected with a crank and flywheel —the latter rotating uniformly. The engine is supplied by a pipe, 100 feet long and 6 inches diameter, from a tank 80 feet above the cylinder. The diameter of the cylinder is 18 inches, and the stroke 24 inches. The revolutions per minute are 15, and $f = 0·0075$. Find the indicated head at each quarter stroke, and also the efficiency. Plot the curves.

Ans.—At ends of stroke, 11 feet and 149 feet.
At centre of stroke, 61·4 feet.

26. In Q. 25, if the mean effective head is reduced to 40 feet by throttling the valve—the speed remaining the same—by how much must the valve be opened (§ 36), and find the indicator heads at the ends and mid-point of stroke. Plot the curves.

Ans.—0·338.

27. In Q. 25, assuming that the motion is harmonic, as before, find the maximum speed at which the engine will run when light, with valves full open; and state, by calculating the inertia heads, at the ends of the stroke, whether motion is possible.

Ans.—Motion impossible.

28. In Q. 25—omitting the speed—find the speed which will cause separation at the commencement of stroke. Find the value of F, the amount the valve is opened (§ 36), and the average effective head.

Ans.— F = 5·1.
Valve opened = 0·45.
Average head = 41·9 feet.

29. A hydraulic machine is supplied with water through a pipe 600 feet long, at a pressure which at the motor end of the pipe is 750 pounds per square inch. The machine is required to work at a maximum of 10 horse-power. Assume a coefficient of friction of 0·017, and a coefficient of resistance of 10, due to other causes than surface friction, and calculate the least diameter of pipe necessary to carry the water. What will be the loss of head when the machine works at 5 horse-power.

30. It is found that a direct-acting ram lift, having a diameter of 6 inches, can lift one ton at a uniform rate of 60 feet per minute when supplied by a pipe 2 inches diameter and 100 feet long, the accumulator pressure being 130 pounds per square inch. If the coefficient of skin friction in the pipe be 0·01, find the coefficient of resistance—other than that due to skin friction in the pipe—referred to the velocity of the ram. If this coefficient remain constant for all speeds of the ram, find the necessary diameter of the supply pipe in order that the speed of the ram should be increased 20 per cent.—the remaining data remaining the same.

31. The piston of a double-acting hydraulic motor, making 50 revolutions per minute, moves with simple harmonic motion. The cylinder is supplied by a pipe 4 inches diameter, 20 feet long, from an accumulator in which the pressure is maintained at 200 pounds per square inch. The cylinder is 8 inches diameter and 12-inch stroke, and the coefficient of friction in the pipe is 0·015. Draw a curve, giving the effective pressure on the piston, neglecting the mass of the reciprocating parts, and the variable mass of water in the cylinder.

32. In a hydraulic intensifier, the larger piston is 10 inches diameter, and the smaller 5 inches. The larger side is supplied with water at 700 pounds per square inch from pumps placed so near that the effect of the connecting pipe may be neglected. The small side delivers water, through a 3-inch pipe 20 feet long, to a hydraulic press 20 inches diameter. The stroke of the intensifier is 5 feet, the mass of the pistons and connecting rod is 1000 pounds, and the velocity during a stroke was observed to follow the simple harmonic law, the time of making the stroke being $2\frac{1}{2}$ seconds. Neglecting the mass of the water in the cylinder and intensifier, draw a diagram showing the available pressure on the piston of the press ($f = 0{\cdot}01$).

CHAPTER IV

33. A pump valve of brass has a specific gravity of $8\frac{1}{2}$ with a lift of 0·1 foot, the stroke of the piston being 4 feet, the head of water 40 feet, and the ratio of the full valve area to the piston area $\frac{1}{4}$. If the valve is neither assisted, nor meets with any resistance to closing, find the time it will take to close the valve, and the slip due to the gradual closing.

Ans.—0·084 seconds.
0·141 slip.

34. Suppose that when a reciprocating pump is working at such a speed as to deliver 1000 gallons of water per minute, the loss of energy due to pump passages is 35 foot-pounds per pound, and per foot of vertical pipe is $\frac{1}{20}$ foot-pounds per pound. Find the efficiency when the head is 20 feet and 200 feet.

Ans.—0·36 and 0·82.

35. A single-acting suction, and lift-plunger pump, the piston diameter being 12 inches, the stroke 2 feet, the revolutions 30 per minute, the height of suction 20 feet, the length of suction pipe 33 feet, and the barometric head 34 feet. Find the minimum diameter of suction pipe in order that separation is prevented at the commencement of the stroke.

Ans.—10·2 inches.

36. In Q. 31, suppose the suction pipe is provided with an air-chamber of large capacity, in which the water level is $1\frac{1}{2}$ feet below piston level, and that the diameter of the suction pipe, and the connecting pipe cylinder is 8 inches. Find approximately the velocity in the suction pipe, the pressure in the air-chamber when at rest and when motion takes place, and the absolute head on the plunger at the commencement of the stroke—the length of pipe to the chamber being 30 feet, and to the cylinder 3 feet.

Ans.—$v = 2{\cdot}26$ feet per second.

37. In a single-acting force pump, the diameter of the plunger is 4 inches, the stroke 6 inches, the length of suction pipe 63 feet, the diameter of suction pipe $2\frac{5}{8}$ inches, and the suction head 0·07 feet. When going at 10 revolutions per minute, it is found that the average loss of head per stroke between the suction tank and plunger cylinder is 0·23 feet. Assuming that the frictional losses vary as the square of the speed, find the absolute heads on the suction side of the plunger at the two ends, and middle of the stroke—the revolutions being 50 per minute, and the barometric head 34 feet of water. Draw the diagram of pressure on the plunger—simple harmonic motion being assumed, and without an air-chamber.

Ans.—7·93, 59·93, 25·31 feet.

38. In Q. 33, find the speed at which the pump has to run so as to cause separation at the commencement of stroke.

Ans.—57·2.

39. A double-acting delivery pump has a plunger of $8\frac{3}{4}$ inches diameter, and a stroke of 30 inches. It is found that, when making 12 double strokes per minute, 150 gallons of water are actually pumped up per minute to a height of 150 feet, through 800 feet of $7\frac{1}{2}$-inch pipe. The efficiency of pump and delivery pipe was found to be 0·80. Find the slip of the pump, and the coefficient of resistance of the valves, valve passages, etc., being referred to the velocity in the pipe—the coefficient of pipe friction of delivery pipe being 0·1. Hence obtain, in the absence of an air-chamber, the indicated head on the plunger at the two ends and mid-point of stroke. Draw the curve of pressure.

Ans.—Heads, 216·6, 207, 83·4.

40. In Q. 35, suppose an air-chamber of very large capacity is placed on the delivery pipe 10 feet from the pump, the mean level of water in chamber being 3 feet above the pump barrel. Obtain, approximately, the head in the air-chamber, and indicated head on the plunger in the two extreme and mean positions of the stroke.

Ans.—152·15, 205·03, 150·5.

41. A hydraulic engine is supplied with water under an effective head of 450 feet, and is transmitted through 2000 feet of 5-inch pipe. The cylinder is $6\frac{1}{4}$ inches diameter and its stroke is $2\frac{1}{2}$ feet. The engine works tandem with a pump which pumps 150 gallons per minute a height of 150 feet through 800 feet of $1\frac{1}{2}$-inch pipe. The diameter of the pump barrel is $8\frac{3}{4}$ inches. Find the net efficiency, both engine and pump being double acting, and slip being neglected.

Assuming a coefficient of resistance in the delivery pipes to be twelve times that due to friction alone, and in the suction pipe 36 times, and that the piston actuates a uniformly rotating crank, from the effective head at the beginning, middle, and end of the stroke on (1) the driving piston, (2) on the pump piston.

Ans.— Driving piston, 274, 329·6, 626.
Pump plunger, 211·2, 217·2, 88·8.

42. Fig. 269 shows one tank resting on the top of a second, both in communication with atmosphere. A bucket pump is arranged as shown to pump from the lower tank, through the suction valve A. Communication between the pump band, and the top tank may also be made by means of the valve V. When the valve V is opened, the head in the pump barrel is considered sufficient to keep the valve A closed. Is this statement necessarily true for all speeds of the pump?

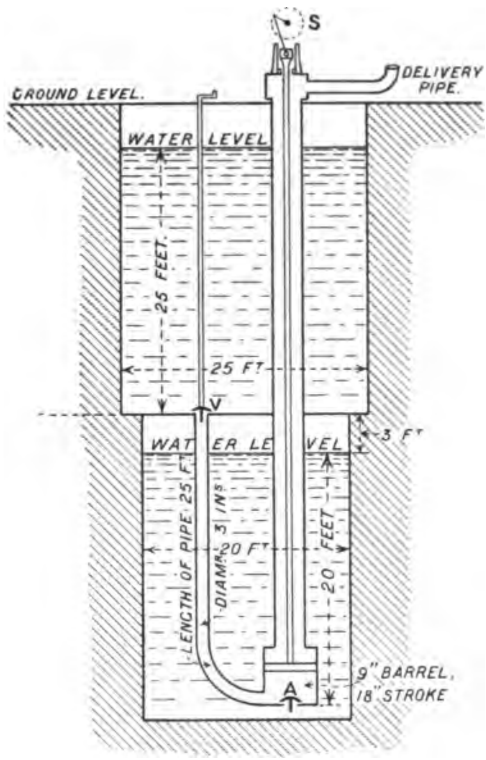


FIG. 269.

If not, at what number of revolutions of the crank shaft S, the valve A will just be on the point of opening when V is wide open, (1) pump piston on the dead centre, (2) pump piston at mid point. Assume that coefficient of friction of pipe = 0·015; motion simple harmonic; valve 20 lbs. weight and 6" diameter.

Ans.—Neglecting weight of valve, 22·1, 24·5.
With weight of valve, 22·7, 25·2.

43. A pump of the duplex kind, in which the steam piston is connected directly to the pump piston, for instance the Worthington direct-acting pump (§ 116),

works against a head of 100 feet of water, the head being applied by the column of water in the delivery pipe which is vertical, and 4 inches diameter. The pump piston and the steam piston are each 4 inches diameter. The difference between the forward and backward steam pressure is constant, and is equal to 45·2 pounds per square inch, stroke 2 feet. Assuming the steam pressure to act to the end of the stroke find the velocity of the pistons at the end of the stroke, and the time taken to perform the stroke. Coefficient of hydraulic friction = 0·015. Neglect the mass of the pistons, and the variable quantity of water in the cylinder (§ 105).

Ans.—$v = 2·7$ feet per second.
$t = 1·76$ seconds.

CHAPTER V

44. A stream, delivering 3000 gallons of water per minute with a velocity of 40 feet per second, by impinging on vanes is caused to be freely deviated through an angle of 10°, the velocity being diminished to 35 feet per second. Determine the pressure on the vanes due. If the vanes be moving in the direction of that pressure, find their velocity, and deduce the useful horse-power.

Ans.— Force = 127·8 pressure.
Useful horse-power = 5·31.
Efficiency = 0·234.

45. In a parallel flow reaction turbine, the guide-angle is 30° and the vanes are radial. Find the proper speed of wheel, the velocity of flow, the pressure head and kinetic head in the chamber—all expressed in terms of v_2—and the angle α, the whirling velocity at exit being zero, the velocity of flow being constant.

Ans.—$\alpha = 20.$
$\eta = 0·938.$

46. In a parallel flow reaction turbine, the angles β and γ (§ 131) are 90 degrees and 20 degrees respectively. Find the proper speed of the wheel, the velocity of flow, the pressure head in supply chamber (all expressed in terms of V_2) and the angle α—the whirling velocity at exit being zero, and the velocity of flow constant.

Ans.—$u_2 = 0·685V_2.$
$f_3 = 0·249V_2.$
$\frac{p_2}{\sigma h_2} = 0·469.$
$\alpha = 20°.$
$\eta = 0·9.$

47. In an inward flow reaction turbine, u, the guide angle is 30 degrees, and the inlet vane angle is 75 degrees—the vane being inclined backwards relative to the direction of motion. The head due to flow is 8 per cent. of the head, and may be taken to be constant. Find the velocity of the wheel, and the whirling velocity at inlet; and, assuming $p_3 = 0$, find the whirling velocity at exit, and also the vane angles. Deduce the efficiency and the pressure head in chamber.

$$\textit{Ans.}\text{—}u_2 = 0{\cdot}414V_2.$$
$$w_2 = 0{\cdot}49V_2.$$
$$\frac{p_3}{\sigma h_2} = 0{\cdot}68.$$
$$\gamma = 20\tfrac{3}{4}°.$$
$$\eta = 0{\cdot}628.$$

48. In an out flow reaction turbine, $n = \sqrt{2}$, crowns parallel, the angles, α, β, γ are 30 degrees, 80 degrees, and 15 degrees, respectively, and head lost in flow is 10 per cent. of the total head. Find the velocity of the wheel, the velocity of whirl at inlet and outlet, and the pressure head at inlet and outlet.

$$\textit{Ans.}\text{—}u_2 = 0{\cdot}696\ V_2.$$
$$w_2 = 0{\cdot}775\ V_2.$$
$$w_3 = 0{\cdot}195\ V_2.$$
$$\frac{p_2}{\sigma h_2} = 2h_2.$$
$$\frac{p_3}{\sigma h_2} = 0{\cdot}6.$$

49. A turbine is found to work efficiently under the following conditions: head, 25 feet; supply in cubic feet per second, 570; revolutions, 50 per minute; diameter, 12 feet. Find the diameter, and revolutions of a similar wheel to work under a head of 140 feet with a discharge of 8·6 cubic feet.

Ans.—Diameter 3·06 feet
N = 465 revolutions per minute.

[**50.** In a parallel flow reaction turbine, the loss in flow is 8 per cent., and the other hydraulic losses are 12 per cent. of the total head. The head is 25 feet, the discharge 570 cubic feet per second, and the pressure head in supply pipe is four-tenths the total. Design the vanes, and calculate the turning moment on the wheel. If the revolutions are 50 per minute, find the mean radius of the buckets, and, neglecting the thickness of the blades, the radial width of bucket.

Ans.—Turning moment = 20 foot-pounds.

51. An inward flow reaction turbine works under a head of 25 feet, and uses 500 cubic feet of water per second. The head lost in flow at exit is

7 per cent. of the head, and the other hydraulic losses 15 per cent. The velocity of flow is constant, and the velocity of the wheel is two-thirds the velocity of whirl. The outer diameter is double the inner. Design the turbine, and find the horse-power.

Ans.— Horse-power = 111.
Diameter of wheel = 4·96 feet.
Width of wheel = 0·301.
Revolutions per minute = 78·5.

52. A Girard impulse turbine works under a head of 5·12 feet with a supply of 112 cubic feet per second. The angles α, β, γ (§ 130) are 33 degrees, 55 degrees, and 23 degrees respectively. Find the velocity of the wheel, the absolute velocity at exit, the efficiency and the horse-power. Find also the ratio of flow areas at inlet and outlet.

Ans.—Horse-power = 42·8.
Efficiency = 0·91.
Ratio of areas = 2·05.

53. An inward flow turbine is required to give 100 horse-power at 150 revolutions per minute on a fall of 25 feet. Eight per cent. of the fall is lost in discharge which is radial, and the vanes are radial at outlet. Neglecting frictional and all other resistances, and assuming a uniform flow through the turbine, find the size of the wheel, the angle of the vanes and the guide blades, and the pressure and kinetic heads in the supply chamber. The outer radius is twice the inner radius, and the depth of the wheel at the outer periphery is a quarter the outer radius. The eye of the wheel is just large enough to take the discharge, neglecting the shaft.

54. An inward flow reaction turbine of 24 inches diameter is supplied with 500 cubic feet of water per minute through an area which measured normally to the radius in 1¼ square feet. The water is directed into the wheel by guide blades which are inclined at an angle of 100° to the radii, and the vanes at inlet are radial. The area of discharge measured normally, is also 1¼ square feet, and the water after leaving the vanes at a radius of 6 inches has an absolute velocity in a direction inclined backwards at 45° to the motion of the wheel. Estimate the turning moment on the wheel, and the work done per pound, and assuming the discharge takes place into atmosphere, find the efficiency, the pressure, and kinetic head.

Ans.—Efficiency = 0·962; $\frac{v_2^2}{2g} = 23$; $\frac{p_2}{\sigma} = 26{\cdot}98$.

55. The distribution of head in an inward flow turbine working under a head of 25 feet, and using 50 cubic feet of water per second, is—

Velocity head in supply chamber = 0·66*h*.
Frictional head = 0·15.
Flow head = 0·07.

The whirling velocity at exit is zero, and the crowns of the wheel are shaped so that the flow is constant. The outer diameter is double the inner. Neglecting the thickness of the vanes, find the proper forms for crowns, vanes, guide-angles, speed of wheel, and available horse-power.

If the diameter of the spindle be 4 inches, the annular area between the spindle and discharging edge of wheel equal to flow area, find the diameter and width of wheel at receiving and discharging edge, and also the revolutions per minute.

56. In a compound four-wheel turbine, the results were—

Pressure gauge at entrance	39·85	38
Pressure gauge at discharge in inches of mercury .	2·36	2·7
Revolutions per minute	2072	1644
Turning moment in foot-pounds	1·3	2·2
Water per minute in pounds	328	366·4

Find the two efficiencies.

Ans.—0·59; 0·73.

CHAPTER VI

57. In a centrifugal pump, the diameter of the wheel is 3 feet 6 inches, breadth of outer periphery 3 inches, vane angle $\cot^{-1} 3$. If 4000 gallons per minute are delivered at a height of 4 feet when running at 120 revolutions, find the efficiency.

58. A centrifugal pump of 4 feet diameter running at 200 revolutions pumps 5000 tons of water from a dock in 45 minutes, the mean lift being 20 feet. The area through the wheel periphery is 1200 square inches, and the angle the vanes make is 26°. Determine the horse-power and the efficiency. Find also the lowest speed to start pumping, against a head of 20 feet, the outer diameter being twice the inner.

Ans.—$\eta = 0·594$. N = 198.

59. An experiment on a four-stage centrifugal pump gave

Head in rising main	= 149·7
Head in suction	= −1·8
Water per minute	= 1329
Turning moment	= 36·7
Revolutions per minute	= 1502

Calculate the useful work, the work put in, and the efficiency.

60. The sectional area of flow from the wheel of a centrifugal pump, normal to the radius, is one square foot, the diameter of the wheel being 2 feet 6 inches. The vanes at the outer periphery are inclined backwards at an angle of

25 degrees with the periphery. Suppose the pump to lift 12 tons of water per minute to a height of 10 feet, when running at 250 revolutions per minute, determine the horse-power required to drive the wheel, and its efficiency.

61. Find the dimensions, and the speed of rotation of the wheel of a centrifugal pump which is required to lift 200 tons of water per minute 5 feet high, the efficiency being 0·6. The velocity of flow through the wheel being 6 per cent. of the lift, and the vanes are curved backwards so that the angle between their directions and a tangent to the circumference of the wheel is 20. The velocity of flow in the eye of the wheel may be taken as 6 per cent. of the lift.

62. A centrifugal pump delivers 1500 gallons per hour with a lift of 25 feet, has an outer diameter of 16 inches, and the vane is inclined backwards at an angle of 30 degrees. All the kinetic head at discharge is lost, and is equivalent to 50 per cent. of the actual lift. Find the revolutions per minute and breadth at outlet, the velocity of whirl being half the velocity of the wheel. Find the efficiency.

Ans.—Revolutions, 705 per minute.
Breadth at outlet, 0·81 inch.

63. Find the speed of rotation of the wheel of a centrifugal pump which is required to lift 200 tons of water per minute 5 feet high, having given that the efficiency 0·6, the velocity of flow through the wheel 4·5 feet per second, and that the vanes are curved backwards so that the angle between their directions, and a tangent to the circumference of the wheel, is 20 degrees.

Ans.—Velocity of wheel = 23·68 feet per second.

64. A centrifugal pump is required to lift 1000 gallons of water per minute to a height of 40 feet. The pump is driven direct from a motor making 800 revolutions per minute. Assuming the total losses in the suction and delivery pipes, wheel casing, and vortex chamber to be 50 per cent. of the velocity of the water leaving the wheel, and the velocity of flow through the wheel to be 5 feet per second, estimate (1) the diameter of the wheel (2) the breadth of the wheel at discharge, and (3) the hydraulic efficiency, when the vanes at outlet are recurved backwards at an angle of 30 degrees to the tangent.

65. Assuming the ordinary laws of friction between a flow and a surface, and neglecting any drag on the water due to friction; show that the work lost per second is $f\frac{2\pi}{a}a^2v^3$, when a disc of radius a rotated in water with a circumferential velocity v.

66. If, in the last question, the disc is surrounded by a free vortex of double its diameter; show that the loss of friction of the vortex on the flat sides of the vortex chamber is $2\frac{1}{2}$ times that by disc friction.

INDEX

A

B

C

D

E

F

G

H

I

J

L

M

N

O

P

T

U

V

W

END OF VOL. I.

PRINTED BY
WILLIAM CLOWES AND SONS, LIMITED,
LONDON AND BECCLES.

cP. 45
6

APR 5 1956